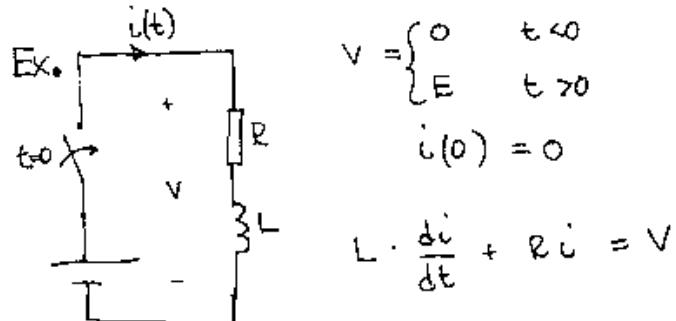


transient = icke-stationär  $\approx$  övergående  
stationär = fortvarig  $\neq$  konstant!



uttryck för erhålls om vi lösen en 1:a ordn. difflik  
för  $t > 0$ :

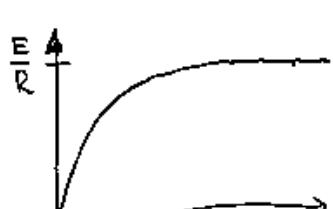
$$L \frac{di}{dt} + Ri = E \Rightarrow \frac{di}{dt} + \frac{R}{L} i = \frac{E}{L} \quad (\text{int. faktor } e^{\frac{R}{L}t})$$

$$\Rightarrow e^{\frac{R}{L}t} \frac{di}{dt} + e^{\frac{R}{L}t} \frac{R}{L} i = \frac{E}{L} e^{\frac{R}{L}t} \Leftrightarrow \frac{d}{dt} (e^{\frac{R}{L}t} i) = \frac{E}{L} e^{\frac{R}{L}t}$$

$$\Rightarrow e^{\frac{R}{L}t} i = \frac{E}{L} \cdot \frac{L}{R} e^{\frac{R}{L}t} + C \Rightarrow i(t) = \frac{E}{R} + C e^{-\frac{R}{L}t}$$

manvillkorset  $i(0) = 0$  ger  $C$ :

$$i(0) = \frac{E}{R} + C = 0 \Rightarrow C = -\frac{E}{R} \Rightarrow i(t) = \frac{E}{R} \left(1 - e^{-\frac{R}{L}t}\right)$$



$\Rightarrow$  stationär del:  $\frac{E}{R}$

transient del:  $-\frac{E}{R} e^{-\frac{R}{L}t}$

- DC eller jω-räkning ger bara den stationära delen!

Laplacetransform

- Jobbigt att lösa diff. ek för hand  
⇒ Laplace-metoden
- går igenom i Tillämpad Matematik M2
- Alla funktioner i tidsplanet  $f(t)$  def. för  $t > 0$   
kan beskrivas med dess Laplacetransform

$$F(s) \hat{=} \mathcal{L}[f(t)] = \int_0^{\infty} f(t) e^{-st} dt$$

funktion i s-plane:  $s = \sigma + j\omega$

Invers transformen:

$$f(t) \hat{=} \mathcal{L}^{-1}[F(s)] = \frac{1}{2\pi j} \int_{\sigma-j\omega}^{\sigma+j\omega} F(s) e^{st} dt$$

- Laplacetransform finns i guida-häftet + Tefyra

Ex.

$$f(t) = t \quad \xrightarrow{\text{nr. 13}} \quad F(s) = \frac{1}{s^2}$$

$$f(t) = \frac{E}{R} (1 - e^{-\frac{R}{L}t})$$

$$= \frac{E}{L} \cdot \frac{1}{1 - e^{-\frac{R}{L}t}} \quad \begin{matrix} \xrightarrow{\text{nr. 2, 3}} \\ \left( \begin{matrix} a=0 \\ b=0 \end{matrix} \right) \end{matrix} \quad F(s) = \frac{E}{L} \cdot \frac{1}{s(s + \frac{R}{L})}$$

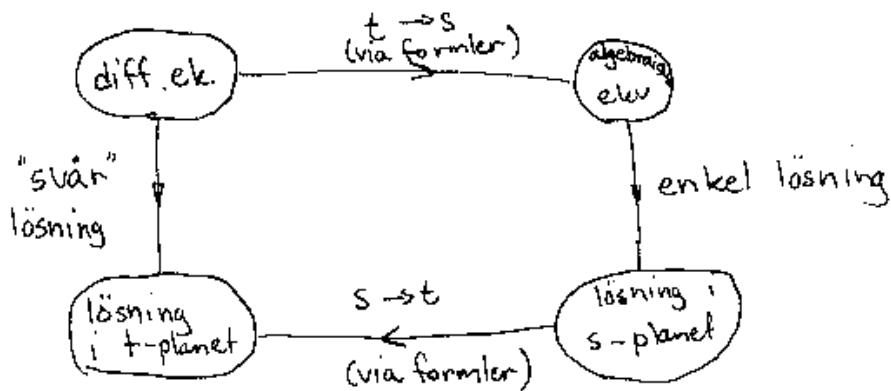
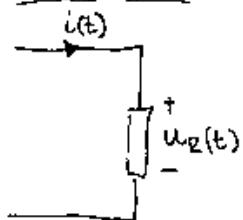
Inverstransform:

$$\begin{aligned} F(s) = \frac{2s+4}{s^2+4s+13} &= \frac{2s+4}{(s+2)^2+9} = 2 \cdot \frac{s+2}{(s+2)^2+3^2} \\ &\xrightarrow{\text{nr. 2, 3}} f(t) = 2e^{-2t} \cos 3t \end{aligned}$$

F6 ③

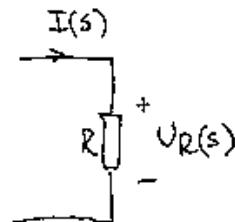
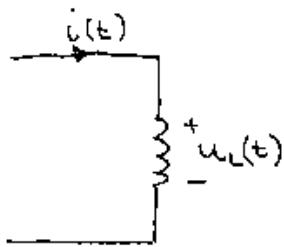
Laplace-transform o Elektriska Kretser

Analysmetoder (16.3, 16.7)

Kretselement i s-planetResistans

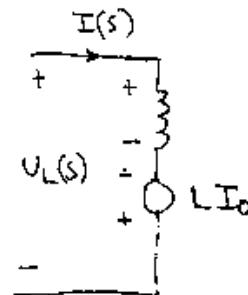
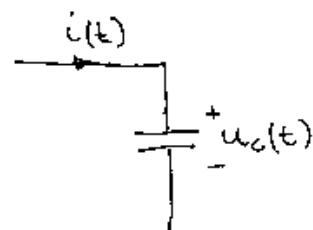
$$u_R(t) = R i(t)$$

$$U_R(s) = R I(s)$$

Induktans

$$u_L(t) = L \frac{di}{dt}$$

$$\begin{aligned} U_L(s) &= L(s I(s) - I_0) = \\ &= s L I(s) - L I_0 \end{aligned}$$

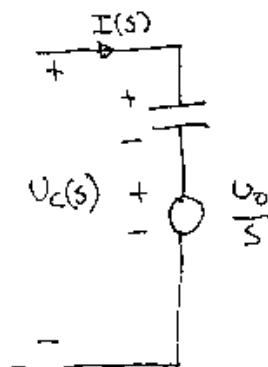
Kapacitans

$$u_C(t) = \frac{1}{C} \int_{-\infty}^t i(\tau) d\tau$$

$$= \frac{1}{C} q_0 + \frac{1}{C} \int_0^t i(\tau) d\tau$$

$$= V_0 + \frac{1}{C} \int_0^t i(\tau) d\tau$$

$$U_C(s) = \frac{V_0}{s} + \frac{1}{sC} I(s)$$



F6 ④

- OBS! Tryckfel i boken s. 389 - 390 ( $\Omega$ )

Alltså:

$$i(t) \rightarrow I(s)$$

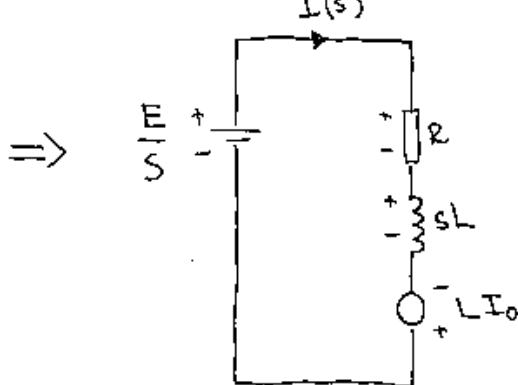
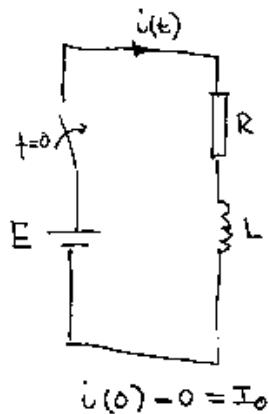
$$R \rightarrow R$$

$$u(t) \rightarrow V(s)$$

$L \rightarrow sL$  + sp. källa motsv.  
lagrad energi vid  $t=0$

$$C \rightarrow \frac{1}{sC} + \dots$$

Exempel



$$\text{KVL: } -\frac{E}{s} + RI(s) + sL I(s) - \underbrace{LI_0}_{=0} = 0$$

$$\Rightarrow I(s) = \frac{E}{s(R + sL)} = \frac{E}{L} \cdot \frac{1}{s(s + \frac{R}{L})}$$

Laplace tabel regel 17  $\Rightarrow$

$$i(t) = \frac{E}{L} \cdot \frac{1}{R/L} \cdot \left(1 - e^{-\frac{R}{L}t}\right) = \frac{E}{R} \left(1 - e^{-\frac{R}{L}t}\right)$$

OK!

## 16.7 CIRCUITS IN THE s-DOMAIN

In Chapter 8 we introduced and utilized the concept of generalized impedance, admittance, and transfer functions as functions of the complex frequency  $s$ . In this section, we extend the use of the complex frequency to transform an RLC circuit, containing sources and initial conditions, from the time domain to the  $s$ -domain.

Table 16-2

Time Domain	s-Domain	s-Domain Voltage Term
$i \rightarrow R$	$I(s) \rightarrow R$	$R I(s)$
$i \rightarrow L$ $\rightarrow i(0^+)$	$I(s) \rightarrow sL$ $L i(0^+)$	$sL I(s) + L i(0^+)$
$i \rightarrow L$ $\leftarrow i(0^+)$	$I(s) \rightarrow sL$ $L i(0^+)$	$sL I(s) + L i(0^+)$
$i \rightarrow C$ $+V_0$	$I(s) \rightarrow \frac{1}{sC}$ $\frac{V_0}{s}$	$\frac{I(s)}{sC} + \frac{V_0}{s}$
$i \rightarrow C$ $-V_0$	$I(s) \rightarrow \frac{1}{sC}$ $\frac{V_0}{s}$	$\frac{I(s)}{sC} - \frac{V_0}{s}$

S. 389

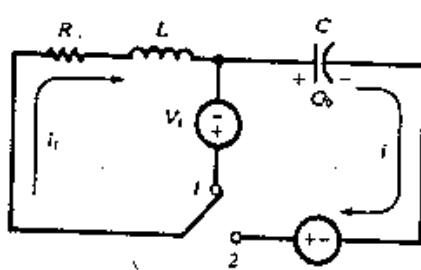
**EXAMPLE 16.5** In the circuit shown in Fig. 16-4(a) an initial current  $i_1$  is established while the switch is in position 1. At  $t = 0$ , it is moved to position 2, introducing both a capacitor with initial charge  $Q_0$  and a constant-voltage source  $V_2$ .

The  $s$ -domain circuit is shown in Fig. 16-4(b). The  $s$ -domain equation is

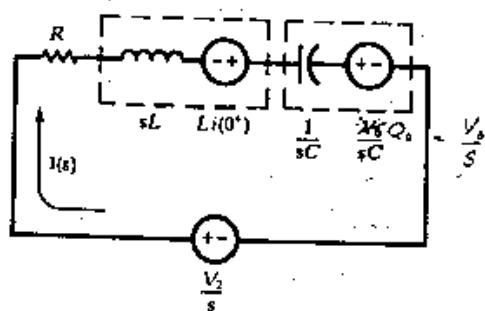
$$RI(s) + sLI(s) - Li(0^+) + \frac{I(s)}{sC} + \frac{V_0}{sC} - \frac{V_2}{s} = 0$$

in which  $V_0 = Q_0/C$  and  $i(0^+) = i_1 = V_1/R$ .

S. 390



(a)



(b)

Fig. 16-4

Initial och Slutvärdesteoremen (16.5)

- I bland besvärligt att inverstransformera

Initialvärdet fås genom:

$$f(0) \stackrel{\wedge}{=} \lim_{s \rightarrow \infty} s F(s) \quad (\text{Tefyra})$$

Slutvärdet fås genom

$$f(\infty) \stackrel{\wedge}{=} \lim_{s \rightarrow 0} s F(s)$$

$$\text{Ex. } I(s) = \frac{E}{L} \cdot \frac{1}{s(s + \frac{R}{L})}$$

$\downarrow I_0=0!$

$$V_L(s) = s L I(s) = \frac{E}{L} \frac{1}{s(s + \frac{R}{L})} \cdot s L$$

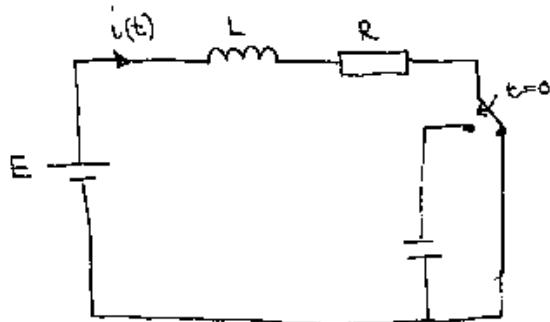
$$= \frac{E}{s + \frac{R}{L}}$$

Ex. Initialvärde

$$V_L(0) = \lim_{s \rightarrow \infty} s V_L(s) = \lim_{s \rightarrow \infty} \frac{s E}{s + \frac{R}{L}} = \lim_{s \rightarrow \infty} \frac{E}{1 + \frac{R}{sL}} = \underline{\underline{E}}$$

Slutvärde

$$i(\infty) = \lim_{s \rightarrow 0} s I(s) = \lim_{s \rightarrow 0} \frac{E}{L} \frac{s}{s(s + \frac{R}{L})} = \frac{1}{RL} = \underline{\underline{\frac{E}{R}}}$$

Ex 2 med initialladdning / ström

$$E = 200 \text{ V} \quad L = 0,2 \text{ H}$$

$$Q_0 = 0,15 \text{ mC} \quad C = 5 \mu\text{F}$$

$$R = 80 \Omega$$

• Beräkna  $i(t)$  för  $t > 0$

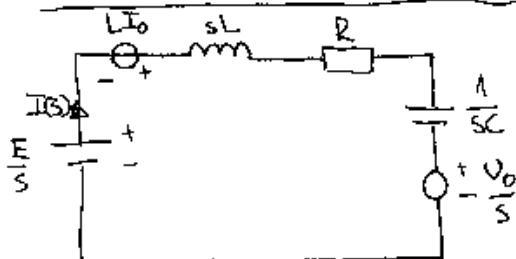
Initialström för induktans:

$$I_0 = \frac{E}{R} = I_0 = \frac{200}{80} = 2,5 \text{ A}$$

Initialspänning för kapacitans:

$$U_C(0) = U_0 = \frac{Q_0}{C} = \frac{0,15 \cdot 10^{-3}}{5 \cdot 10^{-6}} = 30 \text{ V}$$

I splanet för  $t > 0$ :



$$\text{KVL: } -\frac{E}{s} - L I_0 + sL I(s) + R I(s) + \frac{1}{sC} I + \frac{U_0}{s} = 0$$

$$I(s) = \frac{\frac{E}{s} + L I_0 - \frac{U_0}{s}}{sL + R + \frac{1}{sC}} = \frac{E + sL I_0 - U_0}{L(s^2 + \frac{R}{L}s + \frac{1}{LC})}$$

Inverstransformen ges av 17, 18 för reella rötter

Hill  $s^2 + \frac{R}{L}s + \frac{1}{LC}$ , av 30, 31 för komplexa  $\Rightarrow$

använd givna siffervärden!

F6 ④

$$I(s) = \frac{0,5 s + 170}{0,2 (s^2 + 400s + 10^4)} = \frac{2,5 s + 850}{(s+200)^2 + 980} = \\ = 2,5 \frac{s+200}{(s+200)^2 + 980^2} + \frac{350}{(s+200)^2 + 980^2}$$

$$\Rightarrow i(t) = 2,5 e^{-200t} \cos(980t) + \frac{350}{980} e^{-200t} \sin(980t)$$

Fouriertransform utgår:

Hela kap. 17

samt räknetal 17.2  
17.40  
17.41  
17.42