



### i. Diamagnetism

Diamagnetic metals have a weak, negative susceptibility to magnetic fields. In a diamagnetic material, there are no unpaired electrons and all the orbital shells are filled, hence no net magnetic moment. The intrinsic electron magnetic moments cannot produce any bulk effect, thus magnetization arises from electron orbital motion. Under the influence of an applied field,  $H$ , the spinning electrons precess and this motion, which is a type of electric current, produces a weak internal magnetisation,  $M (= \chi H = (\mu_r - 1) \times H = J / \mu_0)$ , that opposes the externally applied magnetic field, thus causing a repulsive effect. The cause of this interaction is in accordance with Lenz's Law by which small, localized currents generated in the material created magnetic fields in opposition to the applied changing field. All materials have a diamagnetic effect, which is often masked by a larger paramagnetic or ferromagnetic component. Although diamagnetism is generally a weak effect in most materials, superconductors exhibit a strong effect, thus repel magnetic fields from their bulk. The dimensionless volumetric susceptibility value  $\chi$  ( $\mu_r = \chi + 1$ ) is independent of temperature. Most elements in the periodic table, including copper, silver, gold, and helium are diamagnetic. The strongest diamagnetic elements are bismuth and carbon graphite, as highlighted in Table 27.1.

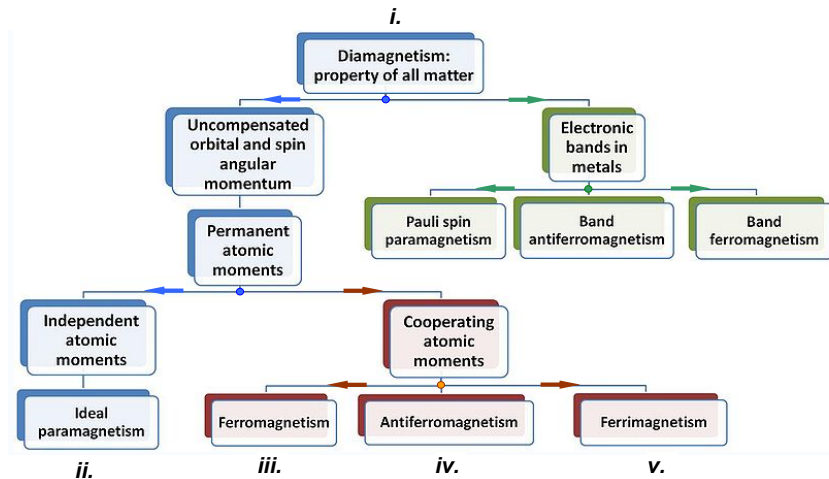


Figure 27.2. Types of natural magnetism.

### ii. Paramagnetism

Paramagnetic metals have a small, positive susceptibility to magnetic fields. They become magnetized in the same direction as the applied magnetic field and the magnetization magnitude is proportional to the applied magnetic field. In a paramagnetic material there are unpaired electrons, specifically, atomic or molecular orbitals with exactly one electron in them. While paired electrons are required by the Pauli exclusion principle to have their intrinsic (spin) magnetic moments pointing in opposite directions, causing their magnetic fields to cancel out, an unpaired electron is free to align its magnetic moment in any direction. When an external magnetic field,  $H$ , is applied, these magnetic moments tend to align in the same direction as the applied field, thus reinforcing it.

Paramagnetic materials are attracted to magnetic fields, hence have a relative magnetic permeability slightly greater than one (that is, a positive magnetic susceptibility,  $\chi > 0$ ). The force of attraction generated by the applied field is linear and weak. Unlike ferromagnets, paramagnets do not retain any magnetization in the absence of an externally applied magnetic field, because thermal motion causes the spins to return to a random orientation. Thus the total magnetization decreases to zero when the applied field is removed. Even in the presence of a  $H$  field, the induced magnetization is small because only a small fraction of the spins are oriented by the field. This small fraction is linearly proportional to the field strength,  $H$ .

The randomly oriented moments result from thermal agitation. A magnetic field slightly aligns these moments whence low magnetisation results, aligned with the applied field. As the temperature increases, the thermal agitation increases and it becomes harder to align the atomic magnetic moments, hence the magnetic susceptibility,  $\chi$ , decreases. This behaviour is known as the *Curie law*, as shown in equation (27.1), where  $C$  is a material-specific constant called the Curie constant.

$$M = H \times \chi = H \times \frac{C}{T} \quad (= H \times (\mu_r - 1)) \quad (27.1)$$

$M$  is the resultant magnetism (with reference to the permeability of a vacuum),  $T$  is absolute temperature, K, and  $H$  is the externally produced magnetic field, A/m.

This law indicates that the susceptibility  $\chi$  of paramagnetic materials is inversely proportional to temperature. Curie's law is only valid under low magnetisation conditions, since it does not consider magnetisation saturation that occurs when the atomic dipoles are all aligned in parallel. After complete alignment, increasing the external field will not increase the total magnetisation since there can be no further alignment. However, such saturation typically requires extremely strong magnetic fields.

The Curie law is a special case of the more general Curie-Weiss law, equation (27.2), which incorporates a temperature constant,  $T_C$ , and derives from Weiss theory, proposed for ferromagnetic materials, that incorporates the interaction between magnetic moments.

$$\chi = \frac{C}{T - T_C} \quad (27.2)$$

In equation (27.2),  $T_C$ , in degrees Kelvin, K, can be positive, negative or zero. When  $T_C = 0$ , the Curie-Weiss law equates to the Curie law. A non-zero  $T_C$  indicates that there is interaction between neighbouring magnetic moments and the material is only paramagnetic above a certain transition temperature. If  $T_C$  is positive then the material is ferromagnetic below the transition temperature and the value of  $T_C$  corresponds to the transition temperature, the Curie temperature,  $T_C$ . If  $T_C$  is negative then the material is antiferromagnetic below the transition temperature, the Néel temperature,  $T_N$ , however the value of  $T_C$  does not relate to  $T_N$ . Equation (27.2) is only valid for a paramagnetic material state and is not valid for many metals, as the electrons contributing to the magnetic moment are not localised. However, the law does apply to some metals, for example, the rare-earth elements shown in figure 27.1, where the 4f electrons, that create the magnetic moment, are closely bound.

Paramagnetic materials include magnesium, molybdenum, lithium, sodium, and tantalum. Paramagnetic materials like aluminium, uranium, and platinum become more magnetic when they are very cold.

### iii. Ferromagnetism

Ferromagnetic materials have a large positive susceptibility to an external magnetic field. Ferromagnetism is the term for the basic mechanism by which specific materials (such as iron) form permanent magnets and/or exhibit strong magnetic interaction with magnets.

A paramagnetic substance has unpaired electrons, but additionally ferromagnetism is only possible when atoms are arranged in a lattice and the atomic magnetic moments can interact to align parallel to each other. The effect is due to the presence of a molecular field within the ferromagnetic material, which is sufficient to magnetise the material to saturation. This creates a net internal magnetic field much greater than the applied field. Even when the applied external field is removed, the electrons in the material maintain each other orientated in the same direction.

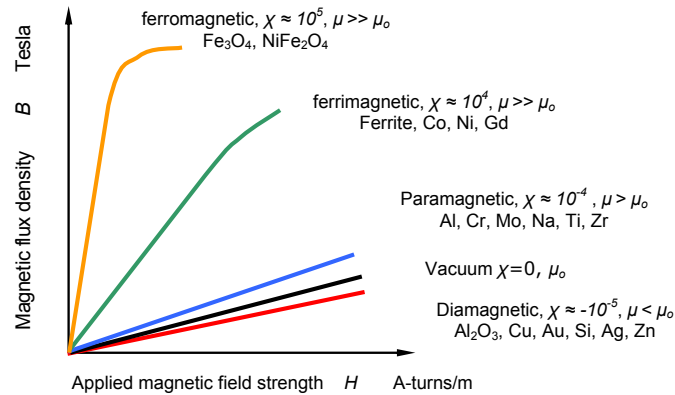
Magnetic domains within the material are the regions of atomic magnetic moments that are aligned. The movement of these domains determines how the material responds to a magnetic field and consequently the susceptibility is a function of the applied magnetic field. Therefore, ferromagnetic materials are usually compared in terms of saturation magnetisation (magnetisation when all domains are aligned) rather than susceptibility.

In the elemental periodic table, only the three consecutive elements Fe, Co and Ni are ferromagnetic at and above room temperature. As ferromagnetic materials are heated, thermal agitation of the atoms decreases the degree of alignment of the atomic magnetic moments, hence saturation magnetisation decreases. Eventually the thermally agitated disorder overwhelms the energy lowering due to ferromagnetic order and the material becomes paramagnetic; the temperature of this critical transition is the Curie temperature,  $T_C$  (Fe:  $T_C = 770^\circ\text{C}$ , Co:  $T_C = 1131^\circ\text{C}$ , and Ni:  $T_C = 358^\circ\text{C}$ ). Above  $T_C$ , susceptibility varies according to equation (27.2).

### iv. Antiferromagnetism

Antiferromagnetic materials are similar to ferromagnetic materials but the exchange interaction between valence electrons of neighbouring atoms leads to the anti-parallel alignment (as opposed to parallel alignment) of the atomic magnetic moments. Therefore the magnetic field cancels out and the material appears to behave like a paramagnetic material. It is difficult to magnetize such materials in the direction of the applied field but they still demonstrate a relative permeability slightly greater than 1. The magnetic susceptibility of an antiferromagnetic material is typically a maximum at the Néel temperature. Like ferromagnetic materials, these materials become paramagnetic above a transition temperature, the Néel temperature,  $T_N$ , where equation (27.2) is applicable. The Néel temperature is similar to the Curie temperature in ferromagnetism. In the periodic table, the only element exhibiting antiferromagnetism at near room temperature is chromium (Cr:  $T_N = 37^\circ\text{C}$ ).

Alloys such as iron manganese (FeMn), and oxides such as nickel oxide (NiO), manganese oxide, and iron oxide (FeO), exhibit antiferromagnetism.

Figure 27.3. Relative susceptibility  $\chi$  ( $= \mu_r - 1$ ) of different materials.

#### v. Ferrimagnetism

Ferrimagnetism is only observed in compounds that have more complex crystal structures than pure elements. Within such materials, the exchange interactions lead to parallel alignment of atoms in some of the crystal sites and anti-parallel alignment of others. The material devolves into magnetic domains, just like a ferromagnetic material and the magnetic behaviour is also similar, although ferrimagnetic materials usually have lower saturation magnetisations. In an optimal geometrical arrangement, there are more magnetic moments from the sublattices of electrons which point in one direction, than from the sublattices which point in the opposite direction. Both ferrimagnetic and ferromagnetic materials retain spontaneous magnetization below the Curie temperature, and show no magnetic order (are paramagnetic) above this temperature. However, there may be a temperature below the Curie temperature at which the two sub-lattices have equal (but opposite polarity) moments, resulting in a net magnetic moment of zero; this is called the *magnetization compensation point*. Examples of ferrimagnetic materials are iron oxide ( $\text{Fe}_3\text{O}_4$ ) and ferrite -  $\text{MOFe}_2\text{O}_3$ .

The different magnetic types are summarised in figure 27.3 in terms of relative susceptibility. There are other types of magnetism, such as spin glass, superparamagnetism, superdiamagnetism, and metamagnetism.

### 27.1 Magnetic properties

The intrinsic properties of a magnetic material are those properties that are characteristic of the material and are unaffected by the microstructure (for example, grain size or crystal orientation of grains). These properties include Curie temperature, saturation magnetisation, and magneto-crystalline anisotropy.

#### i. Saturation Magnetisation

Saturation magnetisation,  $M_s$ , is a measure of the maximum field that can be generated by a material. It depends on the strength of the dipole moments on the atoms that make up the material and how densely they are packed. The atomic dipole moment is affected by the nature of the atom and the overall electronic structure within the compound. The packing density of the atomic moments is determined by the crystal structure (that is, the spacing of the moments) and the presence of any non-magnetic elements within the structure.

For ferromagnetic materials,  $M_s$  also depends on how well the moments are aligned, as thermal vibration of the atoms causes moment misalignment and a reduction in  $M_s$ . For ferrimagnetic materials not all of the moments align parallel, even at zero degree Kelvin, hence  $M_s$  depends on the relative alignment of the moments as well as the temperature. In the case of a single magnetic domain, its saturation magnetisation is referred to as the spontaneous magnetisation.

Table 27.2 gives examples of the saturation polarisation,  $J_s$ , ( $=\mu_0 M_s$ ) and Curie temperature,  $T_C$ , of commonly used magnetic materials.

#### ii. Magnetic Anisotropy

In a crystalline magnetic material, magnetic properties vary depending on the crystallographic direction in which the magnetic dipoles are aligned.

A measure of *magnetocrystalline anisotropy* in the easy direction of magnetisation, is the anisotropy field,  $H_a$ , which is the field required to rotate all the moments by  $90^\circ$  in a saturated single crystal.

Temperature dependant anisotropy is caused by a coupling of the electron orbitals to the lattice, and in the easy direction of magnetisation, this coupling creates orbitals in the lowest energy state.

The easy direction of magnetisation for a permanent magnet, based on ferrite or the rare earth alloys, is uniaxial. However, it is also possible to have materials with multiple easy axes or where the easy direction can lie anywhere on a certain plane or cone surface. The fact that a permanent magnet has uniaxial anisotropy means that it is difficult to demagnetise since it is resistant to rotation from the magnetisation direction. When no preferred crystallographic direction exists within a material, shape anisotropy may arise if there are non-spherical particles present within the material. The long axis of such particles is the preferred axis of magnetization, as with Alnico magnets.

In addition to magnetocrystalline anisotropy, there is another effect related to spin-orbit coupling called stress anisotropy or *magnetostriction*. Magnetostriction (which is temperature dependant) arises from the strain dependence of the anisotropy constants. Upon magnetization, a previously demagnetized crystal experiences a strain that can be measured as a function of the applied field along the principal crystallographic axes. A magnetic material will therefore change its dimension when magnetized. The inverse affect or magnetization change with stress, also occurs. A uniaxial stress can produce a unique easy axis of magnetization if the stress is sufficient to overcome all other anisotropies.

The third type of anisotropy is due to the shape of a grain. A magnetized body will produce magnetic charges or poles at its surface. This surface charge distribution, acting in isolation, is itself a magnetic field source, and is a demagnetizing field, opposing the magnetization that produced it. Anisotropy is temperature dependent, decreasing and vanishing at the Curie temperature.

#### iii. Magnetoresistance

Magnetoresistance (MR) is the effect by which the electrical resistance of a magnetic material changes depending on the relative direction of the current and the magnetisation. In most cases the electrical resistance is highest when the current and magnetisation are parallel and lowest when they are perpendicular. The level of magnetoresistance shown by a material is usually expressed in terms of the percentage change in resistance from the highest to the lowest resistance and is usually of the order of a few percent. The main application for MR sensors is in the read heads of hard disk drives.

Table 27.2. The saturation polarisation,  $B_s$ , and Curie temperature,  $T_C$ , of a range of magnetic materials.

| Material                                  | Magnetic Structure | $J_s$ at 298K<br>$J_s = \mu_0 M_s$ | $T_C$<br>Curie temperature | reversible coefficient     |                           |
|---|--------------------|------------------------------------|----------------------------|----------------------------|---------------------------|
|   |                    | Tesla                              | °C                         | $\alpha$ ( $B_0$ )<br>%/°C | $\beta$ ( $H_a$ )<br>%/°C |
| Fe  | Ferro              | 2.15                               | 770                        |                            |                           |
| Co  | Ferro              | 1.80                               | 1121                       |                            |                           |
| Ni  | Ferro              | 0.62                               | 368                        |                            |                           |
| Alnico 5/8                                | Ferro              | 1.08/0.80                          | 900/860                    | -0.02                      | -0.03                     |
| $\text{Nd}_2\text{Fe}_{14}\text{B}$       | Ferro              | 1.59                               | 312                        | -0.11                      | -0.060                    |
| $\text{SmCo}_5$                           | Ferro              | 1.14                               | 720                        | -0.04                      | -0.3                      |
| $\text{Sm}_2\text{Co}_{17}$               | Ferro              | 1.25                               | 820                        | -0.03                      | -0.3                      |
| FeCrCo                                    | Ferro              | 1.35                               | 630                        | -0.03                      | -0.04                     |
| Fe, 3wt% Si                               | Ferro              | 2.00                               | 740                        |                            |                           |
| Fe, 35wt% Co                              | Ferro              | 2.43                               | 940                        |                            |                           |
| Fe, 78wt% Ni                              | Ferro              | 0.70                               | 580                        |                            |                           |
| Fe 50wt% Ni                               | Ferro              | 1.55                               | 500                        |                            |                           |
| $\text{BaO} \cdot 6\text{Fe}_2\text{O}_3$ | Ferri              | 0.48                               | 450                        | -0.19                      | 0.40                      |
| $\text{SrO} \cdot 6\text{Fe}_2\text{O}_3$ | Ferri              | 0.48                               | 450                        | -0.19                      | 0.40                      |
| $\text{MnO} \cdot \text{Fe}_2\text{O}_3$  | Ferri              | 0.51                               | 300                        | -0.19                      | 0.40                      |





Chemical composition:

| Wt %   | Ni        | Cu  | Cr  | S       | P      | Si    | Mn    | C      | Mo      | Fe      |
|--------|-----------|-----|-----|---------|--------|-------|-------|--------|---------|---------|
| Comp 1 | 79 - 80.6 |     |     | < 0.008 | < 0.02 | 0.42  | 0.95  | < 0.03 | 3.8 - 5 | balance |
| Comp 2 | 75 - 77   | 4-6 | > 3 | < 0.02  | < 0.02 | > 0.5 | > 1.8 | < 0.05 |         |         |

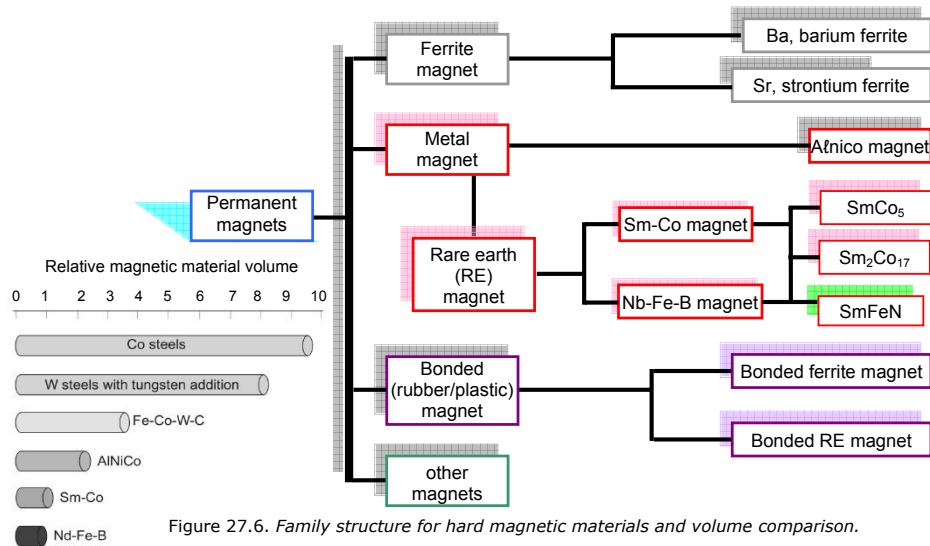


Figure 27.6. Family structure for hard magnetic materials and volume comparison.

## ii. Cobalt alloys

(a) *Alnico* hard magnet material: (alloys based on Al, Ni, and Co)

Chemical composition:

| Wt %   | Al       | Ni      | Co     | Cu    | Ti    | Nb    | Si      | Fe      |
|--------|----------|---------|--------|-------|-------|-------|---------|---------|
| AlNiCo | 6 - 13.5 | 12 - 28 | 0 - 42 | 2 - 6 | 0 - 9 | 0 - 3 | 0 - 0.8 | balance |

A hard magnet that incorporates aluminium, nickel, cobalt, ferrum, and titanium, is called an AlNiCo magnet. Typical weight% is: Fe-35, Co-35, Ni-15, Al-7, Cu-4, Ti-4.

This group of magnets offer far more magnetic hardness than magnetic steels. Their properties rely on the shape anisotropy improvement associated with the two-phase nanostructure comprised of ferromagnetic Fe-Co needles in a matrix of non-magnetic Al-Ni. Due to their high Curie temperature, ~850°C, they have more stable properties around room temperature than some other alloys. This material has the lowest temperature coefficient (-0.02% per degree centigrade) of all permanent magnets, thus producing a constant field over a wide temperature range (-270°C to +500°C).

Alnico is classified as either an isotropic or anisotropic hard magnetic material. Alnico 2, 3 and 4 are unoriented – magnetic properties are isotropic and equal in all directions. Grades 5 to 9 are anisotropic.

There are two manufacturing techniques for AlNiCo hard magnet, namely casting and powder metallurgy (sintering for smaller magnets). If the cobalt content is more than 15%, the introduction of an external magnetic field during heat treatment at 1260°C in a hydrogen (or inert gas or a vacuum) atmosphere creates anisotropy for the sintered magnet, allowing the magnetic property to be increased in the preferred direction. For column crystal or monocrystalline shaped material, a strong external magnetic field parallel to the column crystal axis during heat treatment (annealing), produces optimal properties for the cast (and sintered) hard magnetic material.

Disadvantages of the Alnico materials are relatively low coercivity and mechanical brittleness.

Anisotropic columnar Alnico 9, has an energy product of ~80kJm<sup>-3</sup>. However, although having a relative high  $B_r$ , its main disadvantage is low intrinsic coercivity ( $H_c \sim 50\text{kA/m}$ ) thus it must be made in the form of horseshoes or long thin cylinders/rods, which cannot be exposed to significant demagnetising fields.

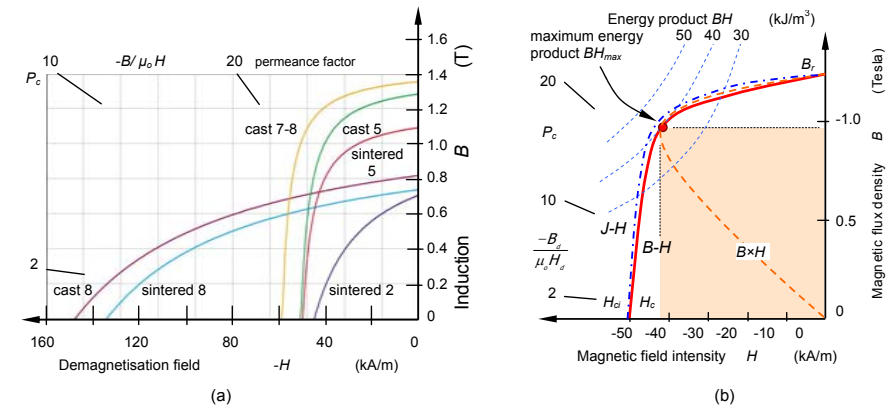


Figure 27.7. Second quadrant hysteresis loop for: (a) Alnico family and (b) Alnico 5.

## (b) CrFeCo hard magnet material

Chemical composition:

| Wt %   | Cr      | Co     | Si, Ti, Mo, Al, V | Fe      |
|--------|---------|--------|-------------------|---------|
| CrFeCo | 25 - 35 | 7 - 25 | 0.1 - 3           | balance |

CrFeCo can be classified into isotropic and anisotropic hard magnetic materials.

There are two production techniques for CrFeCo hard magnet alloy: casting and powder metallurgy. The material is ductile (good machinability) and can be easily hot rolled or cold rolled into strip or drawn into bar shapes, or, after punching, turning or drilling, can be made into the required shape and still maintain its magnetic properties. It has a low coercivity, and is relatively high in cost. Although mostly iron, which is inexpensive, there is significant cobalt and chrome content which is considerably more expensive. Typically strong, hard, tough, but brittle. It is hydrogen resistant.

[ $B_r=1.35\text{T}$ ,  $BH_{\text{max}}=52\text{kJ/m}^3$ ,  $H_c=49\text{kA/m}$ ,  $\alpha=-0.03\%/^\circ\text{C}$ ,  $\beta=-0.04\%/^\circ\text{C}$ ,  $T_c=640^\circ\text{C}$ ,  $T_{\text{op}}=500^\circ\text{C}$  and  $\mu_{\text{rc}}=2.5$ ]

## (c) FeCoVCr hard magnetic material

Chemical composition:

| Wt %    | Co      | V+Cr   | Fe      |
|---------|---------|--------|---------|
| FeCoVCr | 49 - 54 | 4 - 13 | balance |

FeCoVCr hard magnetic anisotropic alloy is produced using a casting process, and can be hot rolled or cool rolled into strip or drawn into wire shapes. Cool deformation (80% to 95%) and subsequent annealing heat treatment at 500 to 600°C is a necessary processing step to obtain the required anisotropic magnetic properties.

[ $B_r=0.85\text{T}$ ,  $BH_{\text{max}}=15\text{kJ/m}^3$ ,  $H_c=28\text{kA/m}$ ,  $\alpha=-0.01\%/^\circ\text{C}$ ,  $\beta=0\%/^\circ\text{C}$ ,  $T_c=720^\circ\text{C}$ ,  $T_{\text{op}}=500^\circ\text{C}$ , and  $\mu_{\text{rc}}=5$ ]

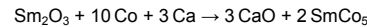
## iii. Samarium Cobalt alloys

Chemical composition, typically:

| Wt % | Sm | Co | Fe and Cu |
|------|----|----|-----------|
| SmCo | 35 | 60 | balance   |

This alloy group combines cobalt, iron (transition metals - TM) and a light rare earth (RE) element to exhibit permanent magnetic properties. They exhibit high-energy hard-magnetic behaviour, but are costly. These magnets have good thermal stability, thus are used where magnets are exposed to high temperatures (>150°C) or low temperatures (<100K). They are brittle and hard to machine.

The combination of a rare earth and a transition metal is ideal as the rare earth provides the anisotropy to the phase and the transition metal provides a high magnetisation and Curie temperature. By varying the percentages of the composition and changing the sintering and heat treatment cycles, two grades are produced, namely  $\text{SmCo}_5$  termed 1:5 and  $\text{Sm}_2\text{Co}_{17}$  termed 2:17. They differ in energy product, temperature coefficient, and magnetisation force required to saturate. In the case of  $\text{SmCo}_5$



The reaction takes place at 1100°C for 1 to 4 hours in a hydrogen (or inert gas) atmosphere or a vacuum. Usually excess Ca is added, typically 20 to 40%, to assure that the reaction goes to completion.  $\text{SmCo}_5$  sintered magnets have energy products of  $\sim 160\text{kJ/m}^3$ . These magnets have excess Sm that forms a smoothing grain boundary phase and coercivity is achieved by preventing the nucleation of reverse domains.

The maximum energy product is increased to  $240\text{kJ/m}^3$ , with a  $\text{Sm}_2\text{Co}_{17}$  based alloy. The theoretical limits are  $230\text{kJ/m}^3$  and  $270\text{kJ/m}^3$  for  $\text{SmCo}_5$  and  $\text{Sm}_2\text{Co}_{17}$ , respectively. These materials are based on the composition  $\text{Sm}_2(\text{Co},\text{Fe},\text{Cu},\text{Zr})_{17}$  and achieve permanent magnetic properties by control of the microstructure. The 2:17 magnets are produced by powder metallurgy and are solution treated at  $\sim 1100^\circ\text{C}$ , where they are single phase, then heat treated at  $500^\circ\text{C}$  to  $800^\circ\text{C}$ . This homogenising stage is followed by aging treatments at lower temperature where a cellular microstructure is formed. The cells are based on the  $\text{Sm}_2\text{Co}_{17}$  type phase, which is enriched in Fe and the cell boundaries comprise a layer of  $\text{SmCo}_5$  type phase, which is enriched in Cu. The intrinsic magnetic properties of the cells and their boundaries vary such that the magnetic domain wall energy is greatly reduced within the cell boundary and hence pin the domain walls, leading to permanent magnetic properties. They can operate at high temperature,  $\sim 500^\circ\text{C}$ , when small quantities of zirconium, hafnium, etc. are added to give better heat treatment response.

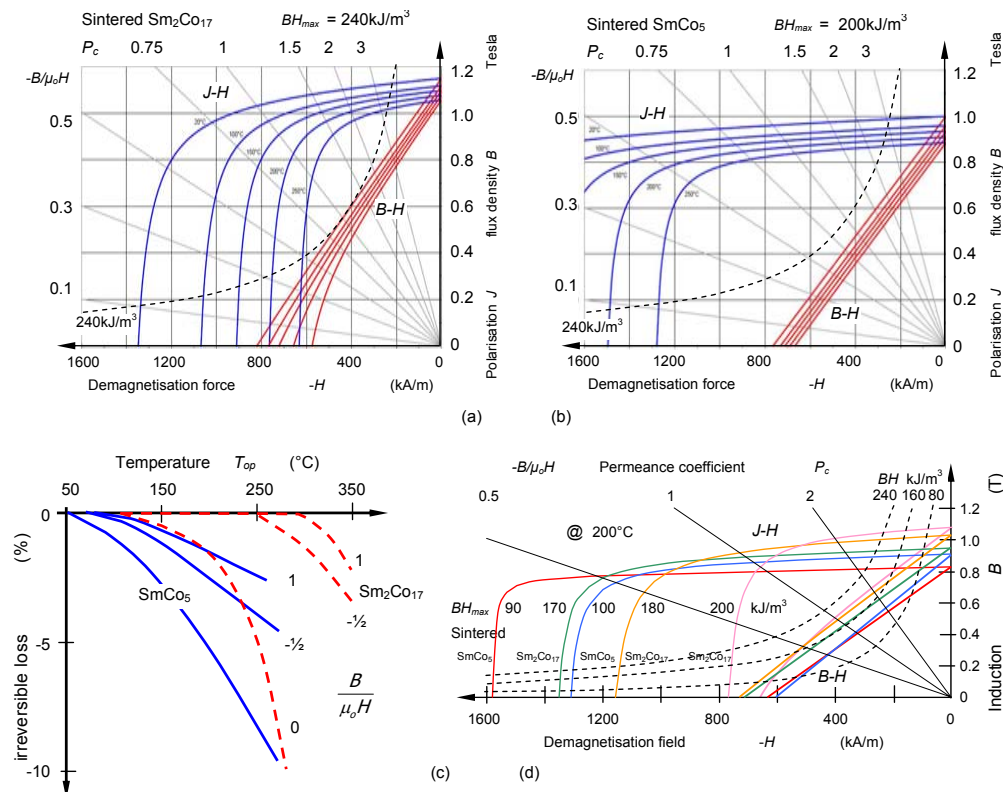


Figure 27.8. Second quadrant hysteresis loop for samarium cobalt alloys: (a)  $\text{Sm}_2\text{Co}_{17}$ , (b)  $\text{SmCo}_5$ , (c) characteristics at  $200^\circ\text{C}$  (with constant energy contours shown), and (d) temperature dependant irreversible losses as a function load line slope.

Both grades exhibit excellent energy, temperature, and corrosion resistant properties, applicable when a compact high energy field is required at temperatures of over  $100^\circ\text{C}$  or in hostile environments. These alloys have the best reversible temperature coefficient of all rare earth alloys, typically  $-0.03\%/^\circ\text{C}$ , as shown in Table 27.2.  $\text{SmCo}$  requires large magnetizing fields (in excess of  $28\text{MA/m}$ ) to saturate and under operational conditions, are difficult to de-magnetize.

A problem with  $\text{Sm/Co}$  based magnets is the expense of the raw materials. Samarium is much less abundant than other light rare-earth elements, such as La, Ce, Pr, and Nd, which account for over 90% of rare-earth metals in typical rare-earth ores.

Samarium-cobalt magnets are manufactured by similar routes as ferrites, being formed into either a fully dense sintered magnet, or a compression - or injection-moulded bonded magnet. Because both samarium and cobalt are relatively expensive elements, anisotropic magnets are usually produced with optimized properties along a pre-determined axis.

Rare-earth magnets have a rather more complicated domain wall mechanism than that for pure magnetocrystalline anisotropy, such that the best magnetic properties are only achieved with the powder milled to a grain size which is an order of magnitude larger than the single domain size. This means that, not only can domain walls exist, but also they can move relatively freely within a grain. While this allows saturation magnetization to be achieved with only a modest applied field, a high intrinsic coercivity will depend upon the grains' ability to resist the formation of a reverse domain when a demagnetizing field is applied. This property is therefore controlled by the grain boundaries, which are composed of deviations from the primary composition that provides a strong pinning of the domain walls at these sites. This mechanism, known as *nucleation*, where a grain undergoing nucleation, occurs in  $\text{SmCo}_5$  magnets.

$\text{Sm}_2\text{Co}_{17}$  differs from  $\text{SmCo}_5$  in that its grains contain a fine cell structure. Heat treatment of this compound promotes the formation of  $\text{Sm}_2\text{Co}_{17}$  cells, separated by thin walls of  $\text{SmCo}_5$  which can provide pinning of the domain walls (rather than the grain boundaries). Pinning, rather than nucleation, is therefore the controlling mechanism in  $\text{Sm}_2\text{Co}_{17}$  magnets, and this requires that a much greater field be applied to initially magnetize it into saturation.

Whether it is a nucleation-type  $\text{SmCo}_5$  or a pinning-type  $\text{Sm}_2\text{Co}_{17}$  magnet, that is, whether the domain walls are pinned at the grain or cell boundaries, they will move quite freely once these pinning forces are overcome, and  $M_{\text{sat}}$  will abruptly flip over into the opposite direction when an applied field of  $-H_{\text{ci}}$  is reached. This effect is seen in the shape of the demagnetization curves for samarium-cobalt type magnets, figure 27.8. The change in  $H_{\text{ci}}$  with temperature is seen in figure 27.8a for  $\text{Sm}_2\text{Co}_{17}$  and largely for  $\text{SmCo}_5$  in figure 27.8b.

The theoretical  $\text{Sm}_2\text{Co}_{17}$  limit is  $270\text{kJ/m}^3$ , N34, with the single phase  $\text{SmCo}_5$  limit is  $230\text{kJ/m}^3$ , N28.8.

#### iv. Samarium Iron Nitride alloys

The rare earth alloy  $\text{Sm}_2\text{Fe}_{17}\text{N}_3$  incorporates Fe rather than the more costly transition metal Co. While its magnetic properties can theoretically exceed those of  $\text{Nd}_2\text{Fe}_{14}\text{B}$ , its processing is more complex. Only bonded magnets (extrusion-flexible and injection-rigid) are made from  $\text{Sm}_2\text{Fe}_{17}\text{N}_3$ , (1 to  $2\mu\text{m}$  powder) which are more easily made in isotropic form than anisotropic. Such permanent magnet alloys offer high resistance to demagnetisation, high magnetisation, lightweight, and increased resistance to corrosion and temperature, compared with neodymium iron boron. They oxidize readily, have high irreversible loss at high temperature and are expensive.

[Extrusion moulded, flexible:  $B_r = 0.78\text{T}$ ,  $BH_{\text{max}} = 111\text{kJ/m}^3$ ,  $H_{\text{ci}} = 812\text{kA/m}$ ,  $H_c = 520\text{kA/m}$ ,  $\alpha = -0.05\%/^\circ\text{C}$ ,  $\beta = -0.45\%/^\circ\text{C}$ ,  $T_c = 498^\circ\text{C}$ ,  $T_{\text{op}} = 80^\circ\text{C}$ , and  $\mu_{\text{rc}} = 1.15$ ;

Injected moulded, rigid:  $B_r = 0.81\text{T}$ ,  $BH_{\text{max}} = 115\text{kJ/m}^3$ ,  $H_{\text{ci}} = 756\text{kA/m}$ ,  $H_c = 533\text{kA/m}$ ,  $\alpha = -0.05\%/^\circ\text{C}$ ,  $\beta = -0.45\%/^\circ\text{C}$ ,  $T_c = 498^\circ\text{C}$ ,  $T_{\text{op}} = 110^\circ\text{C}$ , and  $\mu_{\text{rc}} = 1.15$ ]

Explosion sintered  $\text{Sm}_2\text{Fe}_{17}\text{N}_3$  yields a squarer  $B \times H$  area, which is further improved, Curie temperature wise, with the addition of Ta. [ $B_r = 0.83\text{T}$ ]

#### v. Neodymium Iron Boron alloys

Chemical composition:

| Wt %  | Nd      | Fe          | B         | Al        | Nb      | Dy        |
|-------|---------|-------------|-----------|-----------|---------|-----------|
| NdFeB | 29 - 32 | 64.2 - 68.5 | 1.0 - 1.2 | 0.2 - 0.4 | 0.5 - 1 | 0.8 - 1.2 |

NdFeB combines a high saturation magnetisation with good resistance to demagnetisation. The high cost of samarium and the price volatility of cobalt have lead to NdFeB magnets becoming the preferred material for applications requiring high-energy magnets. Despite the high energy-product, these magnets have a relatively low Curie temperature, typically  $312^\circ\text{C}$ , which prohibits their use in high temperature applications. Additions of Co/Tb and Dy improve temperature and coercivity characteristics but decrease the saturation polarisation and increases production costs. Co improves corrosion resistance. NdFeB is easier to machine than Alnico and SmCo magnets. NdFeB are materials based on the magnetic phase  $\text{Nd}_2\text{Fe}_{14}\text{B}$ , with two different powder metallurgy processing routes being employed.

**Process route #1 Sintering (orient-press-sinter)**

Powder NdFeB based sintered permanent magnets produce a maximum energy product of  $\sim 451 \text{ kJm}^{-3}$ , by accurate heat treatment, controlled processing, and with the use of iron rich compositions. Sintered NdFeB based magnets achieve their coercivity by virtue of an Nd-rich phase at the grain boundaries, which acts to produce liquid phase sintering, smooth the boundaries, hence prevent nucleation of reverse magnetic domains.

The processing sequence for sintered NdFeB based magnets is shown in figure 27.9. The as-cast (untreated) ingot must first be broken into a powder. This is achieved most conveniently by exposing the ingot to hydrogen, which is absorbed at the surface. The hydrogen enters the material in the spaces between the atoms and causes the material to expand. The differential expansion generates stress in the ingot and the alloy breaks down into a fine powder, with an average grain size of  $100 \mu\text{m}$ . This process is known as *Hydrogen Decrepitation*, HD. The HD powder is then broken up further by a jet milling stage, which reduces the grain size to less than  $5 \mu\text{m}$  – the size of a few domains and therefore inherently anisotropic. When the alloy is in a powdered form, it is flammable, thus is handled under an inert gas.

Each particle of the broken down powder is a single crystal, 3 to  $5 \mu\text{m}$ , which can be aligned in a magnetic field; a few 10ms alternating pulses at  $6.4 \text{ MA/m}$ . This alignment is held in place by pressing the powder into a green (unsintered) compact, which is about 60% dense. The compact is then heated in a vacuum (or inert gas) to  $\sim 1060^\circ\text{C}$  for 4 hours. During the heating stage, the hydrogen is driven out of the material and is pumped away. Sintering occurs and the compact densifies, with the assistance of a liquid formed by the melting of the Nd-rich phase. After sintering, then quenching, the magnets are heat-treated, annealed, at  $\sim 550^\circ\text{C}$  to  $800^\circ\text{C}$  for 1 hour, thereby achieving optimum magnetic properties.

The magnet is then machined to the dimensions for the intended application. Due to the large degree of shrinkage that occurs during sintering, which is greater in the direction of alignment, it is not possible to press compacts that will shrink to the exact size required. Machining is an expensive operation and, particularly for small magnets, a large proportion of the material may need to be machined away.

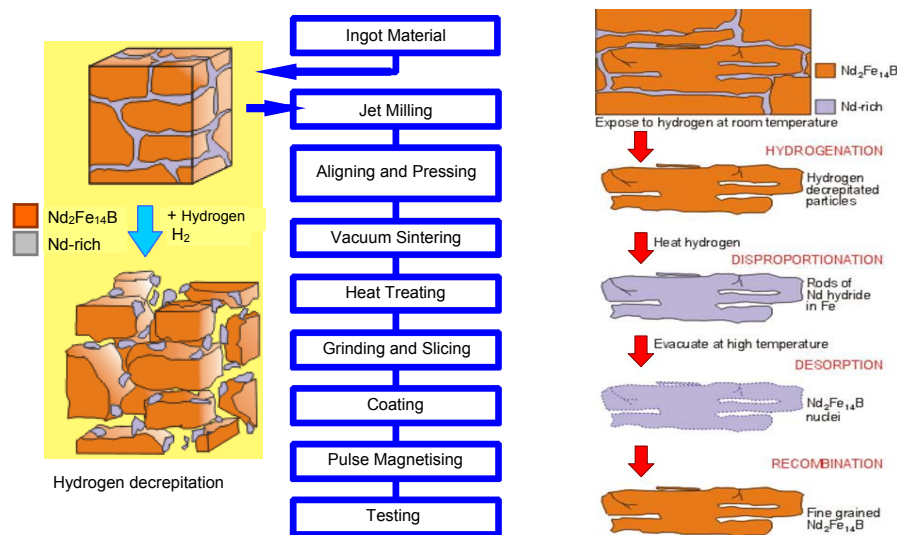
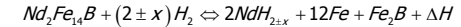


Figure 27.9. The processing route for sintered NdFeB permanent magnets, left HD and right HDDR.

Due to the highly reactive nature of the Nd-rich phase, the magnets tend to corrode rapidly, particularly in moist environments. Therefore, the next processing stage is to provide a protective barrier on the magnet surface, usually a nickel (plus copper) coating, although aluminium, zinc, and epoxy resins are also used. Finally, the magnet material is magnetised and tested.

An extension to the HD process is the *hydrogenation disproportionation desorption and recombination* (HDDR) process. Unlike the more straightforward HD-process, the HDDR route involves heating the bulk alloy in 1 bar of hydrogen to  $\sim 800^\circ\text{C}$ . The first stage is the decrepitation of the alloy due to the initial

absorption of the hydrogen, as described previously. At  $\sim 700^\circ\text{C}$ , the  $\text{Nd}_2\text{Fe}_{14}\text{B}$  matrix phase disproportionates during an exothermic and reversible reaction involving hydrogen absorption



where  $x$  depends on temperature and pressure. Then desorption results in isolated  $0.3 \mu\text{m}$  grains, which exhibit appreciable coercivity. Fully dense isotropic magnets are produced by hot pressing the powder mixed with thermosetting resins in a compression mould, at  $750^\circ\text{C}$  in an inert atmosphere.

In summary, neodymium-iron-boron magnets may be made from alloy powder, which is:

- sintered, nucleation-type;
- rapidly quenched, magnetocrystalline; then
- HDDR, magnetocrystallination.

**Processing route #2 Melt Spinning**

Melt-spinning is used to produce a ribbon like powdered material. Molten alloy is ejected onto the surface of a rotating water-cooled wheel, and cooling rates (rapid quenching) of the order of one million  $^\circ\text{C/s}$  are achieved. The microstructure and magnetic properties of the NdFeB ribbons are sensitive to the quench rate, that is, the speed of the rotating wheel. High quench rates produce essentially amorphous ribbons (that is, no crystal grains) having negligible intrinsic coercivity. Optimum quench rates yield ribbons with high coercivities; they are comprised of roughly spherical  $\text{Nd}_2\text{Fe}_{14}\text{B}$  grains (20 to  $100 \text{ nm}$  in diameter), which are single domain particles thus have a high coercivity,  $\sim 1 \text{ MA/m}^{-1}$ . At wheel velocities below optimum, the slow cooling rate produces ribbons with larger grains and are characterised by low coercivities. This powder cannot be sintered to produce fully dense magnets without destroying the magnetic properties, but is utilised in one of three ways:

**MQ-I**

The melt spun ribbon (typically milled into thin platelets  $200 \mu\text{m}$  wide and  $35 \mu\text{m}$  thick) is blended with a resin to produce a bonded permanent magnet. The crystals of MQ-I material are randomly oriented so that the magnets are isotropic and can be magnetised along any axis to a  $BH_{\text{max}}$  of  $\sim 80 \text{ kJm}^{-3}$ .

**MQ-II**

Improved densification of melt-spun ribbons can be achieved by hot pressing at  $\sim 750^\circ\text{C}$ , without adversely affecting the coercivity of the powder. These MQ-II type magnets exhibit a slight degree of magnetic alignment,  $\sim 10\%$ , and are 100% dense, that is, the magnetic properties are not diluted by a non-magnetic material, such as a resin. This gives MQ-II a higher  $BH_{\text{max}}$ ,  $100 - 120 \text{ kJm}^{-3}$ , than MQ-I.

**MQ-III /sintered**

Substantially greater alignment ( $>75\%$ ), and hence greater maximum energy products, can be obtained by heating the MQ-II material to  $\sim 750^\circ\text{C}$  in a die cavity having a larger diameter and then it is slowly deformed. This second hot press, termed die upset forging, produces plastic flow and a reorientation of the crystals. Such magnets, known as MQ-III, are 100% dense, and because of the alignment of crystals, have maximum energy products of  $\sim 400 \text{ kJm}^{-3}$ .

NdFeB magnets made by rapid quenching, as opposed to sintering, require a much greater applied field to initially align the grains' magnetizations and magnetize this material to its saturation level. Neodymium-iron-boron magnets therefore exhibit similarly shaped demagnetization characteristics to the other classes described, with a well-defined *knee* (Cunic point) at which  $M_{\text{sat}}$  reverses as an applied field of  $-H_{\text{ci}}$  is approached. As an example, figure 27.10b shows the demagnetization characteristics measured at different temperatures for one grade of fully dense anisotropic  $\text{Nd}_2\text{Fe}_{14}\text{B}$ . In all cases the higher the energy density, the lower the Curie temperature.  $\text{Nd}_2\text{Fe}_{14}\text{B}$  with an energy density of  $451 \text{ kJm}^{-3}$  must be operated at a temperature below  $50^\circ\text{C}$ . The theoretical maximum is  $512 \text{ kJm}^{-3}$ , N64.

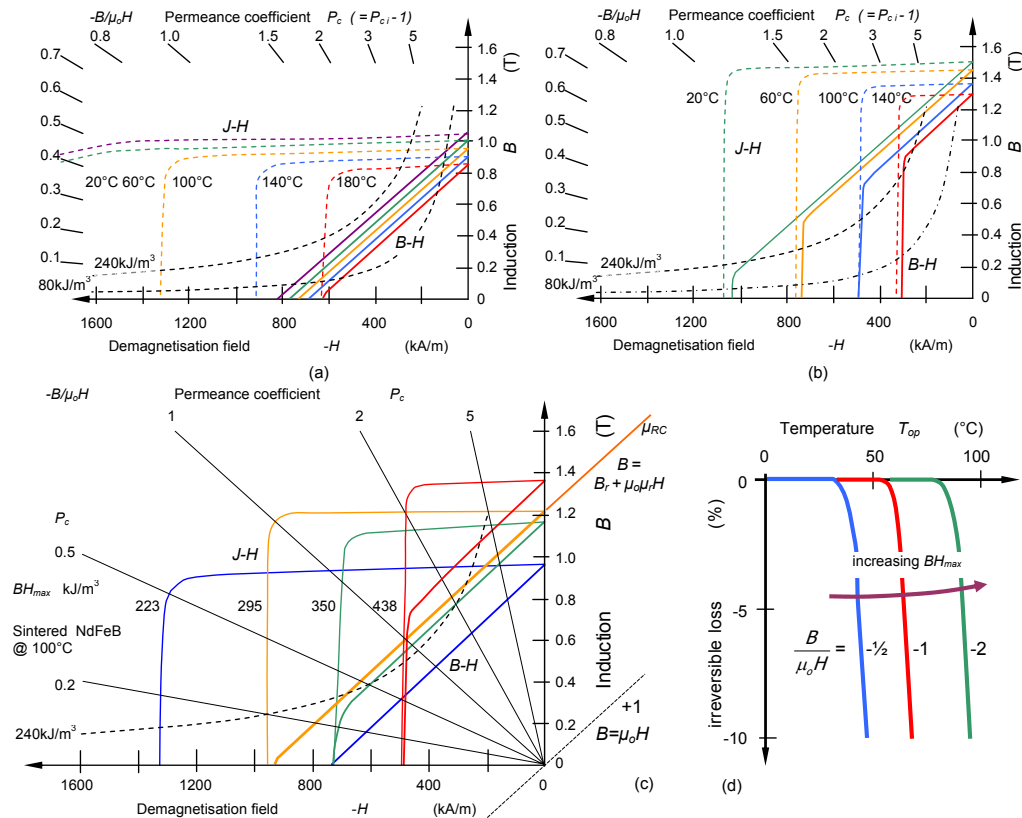


Figure 27.10. Second quadrant hysteresis loop for sintered neodymium-iron-boron magnets: (a) N28, (b) N55, (c) characteristics at 100°C (with a constant energy contour shown), and (d) temperature dependant irreversible losses as a function load line, N55.

Table 27.3 NdFeB magnetic characteristics - sintering results in higher  $B_r$  than MQ processes

| NdFeB Grade                   | symbol           | Melt spin property (Magnequench) |                       |                       | sintered    |             | units                     |
|-------------------------------|------------------|----------------------------------|-----------------------|-----------------------|-------------|-------------|---------------------------|
|                               |                  | MQI                              | MQII                  | MQIII                 | N28         | N55         |                           |
| Maximum energy product        | $BH_{max}$       | 68 – 88/<br>9 – 11               | 111 – 119/<br>15 – 15 | 255 – 335/<br>32 – 42 | 223 /<br>28 | 438 /<br>55 | $\text{kJ/m}^3$ /<br>MGOe |
| Residual induction            | $B_r$            | 0.61 – 0.71                      | 0.8 – 0.83            | 1.16 – 1.31           | 1.06        | 1.50        | T                         |
| Coercive force                | $H_c$            | 412 – 446                        | 557 – 573             | 875 – 979             | 835         | 1082        | kA/m                      |
| Intrinsic coercive force      | $H_{ci}$         | 716 – 1353                       | 1393 –<br>>1433       | 1173 –<br>>1593       | 2705        | 1074        | kA/m                      |
| Rev. Temp. Coefficient        | $\alpha - B_r$   | -0.105                           | -0.10                 | -0.09                 | -0.11       | -0.11       | %/°C                      |
| Rev. Temp. Coefficient        | $\beta - H_{ci}$ |                                  |                       |                       | -0.55       | -0.65       | %/°C                      |
| Magnetising filed             | $H_s$            | 1990 – 2785                      | 3581                  | 2785 – 3581           |             |             | kA/m                      |
| Recoil relative permeability  | $\mu_{rc}$       | 1.15 – 2.31                      | 1.14                  | 1.05 – 1.09           | 1.05        | 1.05        | pu                        |
| Maximum operating temperature | $T_{op-max}$     | 80 – 180                         | 160 – 200             | 150 – 200             | 200         | 80          | °C                        |
| Curie temperature             | $T_c$            | 305 – 470                        | 325 – 370             | 325 – 370             | 365         | 310         | °C                        |

## vi. Amorphous and Nano-Crystalline Alloys

These materials can be produced in the form of a tape by melt-spinning. The alloys consist of iron, nickel and/or cobalt with one or more of the following elements: boron, carbon, phosphorous, and silicon. They have extremely low coercivity, an order of magnitude less than standard Fe-Si, and consequently have lower hysteresis losses. However, they have a relatively low magnetisation and are not suitable for high current applications.

Instead of casting the alloy onto a rotating wheel to produce tapes, it is also possible to direct a stream of molten alloy into a bath of water or oil to produce amorphous wires of typically 50μm thick. These wires exhibit a square hysteresis loop with large changes in magnetisation at low fields, making them ideal for sensing and switching.

Nano-crystalline material is produced by annealing the amorphous material. These alloys can be single phase but are usually comprised of nano-sized grains, in the range 10 to 50nm, in an amorphous matrix. They have relatively high resistivity, low anisotropy, good mechanical strength, and are better suited for soft-magnetic applications.

### 27.2.2 Ceramics

Hard Ceramic hexaferrites: ( $\text{BaFe}_{12}\text{O}_{19}$  or  $\text{SrFe}_{12}\text{O}_{19}$ )

Hard hexagonal ferrite materials, often referred to as ceramic magnets, are ferrimagnetic and considering the proportion of iron within the material, have a low remanence, ~400mT. The coercivity of these magnets is typically ~250kA/m. The low remanence means that the maximum energy product is only ~40kJ/m³, which is lower than the alnicos, but due to the high coercivity, these magnets can be made into thinner sections. The magnets can be exposed to moderate demagnetising fields, and coupled with good mechanical characteristics and low cost, are suitable for applications such as permanent magnet motors.

Ceramic ferrites are made using an iron oxide powder, to which either barium or strontium (carbonate) is added to improve alignment of the crystal lattice structure. The chemical formula is  $\text{M-n}(\text{Fe}_2\text{O}_3)$ , where M is Ba or Sr and  $5.8 < n < 6.0$ . After milling, which produces small grains, <1mm, which is essential for generating coercivity in ferrites, the powder is pressed in a die, with an orienting field applied through the cavity if desired, enhancing the remanence and the maximum energy product. If no field is applied, an isotropic magnet results with no preferred magnetic properties in any particular direction. If an orienting field is applied, an anisotropic magnet results having preferred magnetic properties along a specific axis. The compacted powder is then sintered at a temperature of 1100°C to 1300°C (hence the name *ceramic*) to achieve full densification, cooled at a rate of less than 90°C per hour to avoid thermal shock, and then diamond ground to the required final dimensions. There are no problems with oxidation of the powder during processing, as the material is a stable iron oxide. Alternatively, the powder may be blended with a polymer binder, and then either extruded or formed in a die by compression, or injection-moulding, thus producing a bonded ferrite magnet of near net shape; again, anisotropic properties may be achieved by applying an orienting field through the die cavity. Due to their anisotropic structure, ferrites exhibit relatively high coercivity, however, the energy product is low.

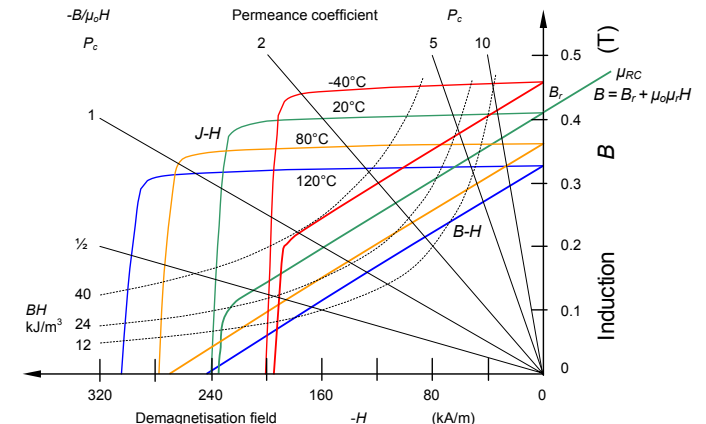


Figure 27.11. Second quadrant hysteresis loop demagnetization characteristics measured at different temperatures for a sintered ceramic ferrite magnet (with constant energy contours shown).



A hexagonal ferrite structure is found in both  $\text{BaO} \cdot 6(\text{Fe}_2\text{O}_3)$  and  $\text{SrO} \cdot 6(\text{Fe}_2\text{O}_3)$ , with Sr ferrite having slightly superior magnetic properties.

Generally, magnet materials respond negatively to heat because thermal energy reduces the flux density and the ability of domains to remain aligned. Increased thermal energy increases the disorder present and at a particular temperature, the Curie point, the material loses its ferromagnetism. Excessive heating of a magnet may cause metallurgical changes to occur; in many magnets the safe operating temperature is well below the Curie point. The one exception is ferrites (ceramic magnets), which, due to their chemical nature, can withstand temperatures significantly past their Curie points. Ferrites also exhibit the unusual characteristic of increased coercivity with temperature, as shown in figure 27.11. Heat treatments during manufacturing are precisely controlled to precipitate desired phases and to control metallurgical changes.

The main advantage of ferrites is low cost, due to the ease of processing and the abundance, non-strategic, and low cost of raw materials, and complex shape possibilities. Ceramic magnet material (ferrite) has modest resistance to corrosion and can operate in moderate heat but demagnetise at low temperatures, typically,  $20^\circ\text{C}$ , as the knee moves up the characteristic line. Ferrites are brittle.

### 27.2.3 Bonded

Rigid bonded magnet materials can be made from Alnico, Ceramic, isotropic NdFeB (MQ), anisotropic NdFeB (HDDR), SmCo, SmFeN, or nano composite powders which are combined (forming a thermoset or thermoplastic) with a variety of plastic binders, a matrix with polyphenylene sulphide (PPS), chlorinated polyethylene (CPE) polyester, PVC, nylon (polyamide), duroplast or nitrile rubber. Rigid bonded magnets are processed by compression, injection, extrusion or calendaring shaping methods.

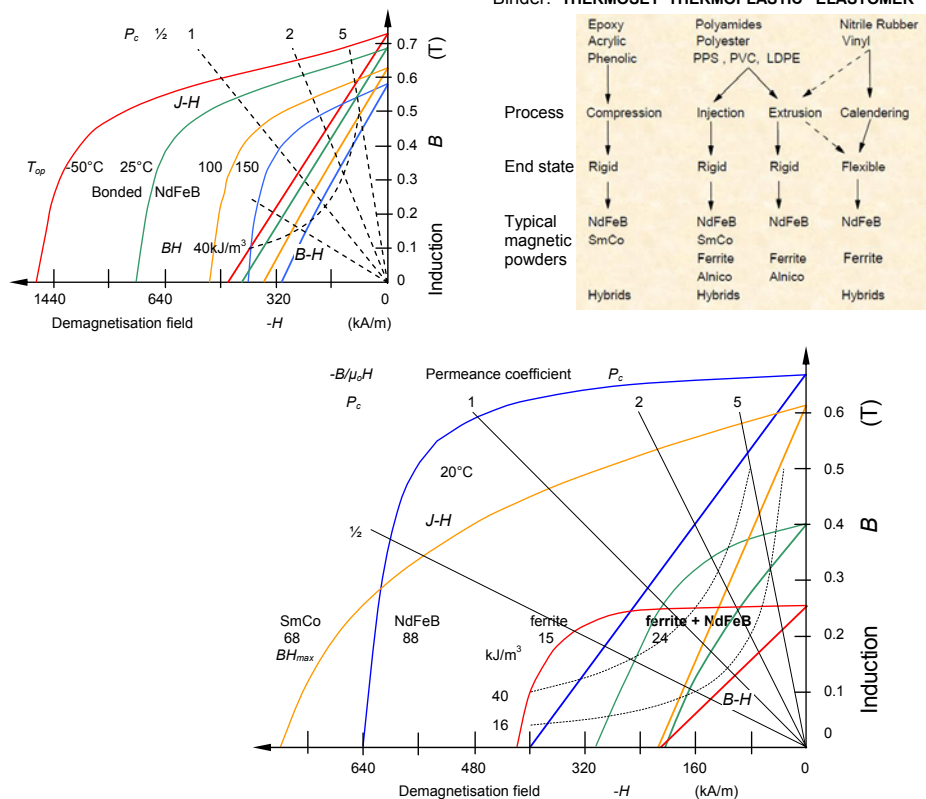


Figure 27.12. Second quadrant hysteresis loop demagnetization characteristics measured at different temperatures for bonded magnets (with constant energy contours shown).

They can be *injection moulded* thermoplastic bonded into mechanically-strong complex magnet shapes with accurate finished dimensions. Bonded magnet materials have a moderate resistance to corrosion and a low tolerance to heat,  $-40^\circ\text{C}$  to  $180^\circ\text{C}$ , because of the binder material. The same binder properties tends to make them chip and break resistant.

Bonded thermoset magnets can also be *compression bonded*, which offers a higher magnetic output (than injected moulded bonded magnets), due to a higher magnetic particle density than either NdFeB or SmCo powders, but are restricted to simpler geometries than injection moulded materials. The use of an epoxy binder and epoxy surface coating prevents oxidation and is resistant to normal industrial solvents and automotive fluids. Because of the compression bonding process, the tooling tolerances and mechanical strength properties are slight less than those materials that have been injection moulded. In the case of NdFeB, the bonded material is isotropic, offering approximately  $80\text{kJ/m}^3$  or 30% of the energy produced by the sintered fully dense material, but can be magnetised in any direction. Typical of bonded magnets, the operating temperature range (specifically the upper limit) is restricted to  $-40^\circ\text{C}$  to  $165^\circ\text{C}$ , with a poor temperature coefficient.

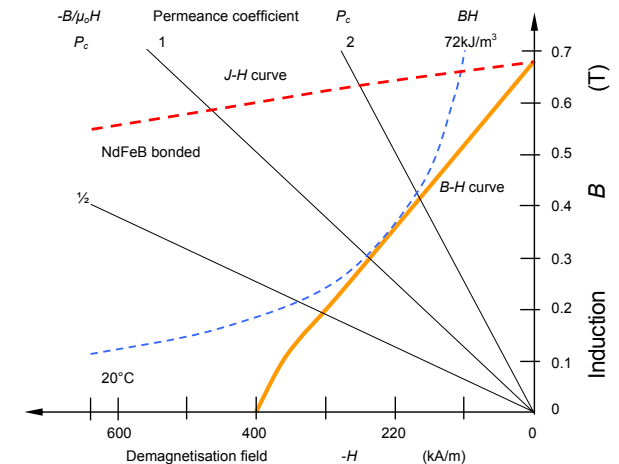


Figure 27.13. Second quadrant hysteresis loop demagnetization characteristics for a flexible bonded magnet (with a constant energy contour shown).

The energy product  $BH_{\text{max}}$  of bonded magnets is also lower than for a fully dense material. This is due to the magnetic material being diluted by the binder. Bonded samarium cobalt bonded materials are typically anisotropic with energy products of  $40\text{kJ/m}^3$  to  $120\text{kJ/m}^3$ .

The polymer matrix associated with bonded magnets weakens magnetic properties and limits thermal properties. Their attribute is freedom of shape and design.

### 27.2.4 Flexible (rubber)

Flexible (rubber) magnets are manufactured by mixing (bonding) ferrite or neodymium magnet powders and synthetic or natural rubber (nitrile, vinyl, etc.) and resin elastomer binders. Flexible (rubber) magnets are manufactured by rolling (calendaring), injection or extrusion methods. Versatility, low cost, and ease of use are among the reasons to choose ferrite based flexible (rubber) magnets. Flexible magnets are usually manufactured in strip or sheet form. There are *isotropic* and *anisotropic* forms. The isotropic rubber magnet is normally magnetized with multipoles magnetization on one side only while the anisotropic form can have magnetics on both sides. The magnet material is low energy, as shown in figure 27.13, and does not usually replace fully dense magnet materials. Flexible neodymium material is higher in strength, but is made in limited quantities because the cost is high.

Table 27.4 Permanent magnet material features

| Permanent magnet material   | advantages                    | disadvantages                                      |
|-----------------------------|-------------------------------|--|
| Alnico                      | Thermal stability             | Brittle  |
|                             | High service temperature      | Low coercive force                                 |
|                             | High flux density             | Required thickness                                 |
|                             | Complex shapes                | Cost variability                                   |
|                             | Easy to magnetise             |  |
|                             | Low tooling cost              |  |
| Hard ferrite                | Low cost                      | Brittle  |
|                             | High coercive force           | Limited shape and tolerances                       |
|                             | Easy to magnetise/demagnetise | Low energy product                                 |
|                             | Stability to oxidation        | Performance varies – hi temperature dependency     |
|                             |                               | High tooling costs                                 |
| Bonded ferrite              | Flexible                      | Low energy product                                 |
|                             | Easily shaped                 | Low service temperature                            |
|                             | Can build subassemblies       | Performance varies – hi temperature dependency     |
|                             | Requires little machining     |  |
| Samarium Cobalt             | High energy product           | Brittle  |
|                             | High coercive force           | High cost  |
|                             | Compact                       | Difficult to magnetise/demagnetise                 |
|                             | High service temperature      | Easy to oxidize                                    |
|                             | Corrosion resistant           | Powders pyrophoric – spontaneous ignition          |
|                             | Thermal stability, low TC     |  |
| Neodymium-iron-boron        | Highest energy product        | Performance varies – medium temperature dependency |
|                             | High coercive force           | Susceptible to corrosion                           |
|                             | Compact                       | Low service temperature                            |
|                             | Low energy cost               | Difficult to magnetise                             |
|                             |                               | May require coating                                |
|                             |                               | Powders pyrophoric                                 |
| Bonded Neodymium-iron-boron | High energy product           | Performance varies – hi temperature dependency     |
|                             | High coercive force           | Susceptible to corrosion                           |
|                             | Low energy product cost       | Low service temperature                            |
|                             | Easily shaped                 |  |
|                             | Can build subassemblies       |  |

### 27.3 Properties of hard magnetic materials

A permanent magnet is a material that when inserted, in a virgin form, into a strong magnetic field will not only begin to exhibit a magnetic field of its own, but also continues to exhibit a magnetic field once removed from the original magnetising field. Permanent-magnet materials are characterized by high values of remanent magnetization and coercivity. Such materials produce significant magnetic flux, even in magnetic circuits with air gaps.

A permanent magnet's magnetization  $M$  [the sum of the individual (unpaired electron spins) magnetic moments per unit volume] provides a magnetizing force  $H$  which establishes a flux density  $B$ ; these being related by  $B = \mu_0(H + M) = \mu_0 H + J$ . This relationship can be used to convert the intrinsic  $M$  versus  $H$  magnetization characteristic into the usual  $B$  versus  $H$  magnetization characteristic shown in figure 27.12. The  $B$  versus  $H$  characteristic is more useful for practical magnet design since  $M$  exists only within the magnet while  $B$  'flows' through the magnet and in the surrounding media.

The  $B$  within a magnet is indicative of the flux density it will deliver into the adjacent air gap, and the point at which a magnet operates on its demagnetization curve relates  $B$  to the demagnetizing force  $-H$  it

experiences. The demagnetization curve shows that as the magnitude of  $-H$  increases, the flux density delivered by the magnet falls, ultimately at  $H = -H_c$  to  $B = 0$ . However, an unconstructive phenomenon begins within the magnet before  $-H_c$  is reached, because the *knee* (Cunic point) in the demagnetization curve represents the onset of a reversal of the material's  $M$  (that is, the threshold for irreversible loss, where the characteristic rapidly leaves the second quadrant and moves into the third quadrant). Thus it is desirable that the operating point of a permanent magnet always remain above the temperature dependant *knee* in the demagnetization curve.

The second quadrant of a hysteresis loop (the *demagnetization curve*) is usually employed for analyzing a permanent-magnet material. Residual flux density or remanent magnetization is  $B_r$ , while coercivity,  $H_c$ , is a measure of the magnitude of the mmf required to demagnetize the material, and the capability of the material to produce flux in a magnetic circuit that includes an air gap.

Magnets are characterized by the three main characteristics.

- Residual Induction or Remanent flux density ( $B_r$ , measured in Tesla).
  - An indication of how strong the magnet is capable of being.
- Coercive Force or Coercivity ( $H_c$ , measured in kA/m).
  - An indication of inherent stability and how difficult it is to demagnetize the magnet.
- Maximum Energy Product per unit volume ( $BH_{max}$ , measured in kJ/m<sup>3</sup>).
  - An indication of the magnet material volume required to produce a given level of magnetic flux. It is where the magnetic field energy density into the air gap surrounding the magnet, is at a maximum, i.e., the operating point that minimises magnet volume.

The temperature dependence of residual induction and coercive force are modelled by

$$\begin{aligned} B_r(T) &= B_{r, 20^\circ\text{C}} (1 + \alpha(T - 20^\circ\text{C})) \\ H_{cl}(T) &= H_{cl, 20^\circ\text{C}} (1 + \beta(T - 20^\circ\text{C})) \end{aligned} \quad (27.3)$$

where reversible temperature coefficient of  $B_r$  and  $H_{cl}$  (and  $H_c$ ), specifically  $\alpha$  and  $\beta$  respectively, are defined as

$$\alpha = \frac{\Delta B_r}{B_r} \times \frac{1}{\Delta T} \times 100\% \quad \beta = \frac{\Delta H_{cl}}{H_{cl}} \times \frac{1}{\Delta T} \times 100\% \quad (27.4)$$

Behaviour of a permanent magnet is referenced to the second quadrant of its  $B$  versus  $H$  curve, where it is termed the *demagnetization curve*. The second quadrant of the curve (b-o-c), shown in figure 27.14c, can be used to demonstrate the demagnetisation characteristics of a permanent magnet material.

Most magnetic materials are classed as *anisotropic*, that is, a preferred magnetic axis is determined during manufacturing so that the magnets can only be magnetised in that predetermined direction. *Isotropic* materials may be magnetized in any direction, but generally have inferior performance than anisotropic grades.

- Remanence is the intercept of the  $B$  versus  $H$  curve on the positive  $B$  axis. For an *ideal* material, its value is  $B_r = \mu_0 M_{sat}$ , but  $B_r$  is always the value of flux density for the condition when a magnet develops no magnetizing force ( $H = 0$ ). Magnetization  $M$ , a bulk material property, is the sum of the magnetic dipole moments  $\mu_m$  per unit volume [ $M_{sat}$  is saturation magnetisation, A/m].  $B_r$  is coincident to both the intrinsic  $J$ - $H$  curve and the normal  $B$ - $H$  curve.
- Coercivity is the intercept of the  $B$  versus  $H$  curve on the negative  $H$  axis. Its value  $-H_c$  is the magnetizing force required to reduce the magnet's flux density  $B$  to zero, which on the *ideal* curve in figure 27.14c, is  $-H_c = M_{sat}$ . There is no observable (external) field because the applied field  $H$  (specifically,  $H_c$ ) is balanced out by the flux  $M$  of the magnet material. Because they are opposing, the net observable induction  $B$ , is zero. Notice that by comparing the *normal* and *intrinsic* characteristics, the values of  $-H_c$  and  $-H_{ci}$  are not the same, that is, the magnetizing force  $H_c$  required to make  $B = 0$  may be less than that required to reduce the net observable flux density  $B$  to zero, namely  $H_{ci}$ , after which the material's magnetization direction is reversed.
- Maximum energy product  $BH_{max}$  is the point on the second quadrant  $B$  versus  $H$  curve at which the product of  $B$  and  $H$  for the magnet is a maximum. On the *ideal* curve in figure 27.14, it is located exactly halfway down the second quadrant line, with a value of  $-BH_{max} = \mu_0 (\frac{1}{2} M_{sat})^2$ .

Generally  $\mu_0 M_s = J_s > B_r$ ,  $B_r \geq \mu_0 H_{ci}$ ,  $H_{ci} \geq H_{cr}$ , and  $\frac{1}{4} B_s^2 \geq \mu_0 B H_{max}$ . The slope of the normal curve at the  $B$ -axis intercept is called the recoil permeability,  $\mu_0 \mu_{rc}$ . The third and fourth quadrants are the same as the first and second quadrants, except have the 'opposite' polarities. The whole curve forms the hysteresis loop.

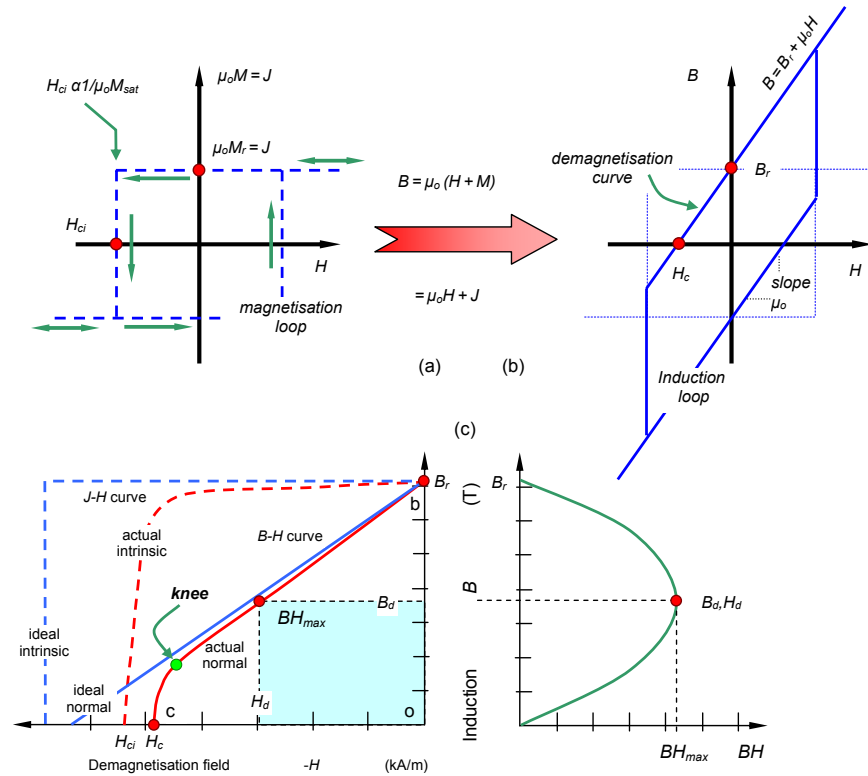


Figure 27.14. Magnetic material curves: (a) ideal magnetisation loop, (b) ideal induction loop, and (c) second quadrant hysteresis loop for a sintered magnet.

Several characteristics have to be taken into consideration when deciding which permanent magnetic material to use, namely:

- Flux related requirements of:
  - energy stored in the material;
  - flux density on the surface of the magnet; and
  - pull required to remove the magnet from a flat piece of steel.
- Maximum operating temperature.
- Cost.
- Availability.
- Degree of corrosion likely to be encountered.
- Magnetic stability required.
- Size and/or weight limitations.

General relative characteristics are shown in figure 27.15 and the following design points are related sequentially to the data columns in Table 27.5.

i. A method of comparing the magnetic performance or the capability of different types and grades of permanent magnet is to consider their maximum energy product  $BH_{max}$ . This is the operating point where the magnet will provide most energy for a minimum volume. When the permanent magnet behaves linearly with a constant recoil permeability  $\mu_{rc}$  the maximum energy product can be expressed simply by:

$$BH_{max} = \frac{B_r^2}{4\mu_{rc}} \quad \text{J/m}^3 \quad (27.5)$$

A material with the largest available maximum energy product results in the smallest required magnet volume. Maximum energy products for various hard magnetic materials are shown in Table 27.5. Ignoring leakage flux, for a permanent magnet with an air gap, the air gap flux density and mmf's are:

$$B_g = \frac{A_m}{A_g} B_m \quad (27.6)$$

$$\frac{H_m \ell_m}{H_g \ell_g} = -1 \quad (27.7)$$

$$\text{Combining gives } B_g^2 = \mu_o \frac{\ell_m A_m}{\ell_g A_g} (-H_m B_m) = \mu_o \frac{Vol_{mag}}{Vol_{air-gap}} (-H_m B_m) \quad (27.8)$$

where  $B_g = \mu_o H_g$ . That is

$$Volume_{mag} = Volume_{air-gap} \frac{B_g^2}{\mu_o} \frac{1}{(-H_m B_m)} \quad (27.9)$$

Equation (27.9) indicates that, to achieve a desired flux density in the air gap, the required magnet volume can be minimized by operating the magnet at the point of maximum energy product.

ii. Table 27.5 shows typical pole face flux densities for the various hard magnetic materials when operating near their  $BH_{max}$  point. This point is not at the remanence  $B_r$ , which is the induction under closed magnetic circuit conditions.

iii. The attraction force  $F$  between a permanent magnet and a soft magnetic body depends on three factors:

$B$  = flux density on the pole face

$A$  = area of the pole face

$\mu$  = permeability of the material being attracted

and the force is given by:

$$F = B^2 A \mu \quad (27.10)$$

iv. There are two distinct temperature related loss categories, namely reversible and irreversible losses. The *reversible* changes with temperature are dependant on material composition and are unaffected by shape, size or working point on the demagnetisation curve. This loss vanishes without need for remagnetisation when the magnet returns to its initial temperature.

*Irreversible* losses do not occur until a certain temperature, the Curie temperature  $T_C$ , has been exceeded. Unrecoverable losses occur when excessive temperatures are reached and metallurgical changes occur within the magnet. This loss can also be limited by operating at a high as possible working point (well above the *knee*, near  $B_r$ ).

v. The maximum working temperature, before irreversible losses commence, is dependant on the magnet working point in the magnetic circuit. The higher the working point up the B-H demagnetising characteristic curve, the higher the temperature at which the magnet can operate. Irreversible losses can be recovered by remagnetising the magnet.

vi. Each magnetic material has a maximum temperature, above the Curie temperature, where metallurgical changes occur within the magnet structure and where the individual magnetic domains randomise due to the high level of thermal agitation. Once these losses occur they cannot be reversed by remagnetising.

vii. The effects of low temperatures are different for each material group and are also related to the magnet shape and therefore its working point on its demagnetization curve.

viii. The temperature has a significant effect on magnet stability, but exposure to high external fields,  $H$ , influences magnets as follows:

- It is possible to ensure magnetic stability, by pre-exposing the magnet to possible detrimental influences by thermal cycling and controlled demagnetizing fields (ageing) techniques.
- Another cause of performance loss is total composition structural breakdown due to corrosion or in the case of NdFeB, exposure to hydrogen.

ix. There are many protective coatings used to help prevent corrosion in magnets.

Samarium cobalt, alnico, and ceramic materials are corrosion resistant, and typically do not require to be coated against corrosion.

Alnico is easily coating for cosmetic purposes with a powder coating or electroplating when required.

Ceramics may be coated to seal the surface, which will otherwise be covered by a thin film of ferrite powder, which does not create a problem in most applications.

Magnets using rare earth (non-precious) elements are highly reactive at elevated operating temperatures in the presence of moisture, due to their strong negative electrochemical standard potential, between -2.2V and -2.5V. Rare-earth-hydroxide is formed, releasing hydrogen, which then reacts with free rare earth metal, forming rare earth metal hydrides. With  $\text{SmCo}_5$  or  $\text{Sm}_2\text{Co}_{17}$ , by adding cobalt, the reaction with water is suppressed.

In the case of NdFeB magnets, which are particularly susceptible to corrosion, the individual magnet grains are held together mechanically and fixed to each other by the so-called neodymium-rich phase, so the neodymium reacts with water to form neodymium hydroxide. Addition of cobalt to the neodymium-rich phase improves the corrosion behaviour by systematically stopping intergranular corrosion in a warm, humid atmosphere. The corrosion behaviour of such alloys is similar to that of pure iron materials (steel). In cases where the humidity turns to condensation, the materials gradually rust forming non-magnetic (antiferromagnetic) metal oxides or hydroxide, similarly to red rust in the case of iron. In applications with possible condensation, hard magnetic parts should be coated. Spray coatings are either metallic (galvanic nickel 10µm thick and/or tin 15 to 30µm thick, or 5µm of electrically isolating aluminium – aluminium chromate vacuum deposited) or organic (5µm and 40µm of epoxy resin, lacquer, parylene or polyethylene polymers).

x. Shape, tolerances, and quantity influence the prices of specific magnets but the most significant factor is the cost of the basic raw material.

Other design factors include time (magnetic creepage, energy loss of typically less than  $1 \times 10^{-5}$  pu per annum at 20°C), radiation, shape, weight (for example, NdFeB magnets are about 13% lighter than SmCo magnets), and shock and vibration. All these are factors affecting magnet stability. Stability can be described as the repeated magnetic performance of a material under specific conditions over the life of the magnet.

## Summary of 2<sup>nd</sup> quadrant demagnetisation characteristics of different hard magnetic materials

Table 27.5 Hard magnetic material characteristics and properties

| Material | structure                | Energy<br>$BH_{\text{max}}$ /<br>recoil $\mu_{\text{re}}$ | flux<br>density/<br>coercivity | pull               | $\alpha_r$                | $T_{\text{op}}$ | $T_C$ | Low $T_{\text{op}}$   | magnetic<br>stability         | corrosion<br>resistant | cost<br>relative    |
|----------|--------------------------|---|--------------------------------|--------------------|---------------------------|-----------------|-------|---|-------------------------------|------------------------|---------------------|
|          |                          | $\text{kJm}^{-3}$   | T / kA/m                       | kg/cm <sup>2</sup> | % / °C<br>20°C -<br>150°C | °C              | °C    |   | Thermal $\alpha/B_r$ ,<br>%/K |                        |                     |
|          | Design sequence<br>stage | i   | ii.                            | iii.               | iv                        | v               | vi    | vii   | viii                          | ix                     | x                   |
| Ferrite  | anisotropic              | 24/1.15   | 0.5/180<br>0.1T typ            | 0.26               | -0.19                     | 250             | 460   | Large<br>irreversible<br>losses below<br>-60 °C                                   | poor<br>-0.2                  | excellent              | low<br>x1           |
| Ferrite  | bonded                   | 3-8/1.1   | 0.2/130                        | 0.06               |                           |                 | 450   |   |                               |                        |                     |
| Alnico   | anisotropic              | 40/3  | 1.0/120<br>0.13T typ           | 0.42               | -0.02                     | 540             | 860   | Permanent<br>losses of no<br>more than<br>10% are to<br>be expected<br>down to 4K | excellent<br>-0.02            | fair                   | medium<br>x5        |
| Alnico   | casted                   | 38/   | 0.7/100                        | 0.42               |                           | 540             | 850   |   |                               |                        |                     |
| SmCo     | Sintered<br>1:5          | 144/1.05  | 1.0/<br>0.28T typ              | 1.50               | -0.04                     | 250             |       |   | excellent<br>-0.035           | excellent              |                     |
| SmCo     | Sintered<br>2:17         | 200/1.1   | 1.2/800<br>0.32mT typ          | 2.08               | -0.03                     | 300             | 750   | Minimal<br>losses down<br>to 4K   | excellent<br>-0.03            | excellent              | very<br>high<br>x20 |
| SmCo     | Bonded<br>1:5            | 55/1.1  | 0.7/750                        | 0.60               |                           |                 | 870   |   |                               |                        |                     |
| NdFeB    | N38H                     | 256/1.05  | 1.3/800<br>0.4T typ            | 3.06               | -0.12                     | 50-<br>200      | 320   | No<br>irreversible<br>losses down<br>to 77K                                       | moderate<br>-0.13             | poor                   | high<br>x10         |
| NdFeB    | bonded                   | 110/1.1   | 1/800                          | 1.20               |                           |                 | 310   |   |                               |                        |                     |
| ferrite  | flexible                 | 5/  | 0.16/110                       |                    |                           | 100             |       |   |                               |                        | low                 |

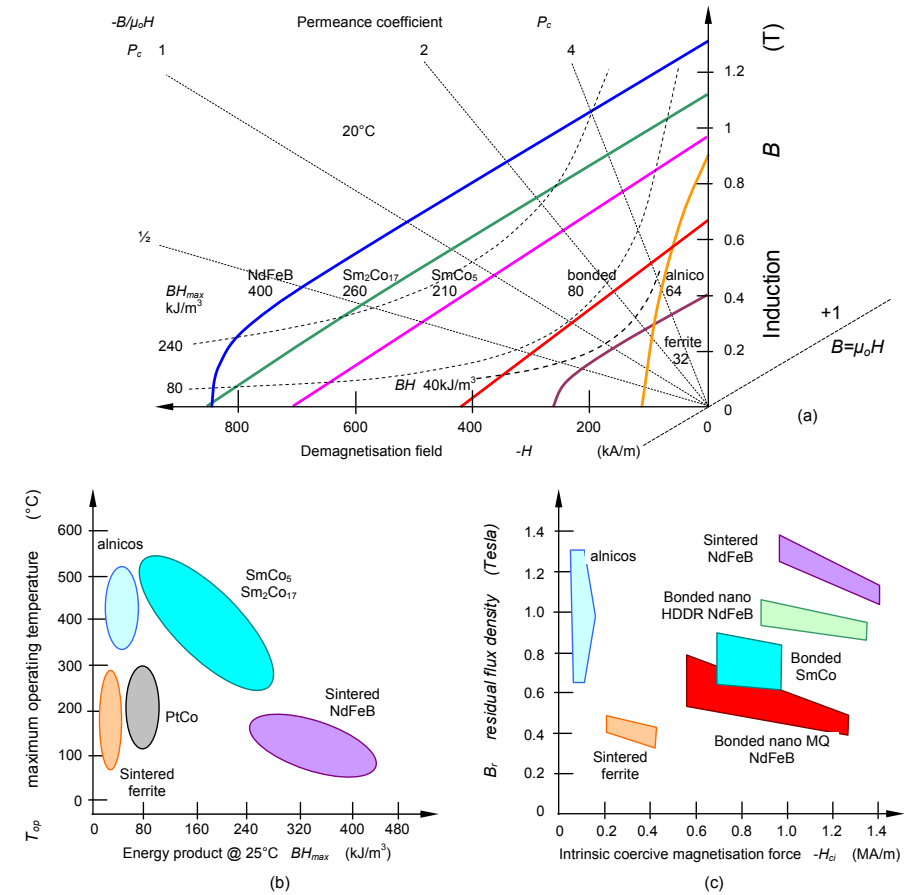


Figure 27.15. Comparison of different magnetic materials: (a) second quadrant hysteresis loop demagnetization characteristics (with constant energy contours shown), (b) energy product versus maximum operating temperature, and (c) remanence versus intrinsic coercive force.

## 27.4 Permanent Magnet Magnetization Curve (hysteresis loop) and recoil

The magnetic circuit in figure 27.16a includes a section of un-magnetised hard magnetic material in a core of highly permeable soft magnetic material and an  $N$ -turns excitation winding. In figure 27.16c, it is assumed that the hard magnetic material is initially un-magnetized (corresponding to point a, the origin, in figures 27.16a and b) and current is applied to the excitation winding. Because the core is assumed to be of infinite permeability, the horizontal axis of figure 27.16c can be considered to be both a measure of the applied current  $i = H \ell_m / N$  as well as the  $H$  field intensity in the magnetic material.

The magnetic circuit of figure 27.16a can be used to magnetize hard magnetic materials. The process requires a large excitation current be provided through the winding, thereby attaining point b, which is then reduced to zero, leaving the material with a remanent magnetization (or residual induction)  $B_r$  (point c in figure 27.16c).

With reference to figure 27.16c, the magnetization process progresses through four stages:

**Stage 1.** The permanent magnet starts in an initially unmagnetized state at field intensity  $H = 0$  and flux density  $B = 0$ , point 'a' in figure 27.16c.



**Stage 2.** As the current  $i$  in the coil is increased to a maximum,  $i_{max}$ , the  $B$ - $H$  trajectory rises from point  $a$  in figure 27.16c to a maximum ( $H_{sat}$ ,  $B_{sat}$ ) at point  $b$ . It is assumed that driving the core well into saturation with field strength  $H_{sat}$  at point  $b$ , fully magnetizes the hard magnetic material.

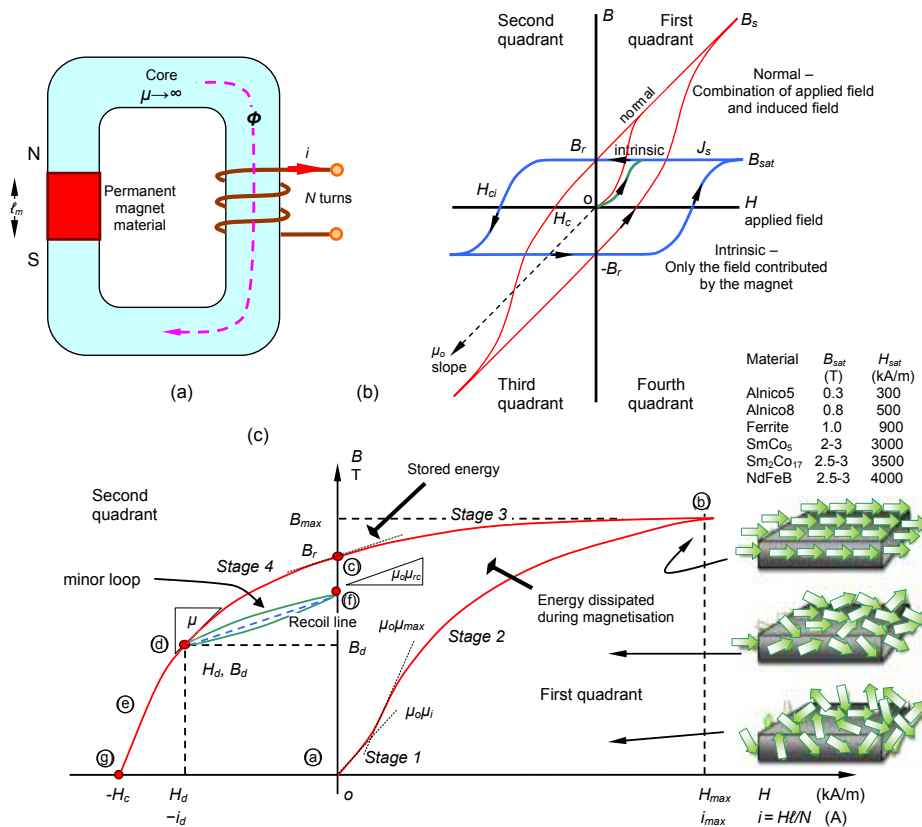


Figure 27.16. Magnetisation of a permanent magnet:

(a) magnetic circuit incorporating a permanent magnet and an excitation winding, (b) four quadrant  $B$ - $H$  characteristics, and (c) portion of a  $B$ - $H$  characteristic showing a minor loop, a recoil line and magnetic induction  $B_{sat}$  required to magnetise various permanent magnet materials.

**Stage 3.** Subsequently, when the coil current is decreased to zero (the external applied magnetising field intensity decreases to zero); the flux density from within the permanent magnet material also decreases. The  $B$ - $H$  characteristic traverses part of a hysteresis loop, moving to point  $c$  at zero current. At point  $c$ , the applied field intensity  $H$  in the material is zero but the flux density  $B$  is at its remanent value  $B_r$ . Through hysteresis mechanisms, the magnet retains flux density  $B_r$  (retentivity if the magnetising force reached saturation - residual flux density if the magnetizing force did not reach saturation).

**Stage 4.** With the introduction of an air gap and/or an external demagnetizing field (current direction in the coil is reversed), the magnet operates within the second quadrant of the demagnetization curve, where  $H$  is negative and  $B$  is positive. The slope of the demagnetization curve in the second quadrant, as shown in figure 27.16c, is  $\mu_m$ . Permeability  $\mu_m$  is typically close to  $\mu_0$ , the magnetic permeability of free space. With increased current, the intersection of the demagnetization curve with  $B = 0$  occurs at  $H = -H_c$ , point  $g$ , where  $H_c$  is known as the coercivity (coercive force) of the magnetic material.

Assuming a linear demagnetising curve, the magnetic circuit second quadrant operating point conforms to the following mathematical relationship between flux density,  $B$ , and field intensity,  $H$ :

$$B = B_r + \mu_m H \quad (27.11)$$

The created (hysteresis) loops are a graphical representation of the relationship between an applied magnetic field and the resulting induced magnetization within a material. The field that is generated by the now magnetized material (orientated domains),  $B_r$ , when added to that of the applied field,  $H$ , is known as the normal induction,  $B_n$ , or simply  $B$ . This induction has two components (normal and intrinsic curves), and they are related by:

$$B_n = B = \mu_0 M + \mu_0 H = J + \mu_0 H \quad (27.12)$$

The  $B$  versus  $H$  curve is known as the *normal* curve, while the  $J$  versus  $H$  curve is called the *intrinsic* curve (polarisation  $J = \mu_0 M$ ). These curves, also called hysteresis loops, are shown in figure 27.16b. The loops show the properties of the magnetic material as it is cycled between magnetization (saturated) and demagnetization (under the influence of an external magnetic field). The second (and third) quadrant of the loops display the magnetic properties as the magnet performs work.

When the current is reversed from zero to  $-i_d$ , the  $B$ - $H$  characteristic operating point traverses along a hysteresis loop with a trajectory from point  $c$  to point  $d$ . If the current is maintained at  $-i_d$ , the magnet operating point is maintained at point  $d$ , where  $H_d$ ,  $B_d$  generally denotes the operating point on the second quadrant demagnetising curve. This same operating point would be attained if the material were to start at point  $c$ , and with zero current excitation, from equation (27.8), an air gap of length  $\ell_g = \ell_m (A_g / A_m) (-\mu_0 H_d / B_d)$  were introduced into the core. If the current becomes more negative, the trajectory continues to trace out the hysteresis loop toward point  $e$ . If instead of increasing the current magnitude, it is reduced from  $-i_d$  to zero, the trajectory does not retrace the hysteresis loop toward point  $c$ . Rather it inscribes a minor hysteresis loop, reaching point  $f$  at zero current. If the current is then varied between zero and  $-i_d$ , the  $B$ - $H$  characteristic operating point traces out the minor loop shown in figure 27.16c. Unlike soft magnetic material, the absence/removal of an external magnetic field does not lead to demagnetization.

The  $B$ - $H$  trajectory between points  $d$  and  $f$  can be represented by a straight line, termed the *recoil line*, with a slope  $\mu_0 \mu_{rc}$ , where  $\mu_{rc}$  is defined as the relative recoil permeability. At the vertex of the minor hysteresis loop, point  $d$ , the material has been partially demagnetized, with the effective remanent magnetization of the magnetic material having been reduced to that represented by point  $f$ , which is less than the remanent magnetization  $B_r$ , point  $c$ . If the demagnetization is increased past point  $d$ , to point  $e$  of figure 27.16c, a new minor loop will be created, with a new recoil line but with a similar recoil permeability (slope  $\mu_0 \mu_{rc}$ ). If the demagnetizing field is increased beyond point  $g$ ,  $H_c$ , the operating point of the magnet now moves into the third quadrant of the normal curve. Recoil still results in a positive remanence flux density. Before the intrinsic coercivity  $H_{ci}$  is reached, which is shown in figure 27.16b, the magnet becomes completely demagnetized, since recoil is back to the origin, point  $a$ . When  $H_{ci}$  is reached and exceeded, the recoil flux density is negative, below point  $a$ . Intrinsic coercivity  $H_{ci}$  is therefore a measure of a magnet's ability to resist demagnetization.

The demagnetization effects of negative current excitation are equivalent to the introduction of an air gap into the magnetic circuit (up to an operating point,  $H_{ci}$ , 0 on the intrinsic curve). If the magnetized material were removed from the core, this is equivalent to creating a large air gap in the magnetic circuit, therein demagnetizing the hard magnetic material. The magnet is effectively weakened, since if it were re-inserted into the magnetic core, it would follow a recoil line and return to a remanent magnetization operating point (to point  $f$ ) less than  $B_r$ . Hard magnetic materials such as Alnico materials, often do not operate stably with varying mmf and geometry conditions, since they can be significantly demagnetized due to improper operation. An advantage of magnetic materials such as ceramic ferrite, samarium-cobalt, and neodymium-iron-boron is that, because of their 'straight-line' characteristic in the second quadrant (with slope close to  $\mu_0$ ), their recoil lines closely match their magnetization characteristic. As a result, demagnetization effects are reduced. At the expense of a modest reduction of the remanent magnetization, hard magnetic materials can be stabilized to operate over a specified field region.

## 27.5 Permanent Magnet model

When a magnetic field is imposed on a permanent magnet, intrinsic induction  $J$  is created. The free poles establish a field potential  $-H_d$  between free poles. The field potential is due to some of the magnetization  $J$  lost internally. The field potential  $-H_d$  associated with a permanent magnet is a due to the magnetization  $J$  and is  $180^\circ$  opposed to  $J$ . The magnitude of  $-H_d$  depends on the geometry of the magnet including the spacing of the poles. These relationships are shown pictorially in figure 27.17. The behaviour of a permanent magnet is described with reference to the second quadrant of its  $B$ - $H$  curve, termed the demagnetization curve. For reference, the *ideal* curves in figure 27.17 are shown in blue. A practical permanent magnet material neither achieves its theoretical intrinsic coercivity  $-H_{ci}$ , nor does the entire magnetization flip over exactly when a reverse field of  $-H_{ci}$  is reached. The practical

intrinsic demagnetization curve, shown dashed red in figure 27.17 does not have an abrupt transition at  $-H_{ci}$ , but rather a *knee* in the curve represents a more gradual reversal of the material's magnetization. The actual normal demagnetization curve, shown solid red in figure 27.17 mirrors this *knee*. The coercivities  $-H_{ci}$  and  $-H_c$  are now defined as the intercepts of the practical intrinsic and normal curves with the  $H$ -axis.

The  $B$  from within the magnet is the flux density that can be delivered into the adjacent air gap, and the point at which a magnet operates on its demagnetization curve relates  $B$  to the experienced demagnetizing force  $-H$ . The demagnetization curve shows that as the magnitude of  $-H$  increases, the flux density produced by the magnet falls, ultimately at  $H = -H_c$  to  $B = 0$ . However, before  $-H_{ci}$  is reached, the *knee* in the demagnetization curve signifies the onset of a reversal of the material's magnetisation  $M$ . It is therefore desirable that the operating point of a permanent magnet remain above any *knee* (Cunic point) in the demagnetization curve - at all operating temperatures.

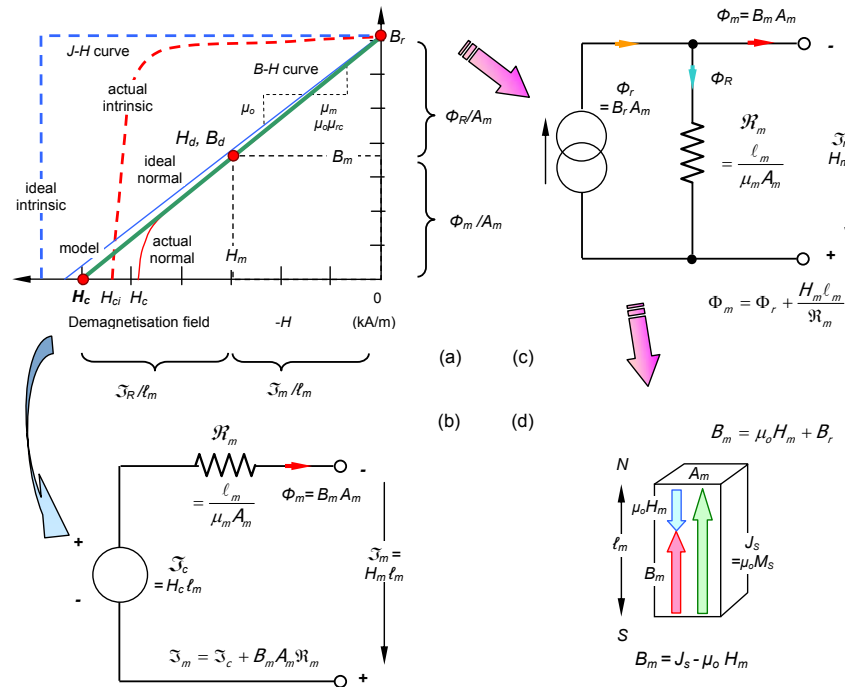


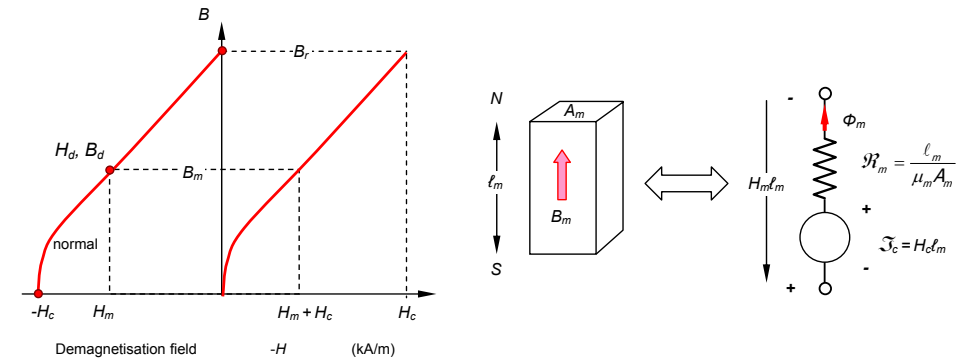
Figure 27.17. Magnetic circuit models of a magnet based on: (a) a linear demagnetization curve, (b) mmf source equivalent series circuit, (c) flux source equivalent shunt circuit, and (d) magnet internal fluxes.

Consider a permanent magnet of uniform cross sectional area  $A_m$  and length  $\ell_m$ . The demagnetization curve of the magnet is a straight line with a coercive force of  $H_c$  and a remanent flux density of  $B_r$  as shown in figure 27.17. A point  $H_d, B_d$  on a linear demagnetization  $B_m - H_m$  curve can be defined by

$$B_m = B_r + \mu_m H_m = \frac{B_r}{H_c} (H_c + H_m) = \mu_m (H_c + H_m) \quad (27.13)$$

where  $\mu_m = B_r / H_c$  is the permeability of the permanent magnet, which is slightly greater than  $\mu_0$ , the permeability of free space. For NdFeB and SmCo<sub>5</sub> sintered magnets,  $\mu_m = 1.05 \mu_0$ , as listed in Table 27.5.

Equation (27.13) shows that the demagnetisation curve can be defined in terms of  $\mu_m$  and either  $B_r$  or  $H_c$ . If  $H_c$  is used as a reference, the model will be based on an mmf source  $H_c \ell_m$  in series with the magnet reluctance  $\mathcal{Y}_m$ . If  $B_r$  is used as a reference, the model will be based on a flux source  $B_r A_m$  in parallel with the magnet reluctance  $\mathcal{Y}_m$ . The two models are equivalent, and in electrical circuit terms, behave as Thevenin and Norton equivalent circuits, as illustrated in figure 27.17.



27.18. Magnetic circuit Thevenin model of a magnet with nonlinear demagnetization curve.

### Thevenin equivalent magnetic circuit

The traditional, less intuitive modelling approach is based on an mmf source in series with the magnet reluctance. From equation (27.13) rearranged, the magnetic mmf across the magnet, using flux  $\Phi_m = B_m A_m$ , can be expressed as

$$H_m \ell_m = \left( \frac{B_m}{\mu_m} - H_c \right) \ell_m = \frac{\ell_m}{\mu_m A_m} \Phi_m - H_c \ell_m = \mathcal{Y}_m \Phi_m - \mathcal{F}_c \quad (27.14)$$

$\mathcal{Y}_m = \ell_m / \mu_m A_m$  is the magnet reluctance and  $\mathcal{F}_c = H_c \ell_m$  the magnetomotive force (energy source) of the magnet. In a magnet,  $B_m$  and  $H_m$ , being in the second quadrant, oppose. The series equivalent circuit satisfying the sum of mmf's around a closed loop summing to zero, is shown in figure 27.17b.

For a magnet with a nonlinear demagnetization curve, the magnetic circuit model in figure 27.17 remains valid, except that the magnetic permeability is

$$\mu_m = \frac{B_m}{H_m + H_c} \quad (27.15)$$

which is a function of the magnetic field in the magnet. The magnetic circuit model derivation of a nonlinear magnet is illustrated graphically in figure 27.18.

The operating point ( $H_d, B_d$ ) does not move along the nonlinear demagnetization curve if a small cyclic external magnetic field (such that the magnet will not be demagnetized) is applied to the magnet. Instead, the operating point moves along a minor loop or for simplicity, a straight line (centre line of the minor loop) as illustrated in figure 27.19.

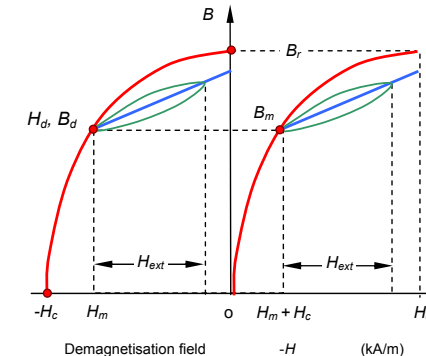


Figure 27.19. Movement of operating point of a nonlinear magnet under an external field  $H_{ext}$ .

### Norton equivalent magnetic circuit

From equation (27.13) rearranged, the magnetic flux from magnet, using flux  $\Phi_m = B_m A_m$ , can be expressed as

$$\mathbf{B}_m \mathbf{A}_m = B_r \mathbf{A}_m + \mu_m \mathbf{H}_m \mathbf{A}_m = B_r \mathbf{A}_m + \frac{\mu_m \mathbf{A}_m}{\ell_m} \mathbf{H}_m \ell_m = B_r \mathbf{A}_m + \frac{1}{\mathcal{Y}_m} \mathbf{H}_m \ell_m \quad (27.16)$$

$$\phi_m = \phi_r + \frac{\mathcal{F}_m}{\mathcal{Y}_m}$$

$\mathcal{Y}_m = \ell_m / \mu_m \mathbf{A}_m$  is the magnet reluctance,  $\mathcal{F}_m = \mathbf{H}_m \ell_m$  the magnetomotive force across the magnet, and  $\phi_r$  is the energy source. In a magnet,  $B_m$  and  $H_m$ , being in the second quadrant, oppose. The equivalent circuit satisfying the three fluxes at a node summing to zero, is shown in figure 27.17c.

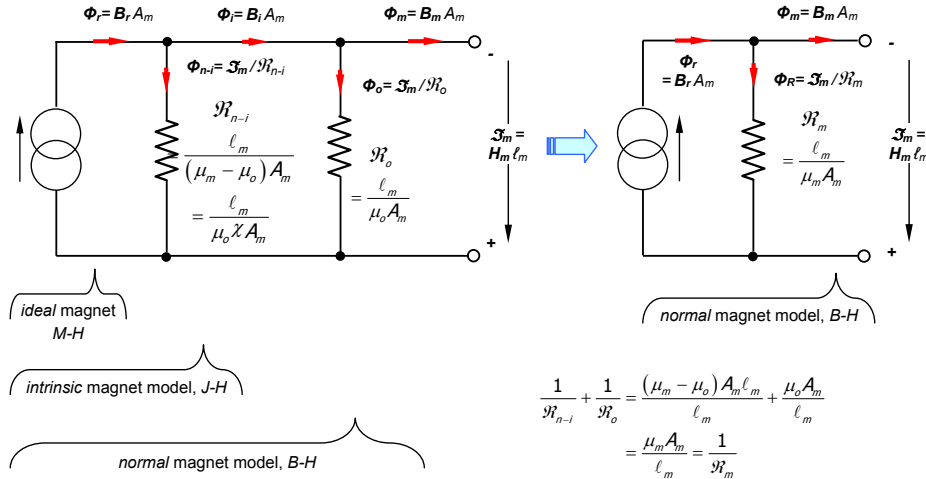


Figure 27.20. Magnet internal model showing an intrinsic magnet model.

In the case of NdFeB and SmCo magnets, there are several advantages in using a flux source model based on  $B_r$  rather than the series mmf model based on  $H_c$ .

- $B_r$  is common to both the intrinsic and normal demagnetisation curves ( $H_c$  and  $H_{c2}$  differ).
- At a given temperature, the demagnetisation curve slope is near constant,  $1/\mathcal{Y}_m$ , for an increasing reverse field, until a knee is reached. (With  $H_c$  as a reference, if a knee exists in the demagnetisation curve, the implied  $B_r$  is under estimated if a straight line demagnetising characteristic is used for modelling or analysis).
- A magnet is normally operated in the near linear region above any knee -  $B_r$  as a reference models this region independent of the existence and effects of any knee.
- The current source model in figure 27.20 clearly illustrates with two reluctances in parallel that the mmf, hence field intensity, must be the same value for the load operating point on the normal and intrinsic demagnetisation curves.

Both models yield the same mathematical solutions, since both yield the same demagnetisation load line equation, when using  $B_r = \mu_m H_c$ :

$$\mathbf{A}_m \mathbf{B}_m = \mathbf{A}_m \mathbf{B}_r + \frac{1}{\mathcal{Y}_m} \mathbf{H}_m \ell_m = \mu_m \mathbf{A}_m \mathbf{H}_c + \frac{1}{\mathcal{Y}_m} \mathbf{H}_m \ell_m \quad (27.17)$$

or  $\mathbf{B}_m = \mathbf{B}_r + \mu_m \mathbf{H}_m$

## 27.6 Load lines

One or more air gaps introduced into a magnetic circuit enable useful work to be exploited. Magnet material can be designed to provide a specific air gap flux density, within a reasonable range. The exact flux density is determined by the magnetic circuit dimensions, particularly those of the permanent magnet and the air gap. The second quadrant of the magnet demagnetization  $B$ - $H$  curve gives magnetic properties per unit magnet volume. The external magnetic circuit, termed the *magnetic load*, together with the demagnetization curve, specify the magnet's operating point on its demagnetisation curve.

Consider the magnetic circuit shown in figure 27.21a, where a permanent magnet, with magnetization  $M$  oriented as shown, drives flux clockwise into an air gap via two steel high permeability pole pieces. The steel high permeability, in conjunction with their shape, confines most of the magnetic flux from the magnet (area  $A_m$ , length  $\ell_m$ ) into the air gap (area  $A_g$ , length  $\ell_g$ ). The lines of magnetic flux are shown in figure 27.21b, and magnified in figure 27.21c, where leakage fluxes outside the air gap volume, termed fringing, are shown.

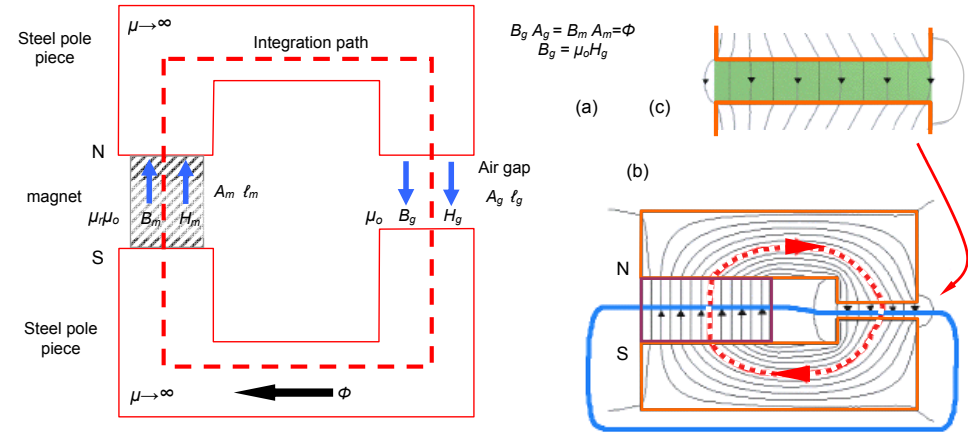


Figure 27.21. Magnet in an iron circuit with an air gap: (a) magnetic circuit, (b) magnetic circuit flux lines, and (c) air gap flux and its leakage.

### 27.6.1 Magnetic Circuit Equations

Flux conservation gives the integral of flux density through a closed surface (equivalent to Kirchhoff's current law), while Ampère's law yields the integral of magnetizing force around a closed loop (equivalent to Kirchhoff's voltage law). These can be applied to the magnetic circuit shown in figure 27.21a, to derive its load line and hence find the flux density levels within the magnetic circuit. Any convenient surface and loop may be chosen to achieve these. The subscript  $m$  denotes the permanent magnet and  $g$  denotes the air gap, while  $s$  is for both steel pole pieces, and  $\ell$  denotes the external regions into which some magnetic field leaks.

The magnetizing force  $H$  is integrated around the closed loop in the direction shown in red. No electrical currents passing through any surface spanning this loop, so  $i = 0$ . From figure 27.21a, as with Kirchhoff's voltage law around a closed loop, the closed integral has three series components:

$$\int_m H \cdot d\ell + \int_s H \cdot d\ell + \int_g H \cdot d\ell = 0$$

$$\mathbf{H}_m \ell_m + \mathbf{H}_s \ell_s + \mathbf{H}_g \ell_g = 0$$

Since the steel pole pieces have a significantly higher permeability than either the magnet or the air gap, their mmf contribution is small,  $\mathbf{H}_s \ell_s = 0$ . It is therefore a simplification to either ignore this small component,  $k_r = 1$  or to lump together the two external terms using a dimensionless *loss or reluctance factor*  $k_r$ , where  $k_r > 1$ . Outside the magnet, the magnetisation is zero, that is,  $M = 0$ . Each individual  $H\ell$  term is a magnetomotive force (mmf),  $H_m \ell_m$  being the mmf across the permanent magnet and  $H_g \ell_g$  the mmf across the air gap.

$$\mathbf{H}_m \ell_m + k_r \mathbf{H}_g \ell_g = 0$$

Rearranging yields

$$\mathbf{H}_m = -k_r \frac{\mathbf{H}_g \ell_g}{\ell_m} \quad (27.18)$$

Integration of the flux density  $B$  through the closed surface shown in blue (Kirchhoff's current law), yields:

$$\mathbf{B}_m \mathbf{A}_m - \mathbf{B}_s \mathbf{A}_s - \mathbf{B}_g \mathbf{A}_g = 0$$





magnet operating point ( $H_d, B_d$ ). This operating condition point of the magnet also leads to the operating point on the *intrinsic* curve (provided current  $i = 0$ ).

The intrinsic operating point ( $H_d, B_{di}$ ), has an intrinsic permeance coefficient  $P_{ci}$  which is obtained by adding 1 to the  $P_c$  value, that is,  $P_{ci} = P_c + 1$ . This is because, from equation (27.12),  $J = \mu_o(\mu_m - 1)H$ . The intrinsic demagnetising factor  $N_i$  is defined as  $N_i = 1/P_{ci}$ , thus both factors are determined by the shape of the magnetic circuit.

The  $P_c$  and  $P_{ci}$  load lines are shown as positive on demagnetisation curves but arithmetically are negative slopes. Figure 27.23 shows the case of the  $P_c$  and  $P_{ci}$  plotted for a NdFeB magnet when  $P_c = 1$  and  $P_{ci} = P_c + 1 = 2$ .

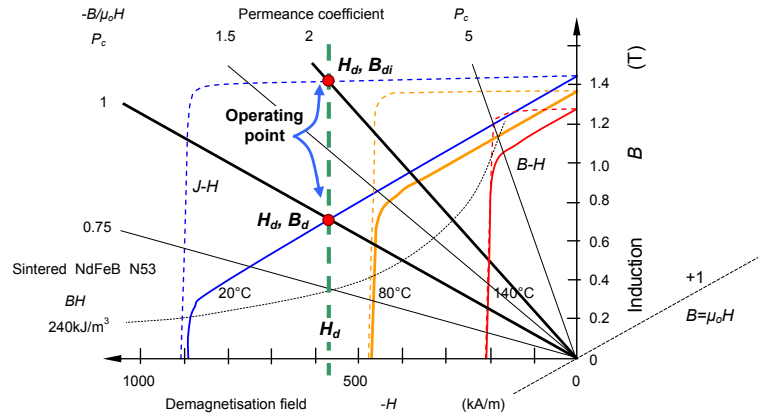


Figure 27.23. Permanent magnet and gapped circuit load lines (constant energy contour shown).

The  $P_{ci}$  and  $P_c$  straight load lines intersect the intrinsic and normal curves respectively with the same  $H_d$  value. This condition also occurs with an externally applied field (magnetising or demagnetizing influence) with or without air gap, which vertically shifts the common intersection origin of the  $P_{ci}$  and  $P_c$  lines by an amount related to the applied field (see section 27.6.3 and figure 27.28).

### Example 27.1: Magnet load dependant operating point

Using an equivalent circuit approach, figure 27.17c; determine the magnet operating point, assuming a linear demagnetization characteristic, when:

- A coil of  $N$  turns carries a demagnetising current  $i$ , wound on a core of infinite permeance, which is also the permeance seen by the magnet, and
- with zero coil current, an air gap of length  $\ell_g$  is introduced.

#### Solution

From equation (27.21) the equation describing the magnet linear demagnetising characteristic in terms of three flux components, shown in figure 27.24, is given by

$$\Phi_m = \phi_r + \frac{\mathfrak{F}_m}{\mathcal{R}_m}$$

where  $\Phi_m$  is the intrinsic flux,  $B_r A_m$ . This three-term equation is common to any magnetic circuit loading the magnet. In order to obtain an operating point, the external magnetic loading circuit is expressed in terms of the magnet flux  $\Phi_m$  and the magnet mmf  $\mathfrak{F}_m$ , that is,  $B_m$  and  $H_m$  respectively.

- When the magnet is loaded by an external mmf  $Ni$ , which can tend to have either a remagnetisation or demagnetisation effect, the load circuit in terms of magnetic variables related to  $H_m$  and  $B_m$  is

$$\mathfrak{F}_m = -Ni \quad (= H_m \ell_m)$$

Substituting  $\mathfrak{F}_m$  into the magnet load line equation yields magnet flux  $\Phi_m$ :

$$\Phi_m = \phi_r - \frac{Ni}{\mathcal{R}_m} \quad (= B_m A_m)$$

These two equations in terms of  $H_m$  and  $B_m$  give the operating point  $H_d, B_d$ , specifically

$$H_m = -\frac{Ni}{\ell_m} \quad (27.28)$$

and

$$B_m = B_r - \mu_m \frac{Ni}{\ell_m} \quad (27.29)$$

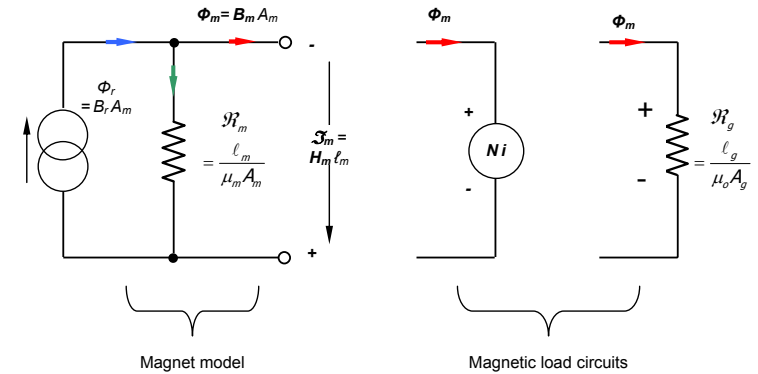


Figure 27.24. Magnetic model for Example 29.1

- When the magnet is loaded by the introduction of an air gap (progressively increased in length from initially zero to  $\ell_g$ ), the reluctance load circuit in terms of magnetic variables related to  $H_m$  and  $B_m$  is

$$\mathfrak{F}_m = -\Phi_m \mathcal{R}_g$$

Solving the magnet load line equation and the load equation for magnet flux  $\Phi_m$  and magnet mmf  $\mathfrak{F}_m$  yields:

$$\Phi_m = \frac{\phi_r}{1 + \frac{\mathcal{R}_g}{\mathcal{R}_m}} = \phi_r \frac{\mathcal{R}_m}{\mathcal{R}_g + \mathcal{R}_m}$$

(This is the same concept as a current (flux) dividing between two parallel-connected resistors (reluctances)).

Next, the magnet mmf is given by

$$\mathfrak{F}_m = -\phi_r \frac{\mathcal{R}_m \mathcal{R}_g}{\mathcal{R}_g + \mathcal{R}_m} = -\phi_r \frac{1}{\frac{1}{\mathcal{R}_g} + \frac{1}{\mathcal{R}_m}}$$

The two operating point expressions in terms of the specific operating point  $H_d$  and  $B_d$  are

$$B_d = \frac{B_r}{1 + \frac{\mathcal{R}_g}{\mathcal{R}_m}}$$

and

$$H_d = \frac{-B_r A_m}{\ell_m} \frac{\mathcal{R}_m \mathcal{R}_g}{\mathcal{R}_m + \mathcal{R}_g} = -H_c \frac{\mathcal{R}_g}{\mathcal{R}_m + \mathcal{R}_g} = -H_c \frac{1}{1 + \frac{\mathcal{R}_m}{\mathcal{R}_g}}$$

$H_b$  and  $B_b$  can be rearranged, using  $S = \mu_m \mathcal{R}_m / \mathcal{R}_g$ , to yield equations (27.22) and (27.23).

For a given  $B-H$  demagnetisation characteristic, the operating point is determined solely by the ratio of the air gap reluctance and magnet reluctance.

The load line solutions are shown in figure 27.25. The operating point on the  $B$ - $H$  characteristics will yield energy change per unit volume,  $B \times H$ , while the operating point on the  $BA_m$  versus  $H\ell_m$  axes yields energy,  $B \times H \times A_m \ell_m = B \times H \times \text{Volume}_m$ .

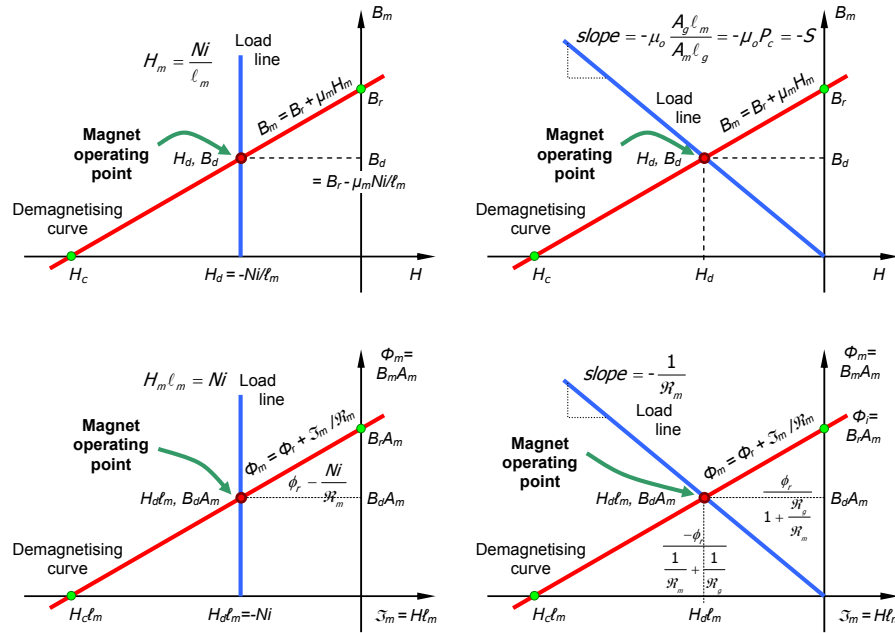


Figure 27.25. Magnetic load line characteristics for problem 27.1.

### 27.6.3 Demagnetizing field

When a permanent magnet provides flux to a single air gap via two steel pole pieces, the specific dimensions of the magnet and air gap yield a unique load line, as shown in blue on figure 27.22. The permanent magnet material is characterized by its demagnetization curve (in red), the intercept between the two giving an operating point with a unique combination of  $B_d$  and  $H_d$  for the magnet in this condition. If the air gap length  $\ell_g$  increases, the permeance, whence permeance coefficient, decrease, and the slope of the load line decreases and the magnet's operating point changes, moving down the curve. Figure 27.26 shows the magnetic circuit in figure 27.21 with the addition of a coil of  $N$  turns carrying current  $i$ , with a direction that opposes (tending to demagnetise) the magnet (of area  $A_m$ , length  $\ell_m$ ) from delivering flux into the air gap (of area  $A_g$ , length  $\ell_g$ ). By integrating the magnetizing force  $H$  around the closed loop shown in red (Ampère's Law), where the loop is linked  $N$  times by the current  $i$ , the derived magnetic circuit equations are

$$\int H \cdot d\ell = Ni$$

$$H_m \ell_m + k_r H_g \ell_g = Ni \quad (27.30)$$

Since

$$B_m A_m = k_r B_g A_g = k_r \mu_o H_g A_g$$

Then, assuming  $k_r/k_m=1$ :

$$B_m = -\mu_o \frac{A_g \ell_m}{A_m \ell_g} \left( H_m - \frac{Ni}{\ell_m} \right)$$

$$B_m = -S \left( H_m - \frac{Ni}{\ell_m} \right) = -\mu_o P_c \left( H_m - \frac{Ni}{\ell_m} \right) \quad (27.31)$$

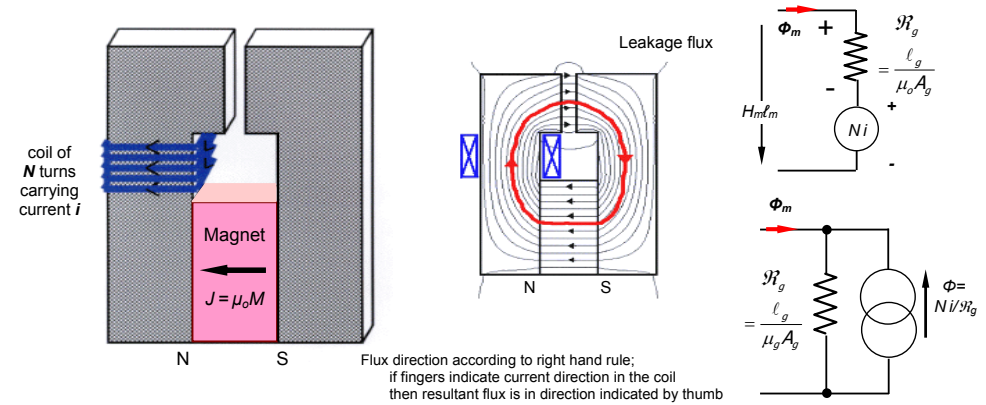


Figure 27.26. Magnetic circuit with a permanent magnet, an air gap, and an excitation coil.

When  $i=0$ , this equation reduces to equation (27.20).

The load line expression now contains an additional term (the last term) representing the demagnetizing field produced by the coil current. The slope  $B_m/\mu_o H_m$ , namely  $P_c$ , of the load line is unchanged, but its parallel position is determined by the coil excitation term  $Ni/\ell_m$  ( $H_m$  when  $B_m=0$  in equation (27.31)) as shown in figure 27.27.

Notice that the demagnetising field,  $H$ , is dependant on the magnet length,  $\ell_m$ . Consequently, the dimensionless  $B$ - $H$  operating point characteristics become dimension dependant when an external demagnetising field is introduced.

The induction at the load line operating point is given

$$B_m = B_r + \mu_m H_m \quad (27.32)$$

Substituting the magnet demagnetising field,  $H_m$  from equation (27.31) for  $H_d$  gives

$$B_m = B_r + \mu_m \left( \frac{Ni}{\ell_m} - \frac{B_m}{S} \right) \quad (27.33)$$

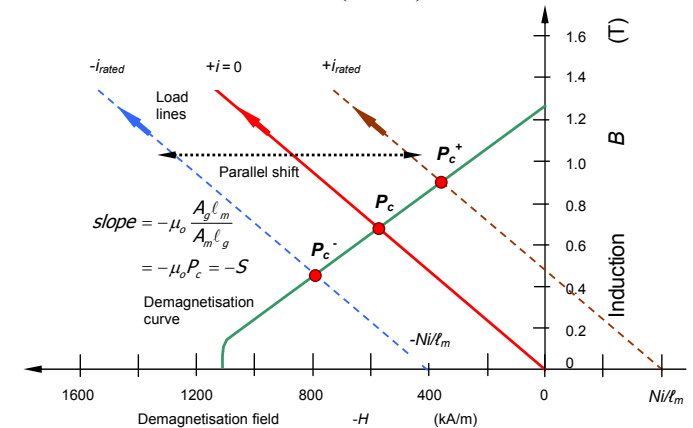


Figure 27.27. Magnetic circuit operating point shift due to coil excitation.

Isolating  $B_m$  and  $H_m$  gives the operating point  $H_d, B_d$ :

$$B_d = \frac{B_r + \mu_m \frac{Ni}{\ell_m}}{1 + \frac{\mu_m}{S}} = \frac{SB_r + \mu_m S \frac{Ni}{\ell_m}}{S + \mu_m} \quad (27.34)$$

$$H_d = -\frac{B_r + S \frac{Ni}{\ell_m}}{S + \mu_m} = -\frac{B_r + \mu_o P_c \frac{Ni}{\ell_m}}{\mu_o P_c + \mu_m} \quad (27.35)$$

Since linear models are used, by superposition, the operating point given by equations (27.34) and (27.35) reduce to:

- equations (27.29) and (27.28) when the air gap is reduced to zero such that  $\ell_g = 0$  and  $S \rightarrow \infty$ .
- equations (27.23) and (27.22) when the current is reduced to zero such that  $Ni = 0$ .

#### Operating point based on the intrinsic demagnetization curve

Applying a magnetising field  $H$  (a demagnetizing or magnetising influence) creates a shift in the load line equal to the applied field, as shown in figure 27.27. A common unnecessary practice is to use the intrinsic magnetisation curve  $J$  as the basis for operating point analysis. The intrinsic curve related load line has a slope  $P_{ci} = P_c + 1$ , but the  $H$  axis intercept is not coincident with the normal load line, unless  $i = 0$ , as is highlighted in the characteristics in figure 27.28. But the  $B$  and  $J$  axis intercepts are coincident.

Applying an  $H$  field creates a horizontal shift of the normal and intrinsic load lines. The horizontal shift of both load lines is related not only to the external  $Ni$ , but also magnetic circuit physical dimensions. The intrinsic operating point  $H_{mi}$ ,  $J_m$  is defined in terms the normal curve operating point related equations, namely equations (27.32) and (27.31), except  $\mu_m$  is replaced by  $\mu_m - \mu_o$  in the magnet equation and  $P_c$  is replaced by  $P_c + 1$  for the slope in the load equation (the load line equation constant term, that is, the  $B$  axis intercept, is unchanged). That is, from the magnet equation, equation (27.32)

$$\mathbf{J}_m = B_r + (\mu_m - \mu_o) \mathbf{H}_{mi} \quad (27.36)$$

and from equation (27.31) the load line is

$$\begin{aligned} \mathbf{B}_m &= -\mu_o P_c \left( \mathbf{H}_m - \frac{Ni}{\ell_m} \right) = -\mu_o P_c \mathbf{H}_m + \mu_o P_c \frac{Ni}{\ell_m} \\ \mathbf{J}_m &= -\mu_o (P_c + 1) \mathbf{H}_m + \mu_o P_c \frac{Ni}{\ell_m} \end{aligned} \quad (27.37)$$

In the load line equations (27.31) and (27.37),  $B_m(H_m = 0) = J_m(H_m = 0) = SNi/\ell_m$ . That is, both load line equations have the same flux density axis intercept, as illustrated in figure 27.28. Also, from the load line equations, the magnetising field  $H$  axis intercepts are  $Ni/\ell_m$  and  $P_c Ni/\ell_m P_{ci+1}$ , respectively, again shown in figure 27.28.

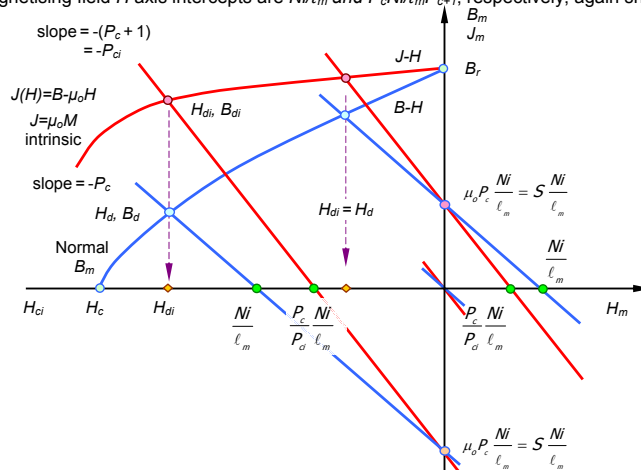


Figure 27.28. Intrinsic demagnetisation characteristic and load lines for a magnet experiencing coil excitation Ampere turns bias.

Solving for the operating point  $H_{mi}$ ,  $J_m$

$$J_m = B_r - \frac{\mu_m - \mu_o}{S + \mu_m} \left( S \frac{Ni}{\ell_m} + B_r \right) = \frac{(\mu_m + S) B_r - (\mu_m - \mu_o) S \frac{Ni}{\ell_m}}{S + \mu_m} \quad (27.38)$$

$$H_{mi} = -\frac{B_r + S \frac{Ni}{\ell_m}}{S + \mu_m} = H_m \quad (27.39)$$

where, comparing equations (27.35) and (27.39), as expected  $H_m = H_{mi}$ , since  $\mathbf{B}_m = \mathbf{J}_m + \mu_o \mathbf{H}_m$ . This equation can be confirmed by substitution of  $B_m$  from equation (27.34) and  $H_m$  from equation (27.35), with the resulting equation for  $\mathbf{J}_m$  being confirmed by equation (27.38).

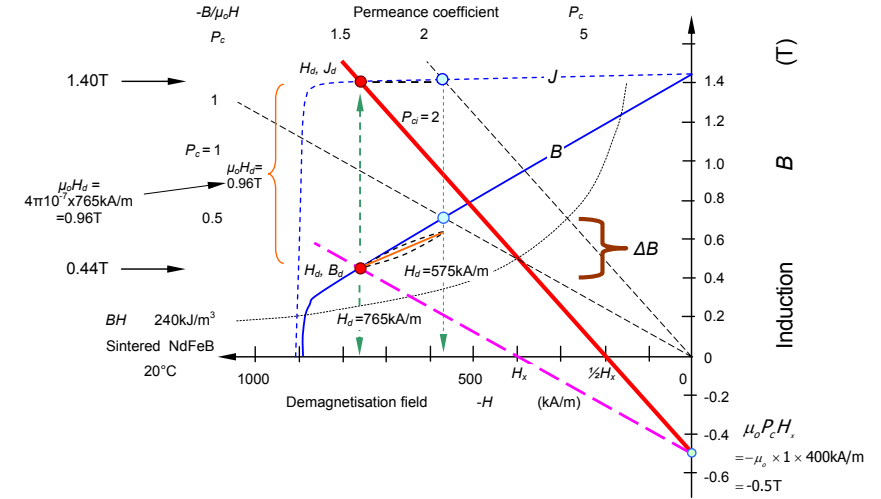


Figure 27.29. Magnetic circuit operating point shift and flux density loss  $\Delta B$  due to coil excitation ( $240\text{kJ/m}^3$  constant energy contour shown).

Figure 27.28 shows load lines for positive and negative coil currents. The intrinsic (magnetization) curve operating point can be used to find the normal curve operating point, and vice versa, since  $\mathbf{B}_m = \mathbf{J}_m + \mu_o \mathbf{H}_m$ , where  $\mathbf{H}_m$  is negative in the second quadrant). These load line and operating point characteristics are illustrated numerically in figure 27.29.

The commonality condition of the  $J/B$  axis intercept to both the normal and intrinsic related load lines is readily extracted (confirmed) using an equivalent circuit approach with  $H_m \ell_m = 0$ , as shown in figure 27.29. Analysis is based on the fact that since the mmf is zero on the Y axis, then the two model current sources must be of equal magnitudes, with one sourcing flux - the other sinking the same flux.

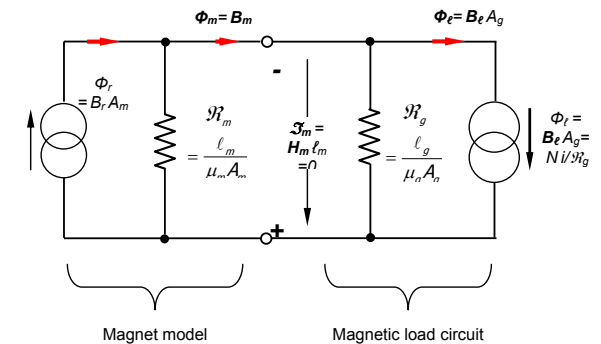


Figure 27.30. Magnetic circuit for demagnetisation and load line characteristic of a magnet experiencing an air gap and coil excitation Ampere turns bias.

That is, with zero net flux  $\Phi_r = \Phi_m$ , (the intrinsic and normal characteristics merge)

when  $H = 0$  then  $\phi_m = \phi_i$  thus  $B_m A_m = B_i A_g$

$$B_m(H=0) = \frac{B_i A_g}{A_m} = \frac{N i / \mathcal{R}_g}{A_m} = S \frac{N i}{\ell_g}$$

The operating point analysis approach adopted for externally applied fields  $H_i$  highlights that use of the intrinsic demagnetising curve  $J$  is redundant. This is expected since  $J$  is artificially generated from the normal demagnetising curve  $B$  by adding  $\mu_o H_m$ . However, use of  $J$ - $H$  does offer two features:

- All practical load line analysis can be performed in the second quadrant. Inspection of the  $B$ - $H$  characteristics in figures 27.39 and 27.40 show that the demagnetising curves project into the third quadrant, down to  $B_m = \mu_o H_m$ , and figure 27.38 shows operating point analysis in the third quadrant.
- The slope of the intrinsic characteristic gives a better visual indication of recoil and thermal properties – the closer to horizontal, the better the magnet. On the other hand, visualising how close the normal characteristic slope is to  $\mu_o$  is more subjective.

### 27.7 Generalising equivalent magnetic circuits in terms of permeances

The magnetic circuits considered thus far are simple circuits comprising a permanent magnet, a single air gap, and a coil – real magnetic circuits are generally more complex. For example, the effect of leakage flux was accounted for by the correction factor  $k_r$ , but this is an important effect, which normally requires calculation using estimates of leakage gap areas and lengths. Leakage flux follows paths in parallel with the main air gap in a magnetic circuit, which complicates the calculation of the load line slope thus  $A_g$  and  $\ell_g$ , must represent the net load experience by the magnet. While it is convenient to find the operating point of a magnet by the intersection of its load line with its demagnetization characteristic, determination of the effective load line for a complex magnetic circuit may be convoluted. Rather than resorting to a finite element CAD approach, an approach is to defining a characteristic equivalent magnetic circuit involving at most two parameters for each circuit component – its permeance and possibly an mmf source. A flux source and its permeance can be converted into an equivalent mmf source and the same permeance, and vice versa.

A magnetic circuit model represents each component in terms of its dimensions and material properties. Unity  $k_r$  and  $k_f$  correction factors are assumed so the load line in equation (27.31) can be rearranged, in terms of flux  $\Phi_m$ , gap permeance  $P_g = \mu_o A_g / \ell_g$ , and the magnets mmf  $\mathcal{F}_m = H_m \ell_m$ , as

$$\phi_m = B_m A_m = -P_g (\mathcal{F}_m - Ni) \quad (27.40)$$

This linear relationship is a *load line* with a slope based on the air gap load. By defining the mmf across the gap as  $\mathcal{F}_g = H_g \ell_g$ , using the magnetic equivalent to Ohms law, the gap mmf  $\mathcal{F}_g$  is obtained from

$$\phi_m = P_g \mathcal{F}_g \quad (27.41)$$

The gap permeance  $P_g$  is the slope of a flux versus mmf load line related either to the air gap or to the magnet. Hence in terms of the magnet's permeance:

$$\phi_m = P_m (\mathcal{F}_m + B_r \ell_m / \mu_o) \quad (27.42)$$

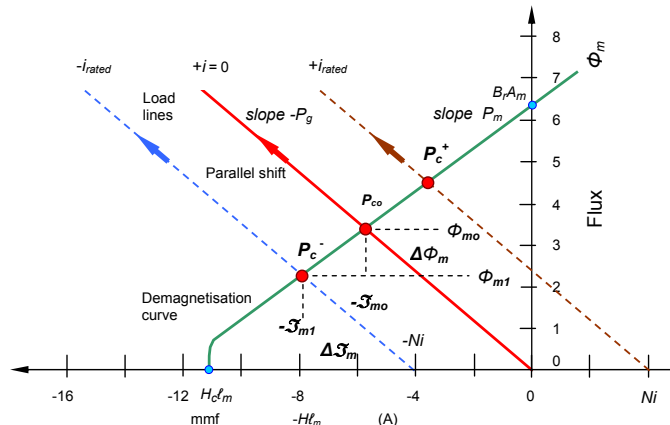


Figure 27.31. Demagnetisation curve and load line for a magnet, expressed as permeances.

Equations (27.40) and (27.42) are plotted in Figure 27.31, which is equivalent to Figure 27.27. The original demagnetization characteristic is scaled, the coercivity being multiplied by magnet length to give the equivalent mmf  $H_c \ell_m$ . The new load line is laterally displaced by the actual coil excitation  $Ni$ .

A coil current  $-i$  moves the magnet's operating point from  $-\mathcal{F}_{m0}$ ,  $\Phi_{m0}$  to  $-\mathcal{F}_{m1}$ ,  $\Phi_{m1}$ , and the following relationships can be deduced from the geometry of Figure 27.31:

$$\begin{aligned} P_m &= \frac{\phi_{m1} - \phi_{m0}}{\mathcal{F}_{m1} - \mathcal{F}_{m0}} = \frac{\Delta \phi_m}{\Delta \mathcal{F}_m} \\ P_g &= -\frac{\phi_{m0}}{\mathcal{F}_{m0}} \\ Ni &= \mathcal{F}_{m1} + \phi_{m1} / P_g \end{aligned} \quad (27.43)$$

The change in magnet flux due to coil excitation  $Ni$  can be found by rearranging the equalities in equation (27.43):

$$\Delta \phi_m = \phi_{m1} - \phi_{m0} = \frac{Ni}{P_m^{-1} + P_g^{-1}} \quad (27.44)$$

The magnetic circuit flux  $\Delta \Phi_m$  can be decreased by decreasing either of the slopes  $P_m$  or  $P_g$ , which can be achieved by raising  $\ell_m$  or  $\ell_g$  respectively – with the advantage that additional magnet length also stabilizes temperature incurred permanent flux changes in the magnet.

In some applications, it is desired to maximise the flux change for a given excitation, which requires increasing  $P_m$  and/or  $P_g$ . This can be achieved not by reducing  $\ell_m$  or  $\ell_g$ , but by raising the permeability of the component materials since the magnet operates on a recoil line of slope  $\mu_o \mu_{rc}$ , the definition of  $P_m$  is changed to include  $\mu_{rc}$ , thus equations (27.27) and (27.42) become

$$P_m = \frac{\mu_o \mu_{rc} A_m}{\ell_m} \quad (27.45)$$

$$\phi_m = P_m \left( \mathcal{F}_m + \frac{B_r \ell_m}{\mu_o \mu_{rc}} \right) \quad (27.46)$$

A magnet with a *high* recoil permeability  $\mu_o \mu_{rc}$ , such as one of the alnico materials can be used to improve  $\Delta \Phi_m$ , equation (27.44). Similarly, a soft magnetic material of relative permeability  $\mu$  can be considered for the *air gap*. The load line equation (27.40) is unchanged provided the gap component is defined by

$$B_g = \mu_o \mu H_g \quad (27.47)$$

$$P_g = \frac{\mu_o \mu A_g}{\ell_g} \quad (27.48)$$

These equations are more general versions of the equations for air, for which  $\mu = 1$ .

The alternative to plotting the two intersecting lines, is to solve equations (27.40) and (27.42) for  $\Phi_m$  and  $\mathcal{F}_m$

$$\phi_m = \frac{Ni + B_r \ell_m / \mu_o}{\mathcal{R}_m + \mathcal{R}_g} \quad (27.49)$$

The load circuit may comprise a number of parallel-connected components, such as a leakage path around a main gap, and because these are in parallel with the same mmf, equation (27.41) shows that the permeance  $P_g$  used as the slope of the load line will be the sum of the parallel component permeances.

$$P_g = P_1 + P_2 + \dots$$

That is, parallel connected reluctance components

$$P_g = \frac{1}{\mathcal{R}_g} = \frac{1}{\mathcal{R}_1} + \frac{1}{\mathcal{R}_2} + \dots$$

The magnetic load circuit may also comprise a number of series connected components, such as pole pieces and the main air gap, and because these experience the same flux, the net load line slope will be the sum of the series component reluctances

$$\mathcal{R}_g = \mathcal{R}_1 + \mathcal{R}_2 + \dots$$

or

$$\frac{1}{P_g} = \frac{1}{P_1} + \frac{1}{P_2} + \dots \quad P_g^{-1} = P_1^{-1} + P_2^{-1} + \dots$$



Note the analogy of magnetic flux, mmf, reluctance, and permeance to electrical current, emf, resistance, and conductance, respectively. Kirchhoff's voltage and current laws are analogous to mmfs around a closed loop and fluxes at a node, both summing to zero.

The electrical equivalent approach involves reducing the magnetic load circuit to a Thevenin or Norton equivalent. With a graphical approach based on fluxes and mmfs, swept areas are energy, in Joules.

Also, magnetic flux density,  $B$ , magnetic induction, and magnetic field are generally interchangeable, as are magnetic field intensity,  $H$ , magnetic field strength, and magnetizing/demagnetizing field.

## 27.8 Permanent magnet stability - Loss of magnetism

The ability of a permanent magnet to support an external magnetic field results from small magnetic domains *locked* in position by crystal anisotropy within the magnet material. Once established by initial magnetization, these positions are maintained until acted upon by forces exceeding those that retain the domains. The energy required to disturb the magnetic field produced by a magnet varies with material type. Permanent magnets can be produced with extremely high coercive forces  $H_c$  that maintains domain alignment in the presence of high external magnetic fields. Stability is described as the repeated magnetic performance of a material under specific conditions over the magnet's operational life.

Factors affecting magnet stability include time, temperature, reluctance changes, adverse fields, radiation, shock, stress, strain, and vibration.

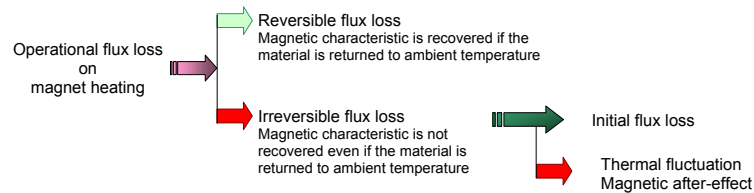


Figure 27.32. Types of magnetic output loss.

There are three types of loss in magnetic output due to temperature effects:

- **Reversible:** Flux output increases or decreases as temperature or mechanical stress changes. When the temperature returns to room temperature, the original flux output is observed. That is, there is no permanent change in the flux output of the magnet. If the magnetic properties vary with temperature without causing the magnet to operate at a  $B-H$  point below any *knee*, then the magnet only experiences a reversible loss, since the original operating condition is restored when the temperature returns to its normal level.

It is because the temperature coefficients of  $B_r$  and  $H_{ci}$  (and  $H_c$ ) are significantly different that the demagnetization curve develops a *knee* at elevated temperatures. Reversible losses cannot be eliminated by magnet stabilization and are described by the Reversible Temperature Coefficients  $\alpha$  and  $\beta$ , shown in table 27.2 and expressed as  $\%/^{\circ}\text{C}$ . To accommodate exposure to temperatures at which mild de-magnetization occurs, magnets can be *pre-stabilized*, that is, partially de-magnetized.

- **Irreversible, Recoverable:** With material temperature change (high or low), a critical operating parameter is reached resulting in the magnet being partially demagnetized. These losses are only recoverable to the original flux level by re-magnetization, and are not recovered when the temperature returns to its original value. These losses occur when the operating point of the magnet falls below the *knee* on the demagnetization curve. An efficient permanent magnet design should have a magnetic circuit in which the magnet operates at a permeance coefficient above the *knee* of the demagnetization curve at expected elevated temperatures. This will prevent performance variations at elevated temperatures.

- **Irreversible, Unrecoverable:** The magnet is exposed to high temperature or corrosion conditions that result in a permanent metallurgical degrading microstructural change. Table 27.2 shows critical temperatures for the various materials, where:

$T_c$  is the Curie temperature at which the elementary magnetic moments are randomized and the material is demagnetized; and

$T_{op}$  is the maximum practical operating temperature in air.

Different grades of each material exhibit values differing slightly from the values shown in Table 27.2. The maximum practical operating temperature is dependent on the operating point of the magnet in the magnetic circuit. The higher the operating point on the demagnetization curve, the higher the magnet allowable operating temperature.

Partially demagnetizing a loaded magnet by exposure to elevated temperatures in a controlled manner stabilizes the magnet with respect to temperature. The slight reduction in flux density improves a magnet's stability because domains with low commitment to orientation are the first to lose their orientation. A magnet thus *stabilized* will exhibit constant flux, with smaller variation between magnets, when exposed to equivalent or lower temperatures.

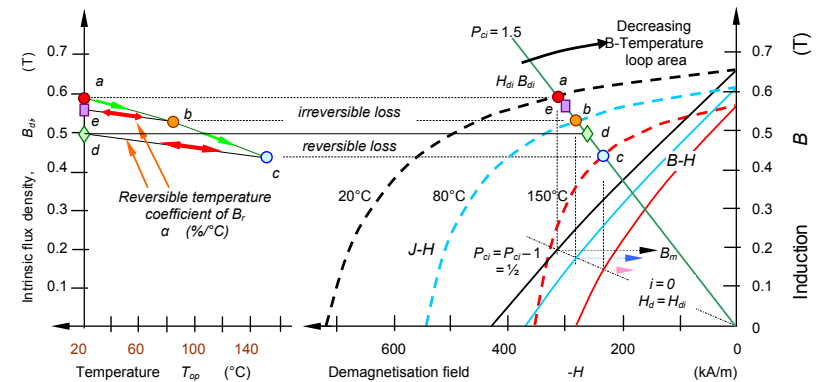


Figure 27.33. Induction reversible and irreversible losses of a bonded magnet after a temperature increase.

Flux losses in a magnet, due to increasing temperature, can be separated into irreversible and reversible loss components.

In figure 27.33, the reversible loss is the portion of loss at the elevated temperature which is recovered when the magnet returns to its initial temperature. The irreversible loss is the part of the flux loss at the elevated temperature that is not recovered when the magnet returns to its initial temperature. To regain the *reversible* portion of the lost flux, the magnet will have to be remagnetized. The irreversible loss in an application can be minimized by cycling the magnet above the expected operating temperature or by magnetically stabilizing (by approximately 1.5 to 2 times the expected irreversible loss).

For a given load condition,  $P_{cl}=1.5$  in figure 27.33, the flux change is partially reversible up to 80°C, although point b represents reduced magnet energy. On cooling to 20°C from 80°C, the operating point flux density is reduced to point e. If the temperature is then increased to 150°C, the magnet operating point shifts from point e through 'b' to point c, well below the characteristic knee, and the reversible change on subsequent cooling to 20°C is to operating point d, as determined by the reversible temperature coefficient,  $\alpha$ , independent of the load line slope. The reversible flux density is the difference between the flux at the operating points c and d. Irreversible loss is the flux density at the operating point 'a' minus the flux density at point 'd'. Continued cycling between 20°C and 150°C will incur minimal additional irreversible (recoverable) loss. The flux-density temperature-dependant loop-area in figure 27.33 decreases as the load line permeance increases.

Characteristic parameters  $B_r$ ,  $H_c$ ,  $H_{ci}$ , and  $\mu_{rc}$  (recoil relative permeability) specify the second quadrant of the hysteresis loop.

Changes in temperature affect both flux output, which is proportional to  $B_r$ , and resistance to demagnetization, which is proportional to  $H_{ci}$ . The amount to which these change are called the *reversible temperature coefficient of induction*  $B_r$ , namely  $\alpha$  or the *reversible temperature coefficient of coercivity*  $H_{ci}$ , namely  $\beta$ . These are sometimes referred to as  $\alpha$  (alpha) or  $\beta$  (beta) respectively and typical values for different hard magnetic material are shown in Table 27.2. Temperature variation of maximum energy product  $BH_{max}$  and intrinsic coercivity  $H_{ci}$  are shown on figure 27.34. The changes in  $B_r$  and  $H_{ci}$  as a function of temperature are not linear. A negative value indicates a loss of  $B_r$  or  $H_{ci}$  with an increase in temperature. Note that intrinsic coercivity for ferrites increases with temperature.

Irreversible-Recoverable loss occurs when the combination of: temperature extreme,  $H_{ci}$ , reversible temperature coefficient, and applied demagnetizing field exceeds the magnet's ability to remain fully magnetized. This occurs when the magnet is at an operating point below the *knee* of the curve.

In addition to the change in  $B_r$  and  $H_{ci}$  values, the demagnetization curve undergoes a subtle temperature induced change. The recoil slope of all magnetic materials increases, to varying extents, with increased temperature. For example, the recoil slope for SmCo increases from 1.082 at room temperature by 4%, up to 1.210 at 200°C.

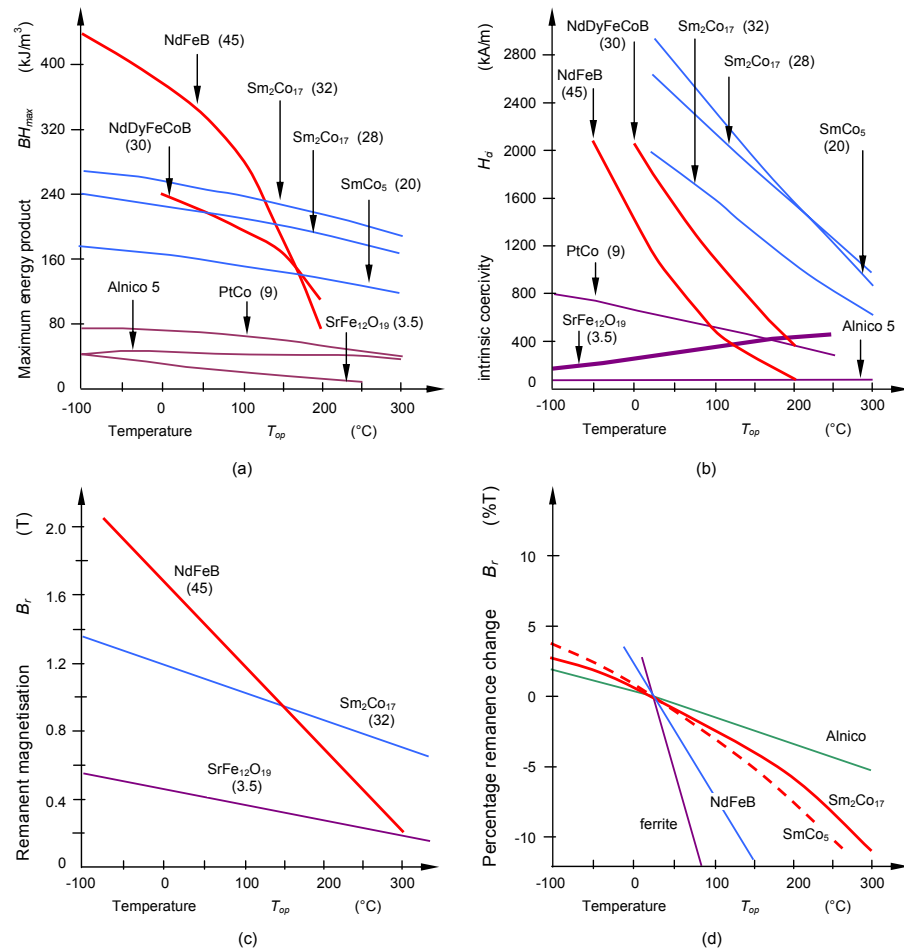


Figure 27.34. Temperature effects on: (a) maximum energy  $BH_{max}$ , (b) intrinsic coercivity  $H_{ci}$ , (c) remanence  $B_r$ , where material room temperature maximum energy product is shown in brackets, and (d) percentage reversible changes in remanence.

#### Time

The effect of time on permanent magnets is minimal. These changes, known as *magnetic creep*, occur as less stable domains are affected by fluctuations in thermal or magnetic energy, even in a thermally stable environment. This variation is reduced as the number of unstable domains decreases. Rare Earth magnets are not as likely to experience this effect because of their extremely high coercivities. Irreversible aging losses increase with increasing temperature and decreasing permeance, with the vast majority of the decrease occurring within the first year after magnetisation. Below 50°C, irreversible loss for magnets over a ten-year period is less than 1/2% for low permeance coefficients, ranging from virtually zero for Samarium Cobalt materials to less than 3% for Alnico 5 materials.

#### Radiation

SmCo materials, and especially Sm<sub>2</sub>Co<sub>17</sub>, withstand radiation up to 40 times better than NdFeB materials. SmCo exhibits significant demagnetization when irradiated with a proton beam of  $10^5$  to  $10^{10}$  rads. NdFeB will lose all of its magnetization at a dose of  $7 \times 10^7$  rads, and 50% at a dose of  $4 \times 10^6$  rads. Magnet materials with high  $H_{ci}$  values, operated at high permeance coefficients,  $P_c$ , should be used in radiation environments, and then shielded from direct heavy particle irradiation. Similar to temperature effects, stabilization can be achieved by pre-exposure to expected radiation levels.

#### Shock, stress, and vibration

Rigid magnet materials are brittle in nature, and can easily be damaged or chipped by improper handling. Samarium Cobalt in particular is a fragile material. Thermal shock when ferrites and Samarium Cobalt magnets exposed to high temperature gradients can fracture within the material.

### 27.9 Recoil operation and associated magnetisation losses

#### 27.9.1 Losses due to reverse magnetic fields

##### i. Increased reverse field – increased air gap

For dynamic systems with changing operating straight lines (for example, motors) shearing should be selected so that the permanent magnet's working point remains within the straight-line range of the demagnetization curve. The reason is to ensure stability from outside field and temperature influences, as shown in figure 27.35. The working point shifts to a larger opposing field strength, for example, from  $P_1$  to  $P_2$ , if the air gap in a magnet system is increased. If the gap change is reversed, the original operating point  $P_1$  can only be recovered if  $P_2$  is within the linear section of the demagnetization curve, as in figure 27.35a. At room temperature, +20°C, the magnetic material suffers no apparent irreversible loss of its magnetization, even if continuously cycled between  $P_1$  and  $P_2$ . Provided the demagnetization curve is linear and  $M$  is constant, there are no irreversible losses and the operating point traverses the same characteristic reversibly as implied by:

$$\begin{aligned} \text{from } B_m &= \mu_o (H_m + M) \\ \frac{dB_m}{dH_m} &= \mu_o \left( 1 + \frac{dM}{dH_m} \right) \\ M &= \text{constant} \rightarrow \frac{dB_m}{dH_m} = \mu_o \end{aligned}$$

However, as the air gap is further increased past  $P_2$ , to operating point  $P_3$ , as shown in figure 27.35b, to below the knee of the demagnetization curve, irreversible losses arise. On reducing the air gap, the working point shifts to  $P_4$  on an inner recoil return path with a correspondingly lower flux density. The slope of this return path is referred to as permanent permeability. If the demagnetization exceeds the intrinsic coercivity  $-H_{ci}$ , then the reversal of  $M$  is complete. An irreversible loss has occurred, and the magnet can only be returned to its original condition if it is fully remagnetised.

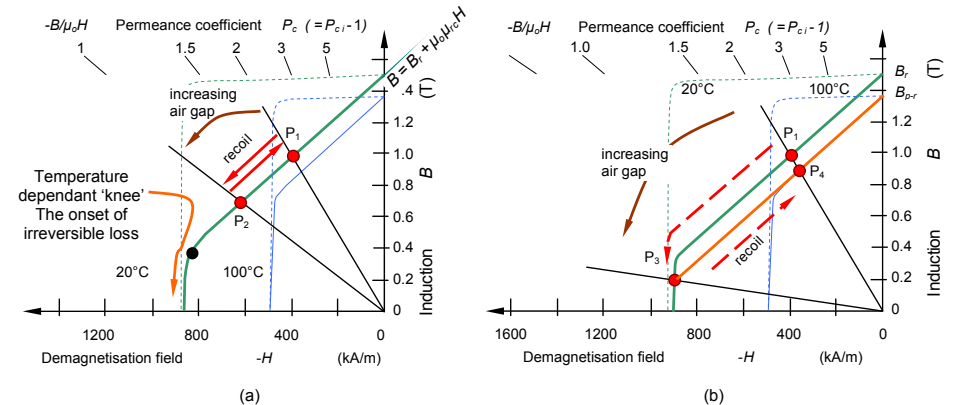


Figure 27.35. Demagnetisation operation for varying air gap.

Remagnetisation is not practical for a magnet installed in a magnetic circuit (a motor), but if the loss of magnetization is minimal, the device can be designed to operate within its *major* demagnetization curve at a lower flux level. In figure 27.35b, the orange line from  $P_3$  that the operating point follows to  $P_4$ , on the original load line, is called a *recoil line*, which is actually part of a *minor* magnetization curve. The excursion of the operating point in recoil does not traverse a line, but a narrow hysteresis loop, as shown in figure 27.16, although a line of slope  $\mu_o \mu_{rc}$  is assumed for calculation purposes. All recoil lines have approximately the same slope, termed *recoil relative permeability*,  $\mu_{rc}$ . In many magnetic materials the slope is near constant whatever the point of origin and is approximated by the slope of the major

hysteresis loop at point  $H = 0$  where  $B = B_r$ . If a recoil line is projected to intercept the  $B_m$  axis at a reduced 'remanence' of  $B_{r-r} = \mu_o M_r$ , then the equation (similar to that for the demagnetization curve) describing it is:

$$B_m = \mu_o (\mu_r H_m + M_r)$$

A permanent magnet's uniform magnetization  $M$  produces a magnetizing force  $H$  which establishes a flux density  $B$ . For operation in recoil, it is the recoil line (rather than the demagnetization curve) and the load line which are the two characteristic equations describing the magnet in a particular magnetic circuit, their intercept giving a unique magnet operating point with a specific  $B_m$  and  $H_m$  for the magnet. In terms of the  $B_m$  versus  $H_m$  graph in figure 27.35b, the recoil line replaces the demagnetization curve.

## ii. Reverse applied field - Ampere turns bias

Irreversible loss caused by the same effect as an excessive air gap, can result if an excessive demagnetizing field is applied to a magnet via current in a coil wound as shown in figure 27.26. When a coil biasing current flows, creating  $-400\text{kA/m}$ , the operating point moves from  $P_1$  under zero current to  $P_2$ , as shown in figure 27.36a. When the current ceases the operating point recoils, without magnetisation losses to the original operating point  $P_1$ . As shown, a higher current, equivalent to  $-800\text{kA/m}$ , causes the operating point to move past  $P_2$ , around the *knee*, down to the working point  $P_3$ . On winding current cessation, recoil to point  $P_4$  results, such that the magnet has suffered irreversible loss of its magnetisation, with the remanence flux density decreasing from  $B_r$  to  $B_{r-r}$  on figure 27.36a.

Since  $H = Ni/\ell_m$ , the  $H$  axis excursion of the load line is proportional to the coil's excitation  $i$  and is also inversely proportional to the magnet's length  $\ell_m$ . Additional magnet length  $\ell_m$ , hence additional magnet material volume, can be used to stabilize a magnet against irreversible loss in two ways:

- it raises the slope of the load line, so the magnet's operating point at any temperature or ampere turns bias, is further away from the *knee*; and
- for a given coil excitation, it reduces the rate at which the load line approaches the *knee* (with its associated loss of magnetization).

The penalty of such temperature stability mechanisms is the use of additional magnet material. Since an irreversible loss has occurred in operating at point  $P_3$ , the magnet can only be returned to its original condition if it is fully re-magnetized. Figure 27.36b show a  $J$ - $H$  curve recoil calculation.

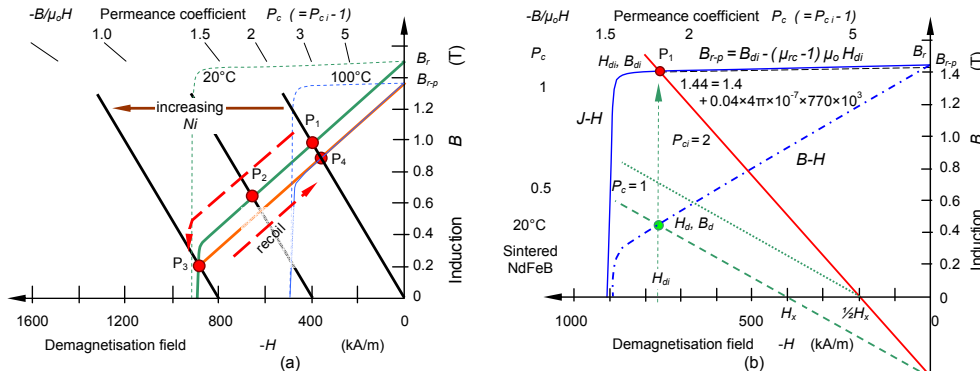


Figure 27.36. Demagnetisation for varying Ampere turns demagnetising bias.

## 27.9.2 Demagnetisation due to temperature increase

### i. Increasing air gap – no external reverse field

Both induction  $B$  (and specifically remanent flux density  $B_r$ ) and intrinsic coercivity  $H_{ci}$ , whence maximum energy product, are temperature (non-linear) dependant. Temperature variation of maximum energy product, remanence  $B_r$ , and intrinsic coercivity  $H_{ci}$  are shown in figure 27.34 for various hard magnetic materials. The relationships are referred to as the Reversible Temperature Coefficients,  $\alpha$  for  $B_r$  and  $\beta$ ,  $\beta$  for  $H_{ci}$ , in % change per  $^{\circ}\text{C}$ , over a specified temperature range, as indicated by equations (27.3) and (27.4). Both coefficients are negative, except in the case of ferrite ( $\text{SrFe}_{12}\text{O}_{19}$ ), where  $\beta$  is positive, as shown in figure 27.34b. Consequently, the demagnetization curves of permanent magnets are temperature dependent, as shown in figure 27.37. Notice that each diagonal magnet line has a bend in it, referred to as the *knee* in the curve, and the knee gets higher with increased temperature.

An increase in temperature causes the working point to shift along the load line radially towards the origin. In figure 27.37a, for a high permeance,  $P_{c1}$ , as the temperature increases the operating point progressively moves from point  $P_w$ , to  $P_x$  then to  $P_y$  and finally to  $P_z$ . Since the operating points stay within the linear region (above the *knee*) of each demagnetization curve, each reduction in flux density is reversible, that is, after cooling the flux density returns to its original value associated with operating point  $P_w$ . The magnet is operating in its safe linear region and performs as expected.

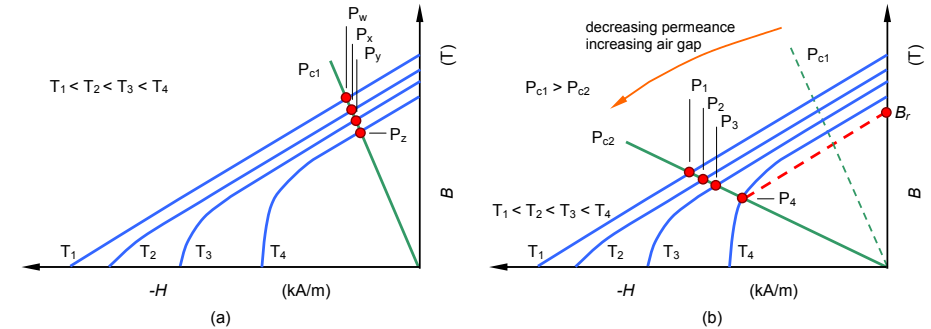


Figure 27.37. Load line and operating point temperature dependence.

In figure 27.37b, for the load line with decreased permeance  $P_{c2}$ , as the temperature increases the operating point moves towards the origin, from point  $P_1$ , to  $P_2$  then to  $P_3$  and finally to  $P_4$ . With an increased magnet temperature  $T_4$ , the operating point  $P_4$  is below the *knee* of the demagnetization curve and the change in flux density is irreversible, that is, after cooling the flux density does not return to its original value associated with  $P_1$ . Only a portion of the magnetisation is reversible, as shown in figure 27.33. If the air gap is closed, the operating point recoils to a lower remanence flux density,  $B_r$  at  $T_4$ . As in figure 27.33, equation (27.3) can be used to estimate the remanence flux density for temperatures  $T_1$ ,  $T_2$ , and  $T_3$ , based on the projected  $B_r$  for temperature  $T_4$ . With any operating point below the *knee* on the demagnetising curve,  $P_4$ , the irreversible component (demagnetisation - irreversible magnetic losses) and can only be recovered by remagnetisation. The knee flux density level increases with increasing temperature, reflecting the material's increasing vulnerability to demagnetization at higher temperatures.

To avoid irreversible changes in the flux density through temperature fluctuations, the operating point must remain within the upper linear section of the demagnetization curve over the entire temperature range in which the magnet is to operate. A permanent magnet can be completely demagnetized by heating to a temperature above the Curie temperature  $T_c$ . After cooling to the initial temperature, the initial state of magnetization can be reproduced by remagnetising, provided heating has not caused changes in the material microstructure.

### ii. Reverse field – constant air gap

Consider a coil excitation that cycles the load line between zero and  $H_x = -320\text{kA/m}$ , but with the magnet now operating at  $+150^{\circ}\text{C}$  rather than  $+20^{\circ}\text{C}$ . The diagram in figure 27.38b shows that, at higher demagnetizing fields, the operating point passes the *knee* and the magnet suffer an irreversible loss of magnetization. The operating point does return up along the major demagnetization curve, but follows a recoil path within (below) this characteristic. Since an irreversible loss has occurred, the magnet only returns to its original condition if it is fully remagnetized.

In figure 27.38a,  $N_o$  is the operating point on the  $B$ - $H$  (normal)  $20^{\circ}\text{C}$  curve for the operating line  $P_c = 1$  derived from the magnetic circuit and the corresponding point on the  $J$ - $H$  (intrinsic)  $20^{\circ}\text{C}$  curve, with  $P_{ci} = P_c + 1 = 2$ , is point  $I_o$ . Both operating points have the same magnet field co-ordinate,  $H_{io} = H_o$ , when no external bias field is applied, that is,  $i = 0$  in any coil.

When a demagnetizing field  $H_x = -320\text{kA/m}$  is applied to the magnet (via current in a coil), at  $20^{\circ}\text{C}$ , the operating point moves from  $I_o$  to the operating line  $P_{ci} = 2$  at point  $I_1$  on the  $J$ - $H$  curve, equivalent to point  $N_1$  on the  $B$ - $H$  curve. The field intensities for the normal and intrinsic operating points remain the same even when  $i \neq 0$ . The operating points shift (virtually) back to  $I_o$  and  $N_o$  as the demagnetizing (current) field is reduced to zero, that is,  $i = 0$ . The projected remanence flux density decreases slightly during what is effectively a stabilisation process.

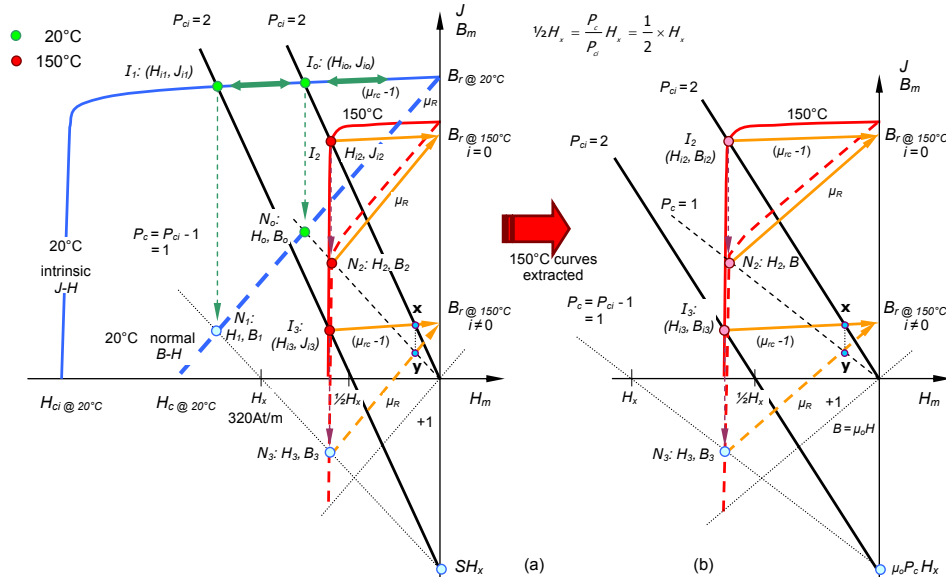


Figure 27.38. Losses at elevated temperatures with demagnetisation in an external field at: (a) 20°C and 150°C with (b) 150°C operation extracted and expanded.

Figure 27.38b shows the operating point trajectories at a higher temperature, 150°C, where coil excitation causes the operating points to locate well below the knee, in fact in the third quadrant. With no field bias,  $i = 0$ , operation at points  $I_2 : N_2$  results in a small reduction in  $B_r$ . Having applied a coil current corresponding to  $H_x = -320 \text{ kA/m}$ , removal of the demagnetising bias field leaves the magnet with severe irreversible loss, with the operating points having reduced from  $I_3 : N_3$  to  $x : y$ , respectively. When a bias field is applied, either the intrinsic or normal demagnetising curves readily illustrate the loss.

#### Magnet Stability: irreversible thermal losses

The variation in a magnet's remanence  $B_r$  in figure 27.34c is approximately linear with increasing temperature and reversible up to its specific transition temperature. Once the change becomes non-linear, an irreversible loss of  $B_r$  occurs. This transition is associated with the onset of a reversal of the material's magnetization, and is represented by a *knee* in the demagnetization curve. The load line intersection with the demagnetization curve identifies the operating point ( $B_d$ ,  $H_d$ ) for the magnet supplying flux to a given magnetic circuit. As with the remanence, if the operating point is above the demagnetization curve *knee*, changes in the magnet's condition are reversible, but if the operating point falls below the *knee*, irreversible loss of part of  $B_r$  occurs.

The position of the *knee*, that is, the threshold for a irreversible loss component arising, depends on temperature, and as the temperature increases, the operating point may readily fall below the *knee* of the applicable demagnetization curve.

#### Example 27.2: Magnet load and temperature dependant operating point

Consider the following two magnets to be operating in a magnetic circuit with a load line as specified:

- iii. A fully dense anisotropic neodymium-iron-boron magnet with demagnetization curves as shown in figure 27.39, is temperature cycled between 20°C and 120°C. Determine the subsequent remanence flux density at 20°C and 120°C, if the load line slope is -1.5; and
- iv. A ceramic ferrite magnet with demagnetization curves as shown in figure 27.40, is temperature cycled between -60°C and 60°C. Determine the subsequent remanence flux density at -60°C and 60°C, if the load line slope is -1.

#### Solution

i. Sintered NdFeB

a. Reversible loss:- magnet operating temperature is cycled between +20°C and +80°C.

The second quadrant demagnetization  $B$ - $H$  curves in figure 27.39 develop a *knee* in the second quadrant above about 0°C, with a knee apparent for a temperature of +20°C. The magnet's operating point at the intersection of the load line,  $P_{ci} = -1.5$ , and the +80°C curve is above the *knee*, so no irreversible loss occurs for temperatures up to about 85°C. This reversible change is illustrated on the right of the  $B$ -axis, with magnet flux cycling between point 'a' at 20°C, to point 'b' at +60°C and point 'c' at +80°C.

b. Irreversible loss:- magnet operating temperature is cycled between +20°C and +120°C.

The magnet begins again at 20°C, point 'a', is heated first to +60°C, point 'b', then to +80°C, point 'c', and then up to +100°C - point 'd'. However, the intersection of the load line and the +100°C demagnetisation curve is now below the *knee*, and an irreversible loss of magnet flux occurs (starting just above +80°C). When the magnet is cooled to 20°C, the magnet's operating point on the load line, point 'd'', is no longer on the 20°C demagnetization curve, but at some point within the major  $B$ - $H$  curve in the second quadrant. If rather than cooling from 100°C, the magnet is heated to 120°C, the operating point moves to point 'e'. Now when the magnet is cooled to 20°C, operating point 'e'' results. The magnet is no longer fully magnetised, having suffered irreversible loss. Operation up to 120°C is stabilised but it must be re-magnetized to saturate the material's magnetization once again, and to regain operation on the major  $B$ - $H$  curve.

In summary:

'a' to 'b' to 'c' is a linear change and is reversible, as described in part a.

'c' to 'd' then to 'e' are non-linear changes and represent partial irreversible loss.

'e' to 'e'' is a linear change and is reversible, but with reduced magnetic properties.

'e'' to 'a' illustrates restoration of full magnetic properties after re-magnetizing the magnet.

The recoil lines in figure 27.39 from 'e' and 'e'' predict remanence fluxes of 0.87T and 1.06T at +120°C and +20°C, respectively. The magnet is now temperature stabilised for operation up to 120°C.

As the load line slope is increased, given the shape of these demagnetization curves, the transition temperature to an irreversible loss also progressively increases. A greater load line slope raises the flux produced by the magnet to the magnetic circuit and helps to stabilize the magnet against irreversible loss thermal effects.

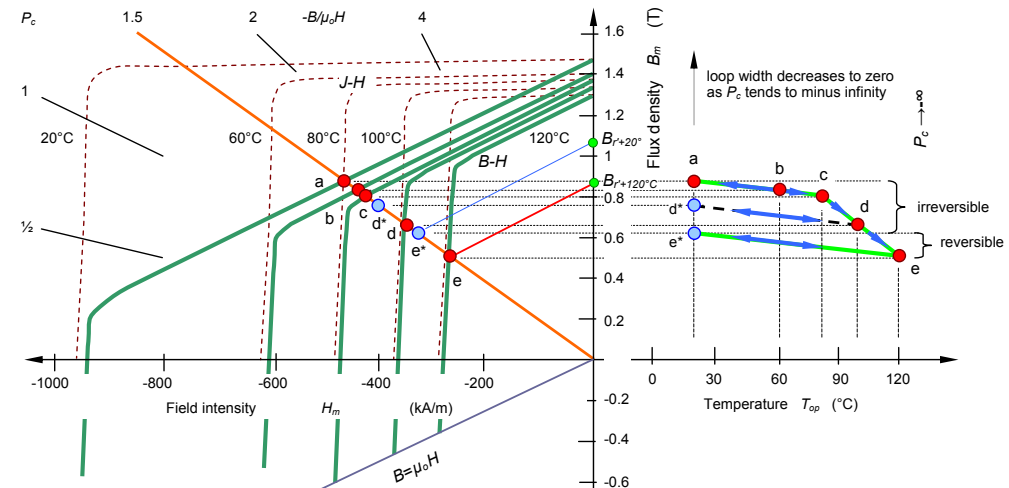


Figure 27.39. Interpretation of irreversible and reversible temperature losses for a NdFeB magnet.

ii. Ferrite

When considering the change in coercivity,  $H_{ci}$  decreases with temperature both for samarium-cobalt and for neodymium-iron-boron magnets;  $H_{ci}$  increases with temperature in the case of ceramic ferrites (being based only on magneto-crystalline anisotropy). This means that the *knee* of the demagnetization curve can arise as the temperature falls. The magnet is cycled from +20°C to -60°C and back again. Figure 27.40 illustrates that, with a load line slope of -1, the transition to an irreversible loss occurs when the temperature falls below about -20°C.



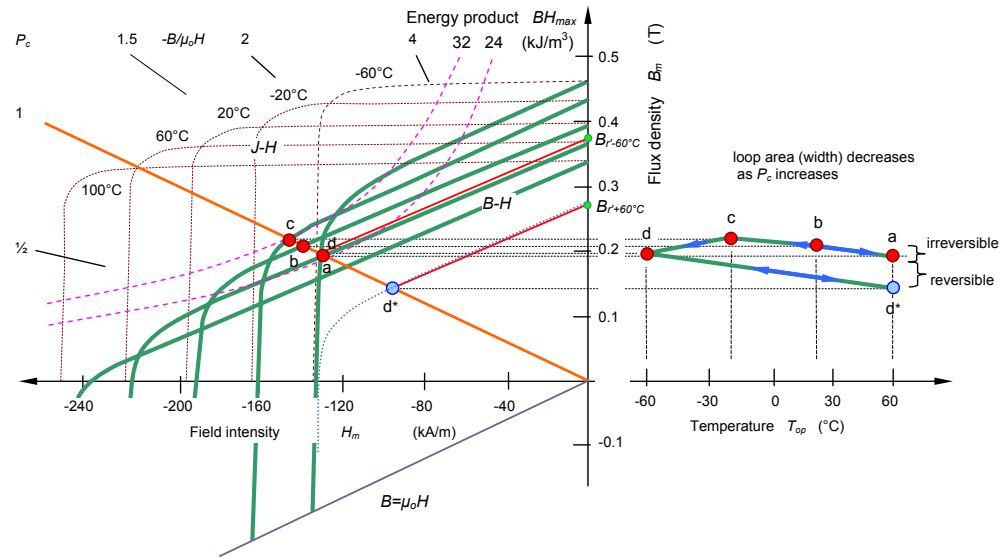


Figure 27.40. Interpretation of irreversible and reversible temperature losses for a ferrite magnet.

The sequence of thermal events is:

- a to b to c is a linear reversible change between +60°C and -20°C.
- c to d is a non-linear irreversible change from -20°C down to -60°C.
- c to d to d\* is a linear reversible change from -60°C back to +60°C, but with reduced magnetism.

The recoil lines in figure 27.40 from 'd' and 'd\*' predict remanence fluxes of 0.38T (reduced from 0.46T) and 0.275T (reduced from 0.37T) at -60°C and +60°C, respectively.

If operation of a magnet over its working temperature range is predicted to introduce unacceptable irreversible loss, then the magnet application should be reassessed to increase the load line slope and stabilize its operation without degrading the properties.

### 27.10 Energy transfer

One or more air gaps introduced into a magnetic circuit enable useful work to be performed. The mechanical energy used to separate a magnet from soft iron, there-in creating an air gap, is stored as potential energy within that air gap and the magnet. This moves the point of operation on the intrinsic curve in the second quadrant. The normal curve represents the energy output of the magnet and is used during magnet design. If the iron in the circuit is completely removed, the air gap becomes large and the operating point of the curve approaches  $H_c$  (the normal coercivity) in the second quadrant and the induction  $B$  approaches zero.

If the air gap is closed again, the stored potential energy is used to perform the work of bringing the magnet and the iron together. However, the operating point does not return to  $B_r$ . The magnet recoils along a so-called minor hysteresis loop to a point below  $B_r$ , figure 27.16c. Repeated opening and closing of the air gap will result in the magnet cycling along this minor hysteresis loop. The average slope of the minor loop is the recoil permeability,  $\mu_0\mu_r$ .

Section 27.6 presented the method for determining a magnet's operating point ( $B_m$ ,  $H_m$ ), with which is associated stored energy, that may be instrumental in the conversion of electrical and mechanical work. In figure 27.41, the energy stored in volume  $V_m$  of a magnet, leading to the change in energy is:

$$-\int_a^b B dH - \int_a^b H dB = -[BH]_a^b \quad (27.50)$$

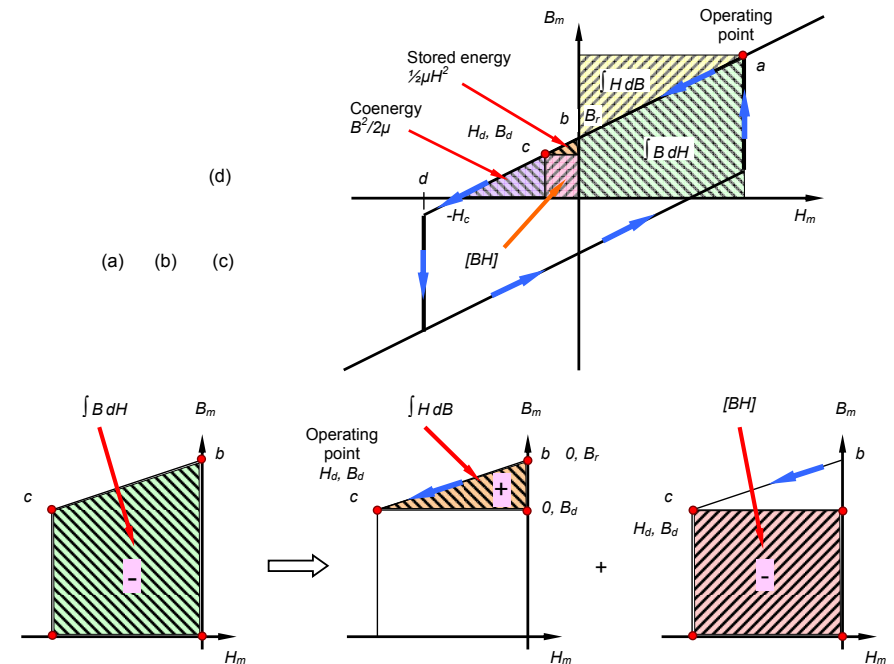


Figure 27.41. Graphical demagnetisation change per unit volume: (a) applied field energy, (b) magnet kinetic energy, (c) total potential energy, and energy changes around the B-H loop.

The first term is the work done (energy) by the applied field, the second is the internal kinetic energy stored, and the sum equals the total potential energy per unit magnet material volume.

After being magnetized to saturation, point a, the potential energy is reduced to zero at remanence ( $H_m = 0$  at point b); the magnet conditions must move into the second quadrant to deliver its stored energy. Figure 27.41 shows how the three energy density components of equation (27.50) develop as the magnet moves from point b to point c. Considering the direction of integration, the three areas sum according to equation (27.50), namely, the work done as a result of the applied field  $[BdH]$  equalling the change in potential energy  $[BH]$  plus the release of kinetic energy from the magnet  $[HdB]$ . The higher the potential energy of a permanent magnet, the greater the release of its kinetic energy in establishing an external field. While magnet developments have focussed on improving the available (potential) energy density, it also requires a corresponding increase in the work to change the operating condition of the material. Consequently, the basis of most devices using high-energy magnets is a design that keeps the magnet close to a unique operating point with a minimum of dynamic operation.

The kinetic energy released by a magnet of volume  $V_m$ , which is operating at a typical point c, ( $H_c$ ,  $B_c$ ), in figure 27.41, is

$$W = V_m \int H dB \quad (= \int F d\ell) \quad (27.51)$$

This triangular area in figure 27.41b, the kinetic energy being released by the magnet, is the magnetic energy stored in the magnet reluctance, equivalent to  $w = \frac{1}{2}LI^2$ . For a given operating point, ( $H_c$ ,  $B_c$ ), this energy can be derived from the Thevenin or Norton equivalent circuits, specifically the magnetic energy in the magnet reluctance:

$$w = \frac{1}{2}\phi_m^2 \mathfrak{R}_m = \frac{1}{2}\mathfrak{I}^2 / \mathfrak{R}_m$$

Energy being delivered, is readily derived from the Norton equivalent circuit in figure 27.17c, where

$$w = \frac{1}{2}\phi_m^2 \mathfrak{R}_m = \frac{1}{2}(B_m A_m)^2 \frac{\ell_m}{\mu_m A_m} = \frac{1}{2} \frac{B_m^2}{\mu_m} A_m \ell_m = \frac{1}{2} B_m H_m \times \text{volume}$$

This energy is only released if work is done, that is, power is released or absorbed. Mechanically this could be an air gap change or electrically a voltage developed across a current carrying coil. Note coil current alone is insufficient, the voltage component can only be produced by a changing flux, from  $v = N \times d\Phi/dt$ , which implies a magnet operating point flux change,  $d\Phi$ .

#### Energy delivered to an air gap

The following equation may be applied to the field in any volume, even an air gap  $V_g$ , where  $B_g = \mu_0 H_g$ , that is:

$$\begin{aligned} W_g &= V_g \int_{\mu_0} B_g dB \\ &= \frac{B_g^2}{2\mu_0} V_g = \frac{1}{2} B_g H_g V_g = \frac{1}{2} \mu_0 H_g^2 V_g \end{aligned} \quad (27.52)$$

If this is the air gap in the magnetic circuit of figure 27.21, then equations (27.18) and (27.19) may be used in equation (27.52) to show that all the energy released from the magnet is delivered into the gap:

$$W_g = \frac{1}{2} B_g H_g V_g = -\frac{1}{2} B_m H_m V_m \quad (27.53)$$

Note the energy density delivered to the gap is half the magnet's energy product. The work done by the externally applied field in establishing the air gap energy has two components as shown in figure 27.42a. The first component is the pu volume energy released into the air gap, equation (27.53), the second is the pu volume energy associated with establishing the operating point, namely the energy associated with increasing the air gap from zero at point b to length  $\ell_g$  at point c.

Once the magnet operates on a recoil line of relative permeability  $\mu_R$  and recoil remanence  $B_r$ , this equation is modified by  $B_m = \mu_0 \mu_R H_m + B_r$  to

$$W_g = \frac{1}{2} B_m H_m V_g = \frac{1}{2} (-\mu_0 \mu_R H_m - B_r) H_m V_g = \frac{1}{2} (-\mu_0 \mu_R H_m^2 - B_r H_m) V_g \quad (27.54)$$

The further down its demagnetization curve a magnet is driven, the greater the area swept by the load line, by which the applied field within the magnet does more work. This energy is recaptured by the magnet if the gap is re-closed and the load line returns to its original position. However, if this cycle includes driving the magnet beyond a *knee* in its characteristic, a return to the original load line along a recoil line involves the irreversible loss of magnet field energy, as shown in figure 27.42. Large dynamic excursions of a high-energy magnet can be restricted by increasing  $\ell_g$  during magnet installation. Subsequent cycling along the recoil line incurs no further irreversible loss, only reversible energy changes. While  $BH_{\max}$  is the most common figure of merit for a permanent magnet, operation at this point maximizes the release of magnet energy into the gap, namely  $-\frac{1}{2} V_m B H_{\max}$ . If the magnetization  $M$  is constant and the linear demagnetization characteristic is represented by  $B_m = \mu_0 (H_m + M)$ , then the air gap energy, equation (27.53), may be written as

$$W_g = -\frac{1}{2} \mu_0 V_m (H_m^2 + M H_m) \quad (27.55)$$

Differentiating,  $BH_{\max}$  occurs at  $B_m = \frac{1}{2} \mu_0 M$ ,  $-H_m = \frac{1}{2} M$ , for which

$$W_g^{\max} = \frac{1}{2} \mu_0 V_m \left( \frac{1}{2} M \right)^2 \approx \frac{1}{2} \mu_0 V_m \left( \frac{1}{2} B_r \right)^2$$

The magnet energy released is lower either side of the  $BH_{\max}$  point on the major demagnetization characteristic, and by virtue of a reduced alignment of the magnetization  $M$ ,  $W_g$  is also smaller for operation on recoil lines within the major  $B$  versus  $H$  curve. The superimposed constant energy contours on the characteristic in figure 27.42 illustrate the energy penalty of non-optimal operation. Convention is to express these as constant energy product  $B_m H_m$  (as also seen in figures 27.8, 27.11, and 27.15, amongst others), rather than the actual energy density  $\frac{1}{2} B H_{\max}$ .

A more realistic representation of the soft iron pole pieces requires the inclusion of the flux leakage coefficient  $k_f$  and mmf loss factor  $k_r$  via equations (27.19) and (27.18) within equation (27.52). Both loss factors reduce the amount of magnet energy delivered in to the gap

$$W_g = \frac{1}{2} B_g H_g V_g = -\frac{1}{2} \frac{B_m H_m}{k_f k_r} V_m$$

By adding a coil of  $N$  turns with current  $i$ , as in figure 27.26, and using equation (27.30), the air gap energy becomes

$$W_g = -\frac{1}{2} \frac{V_m B_m}{k_f k_r} \left( H_m - \frac{Ni}{\ell_m} \right)$$

The last term represents the energy contribution from the coil. Since the flux in the circuit is  $\Phi = B_m A_m$ , and the total flux linkage with the coil is defined as  $\Lambda = N \Phi$ , the gap energy can be written as:

$$W_g = \frac{1}{2} \frac{1}{k_f k_r} (-V_m B_m H_m + \lambda i)$$

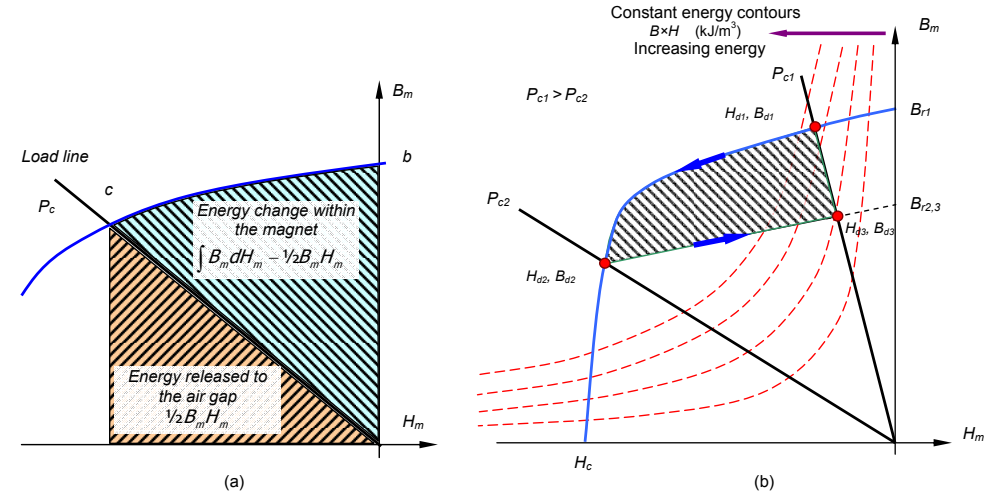


Figure 27.42. Per unit volume change: (a) of the magnet's applied field energy and (b) magnetic field energy with load line slope.

#### Coenergy

Coenergy can be considered a dual of stored energy that is often used to compute forces on electromechanical systems with permanent magnets and current-carrying coils. Force and torque can be deduced by changes in energy with respect to position, but the process is complicated by the need to compute the energy that is sunk or sourced by the supplies driving the coils over the change in position. Conversely, computing the change in coenergy for constant currents directly yields the mechanical work done on the system.

An expedient lower bounds for coenergy computation, for the integral is  $H_0 = -H_c$ . The coenergy density in the magnet  $w'_m$  is, for the case in which the flux density is aligned with the magnetization:

$$w'_m = \int_{-H_c}^H B(H) dH = B_r H + \frac{1}{2} \mu H^2 \Big|_{-H_c}^H = \frac{1}{2} \mu (H + H_c)^2$$

The definition of magnet flux density  $B$  from equation (27.11) yields a simpler definition of coenergy within the magnet:

$$w'_m = \frac{B^2}{2\mu}$$

Stored energy and coenergy can be shown as regions associated to the demagnetization curve as shown in figure 27.43.

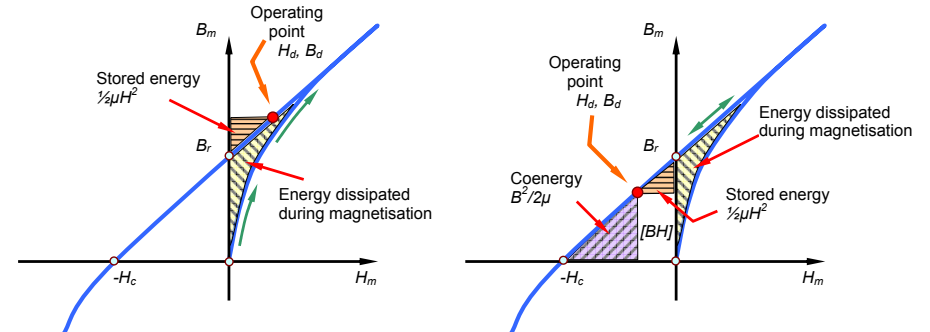


Figure 27.43. Graphical representation of permanent magnetic energy components: (a) with applied bias  $+H_d$  to enable operation in the first quadrant and (b) demagnetisation quadrant operation energy components.

As shown, the area swept in the first quadrant represents the energy per unit volume (the integral of  $H(B)$  dB) that was needed to magnetize the material. This energy can never be recovered, even if a large field intensity is applied to the magnet to push its operating point back into the first quadrant, as indicated in both parts of Figure 27.43. Remagnetisation only stores energy ( $\frac{1}{2}\mu H^2$ ) in the magnet without possibility of extracting the energy associated with the magnet's magnetisation.

### 27.11 Force of attraction within an air gap

The equation  $F = -dW/dx$  can be used to calculate the force of attraction between two pole faces bounding the air gap of figure 27.21 (figure 27.44) of area  $A_g$  and length  $\ell_g$ . The energy in this gap is given by equation (27.52), which, with  $B_g = \mu_o H_g$ , becomes

$$W_g = \frac{1}{2} \frac{B_g^2}{\mu_o} V_g = \frac{1}{2} \frac{B_g^2}{\mu_o} A_g \ell_g \quad (27.56)$$

Using Cartesian axes in the air gap, with flux along the gap length  $\ell_g$  being in the x-direction, force is calculated by differentiating equation (27.56) with respect to  $\ell_g$ , which yields

$$F_x = -\frac{dW_g}{d\ell_g} = -\frac{1}{2} \frac{A_g B_g^2}{\mu_o} = -\frac{1}{2} \frac{\phi^2}{\mu_o A_g} \quad (27.57)$$

The definition of  $-F_x$  acting into the air gap confirms this as a force of attraction, which will increase as the gap closes. Although  $\ell_g$  is not a parameter in equation (27.57),  $F_x$  is proportional to  $\Phi^2$ , where a flux increase results from decreasing  $\ell_g$ , which causes an increase in load line slope as the magnet's operating point moves up its demagnetization characteristic. Similarly, there are forces acting to retard the lateral displacement of one pole piece with respect to the other ( $F_y$ ,  $F_z$ ), which may also be determined from equation (27.56) by splitting  $A_g$  into its y and z components and differentiating with respect to the appropriate axis.

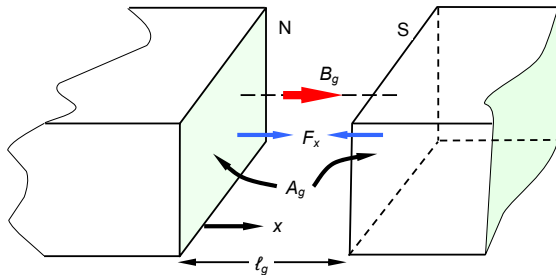


Figure 27.44. Force of attraction between opposite pole faces bordering a uniform air gap.

### 27.12 Appendix: Magnet processing and properties

#### Permanent magnet manufacturing Process

Sintered / fully dense, anisotropic magnets - maximum output: (Rare Earths, Ceramics, and Alnicos)

- Maximum energy product for magnet size and weight
- Restricted to simple geometries
- Brittle thus requires careful handling
- No dilution effect due to non-magnetic phase

Injection moulded - shape flexibility: (Rare Earths and Ceramics)

- Complex geometries
- Tight geometric tolerancing without finishing operations
- Relatively robust, resistant to chipping
- Insert and over moulding to reduce assembly costs
- Variety of pole configurations are possible
- Multistep and multi-component moulding to produce assemblies
- Dilution of magnetic phase produces lower energy product
- Anisotropic and isotropic powders offer a wide range of magnetic alignment and output options
- Relatively high tooling costs make them suited to high volume manufacturing

Compression bonded - low cost manufacturing: (Rare Earths and Ceramics)

- Higher loading than injection moulded, but lower than fully dense creates a compromise in energy product
- Limited to simple geometries: rectangles, cylinders, arcs – thin walled cylindrical shape possible
- Tight geometric tolerancing except in pressed thickness
- Brittle thus requires careful handling
- Isotropic powder allows complex magnetizing patterns

Casting: (Alnico)

Extruded: (Bonded NdFeB and Flexible)

Calendering: (Flexible)

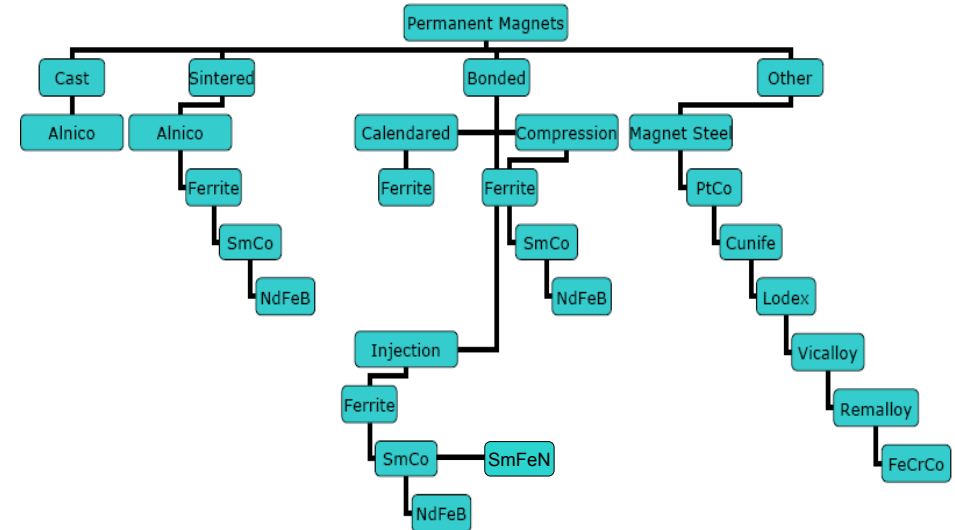


Table 27.6 Physical and mechanical properties of AlNiCo, FeCrCo, FeCoVCr, SmCo, NdFeB, and hard ferrite

| Material and production method   |                          |                      | Physical properties              |                      |                      |         | Mechanical properties      |                      |                 |                        |
|----------------------------------|--------------------------|----------------------|----------------------------------|----------------------|----------------------|---------|----------------------------|----------------------|-----------------|------------------------|
| Material                         | isotropic or anisotropic | Production method    | Coefficient of thermal expansion | Thermal conductivity | Resistivity          | Density | Tensile strength           | Compressive strength | Young's modulus | Hardness               |
|                                  |                          |                      | $10^{-6}/K$                      | W/m.K                | $\mu\Omega \times m$ | g/cc    | MPa                        | MPa                  | MPa             | HV                     |
| AlNiCo                           | i or a                   | casting or sintering | 11                               | 10 - 15              | 0.45 - 0.55          |         | 80 - 300                   | 300 - 400            | 100 - 200       | 300 - 400              |
| FeCrCo                           | i or a                   |                      | 10                               | 10 - 13              | 0.7 - 0.8            |         | 1200 - 1400<br>600 - 700   |                      |                 | 300 - 350<br>400 - 500 |
| FeCoVCr                          | a                        | casting              | 11.5                             |                      | 0.55 - 0.65          |         | 2000 - 5000<br>2500 - 3500 |                      |                 |                        |
| SmCo <sub>5</sub>                | a                        | sintering            | //6 $\perp$ 13                   | 10 - 13              | 0.5 - 0.6            | 8.2     | 30 - 40                    | 900 - 1000           | 100 - 150       | 500 - 600              |
| Sm <sub>2</sub> Co <sub>17</sub> | a                        |                      | //8 $\perp$ 11                   | 10 - 13              | 0.75 - 0.85          | 8.4     | 40 - 50                    | 800 - 900            | 150 - 200       | 600 - 700              |
| NdFeB                            | a                        |                      | //3.4 $\perp$ -5                 | 8 - 10               | 1.4 - 1.6            | 7.4     | 80 - 90                    | 1000 - 1100          | 150 - 200       | 500 - 600              |
| hard ferrite                     | i or a                   |                      | 9, 12, 11                        | 4                    | $> 10^4$             | 4.9     | 50 - 60                    | 600 - 700            | 15 - 200        | 500 - 600              |

NdFeB: Temperature Coefficient of resistivity  $2 \times 10^{-4} /K$ , specific heat capacity 0.12 kJ/K, Poisson's ratio 0.24

### Magnet Material

#### Neodymium-Iron-Boron - high energy

- Relatively abundant resource with large proven reserves
- Refining costs are moderate
- Manufacturing technology is established
- Highest energy product output of all commercially available PM materials
- High temperature, > 150°C, applications require a compromise in energy product
- Tendency to corrode requires protective coating, chips, cracks, and brittle
- PrFeB variant for temperatures between a few degrees Kelvin and 135K

#### Samarium Cobalt - stable

- Relatively abundant resource with large proven reserves
- Manufacturing technology is established; dominated by 2-17 grades
- Second to NdFeB in magnetic output – high energy product and coercive force
- Excellent high temperature performance with grades available for use to 550°C
- Corrosion resistance superior to NdFeB, but coatings generally advisable
- Brittle, chips, cracks easily, and hard to machine
- Refining costs are higher than for NdFeB

#### Ferrite (Ceramic) - low cost

- Abundant, low cost raw material
- Magnets are lowest cost option
- Manufacturing technology is well-established
- Lower magnetic output than the rare earth materials, but high coercive force
- Excellent high temperature performance with grades available for use to 250°C
- Limited low temperature performance, generally to -40°C
- Corrosion resistance is outstanding, but brittle and chips easily

#### Alnico – stable properties

- High corrosion resistance
- High mechanical strength, tough but brittle
- High temperature stability
- High cost
- Low coercive force and energy product

### 27.13 Appendix: Magnetic Basics

$$B = \mu_o H$$

$B$  = magnetic flux density or magnetic induction, Vs/m<sup>2</sup>, 1 Tesla

$\mu_o$  = magnetic permeability of a vacuum =  $4\pi \times 10^{-7}$  Vs/Am

$H$  = magnetic field strength, A/m

If a material is present, the relation between magnetic field strength and magnetic flux density becomes

$$B = \mu_o \mu_r H$$

$B$  can be split into the flux density in the vacuum plus the material part according to

$$B = \mu_o H + J$$

$$J = \mu_o M$$

This gives

$$B = \mu_o (H + M)$$

The definition of  $B$  yields

$$M = (\mu_r - 1)H = \chi_{mag} H$$

with  $\chi_{mag} = (\mu_r - 1)$  = magnetic susceptibility. With superconductors (= ideal diamagnets),  $\chi_{sc} = -1$ .

### 27.14 Appendix: Magnetic properties for Sintered NdFeB and SmCo Magnets

Neodymium magnets are graded by the material they are made of. The higher the grade (the  $BH_{max}$ , following the 'N' in Table 27.3), the stronger the magnet.

Table 27.7 can be used to determine whether it is better to use sintered Neodymium Iron Boron or sintered Samarium Cobalt in a particular application.

Table 27.7 Typical magnetic properties for NdFeB and SmCo

|   | Neodymium Iron Boron (NdFeB)   | Samarium Cobalt (SmCo)   |
|---|--|--|
| High temperature applications   | Higher $H_{ci}$ materials can be used up to 200°C, moderate $H_o$ materials used up to 150°C. Low $H_{ci}$ materials limited to 100°C  | SmCo can be used at substantially higher temperatures than NdFeB. Continuous operation at temperatures above 250°C<br>Also significantly better at temperatures below 100K   |
| Loss of flux density at elevated temperatures                                       | Loses 0.11% of $B_r$ /°C.<br>See NdFeB temperature effects on $B_r$ & $H_{ci}$   | Loses about 0.03% of $B_r$ /°C   |
| General environmental conditions  | Neodymium Iron Boron is highly reactive to environmental conditions, while Samarium Cobalt magnets are very corrosion resistant.   |  |
| Humid environments  | Surface treatment advisable, due to oxidation<br>Options are nickel, IVD or polymer coatings.  | Surface treatment is not required.<br>Does not oxidize.  |
| Hydrogen rich atmosphere  | Not recommended, hydrogenation occurs, causing the magnet material to disintegrate   | No known adverse effects   |
| Cost of part  | Lower cost generally   | Higher cost than NdFeB because of Co content   |
| Radiation environment   | Damaged by radiation, mainly gamma rays  | Higher grades are stable to radiation  |
| Mechanical strength   | Mechanically strong, not as brittle as SmCo  | Brittle. Both alloys require diamond tooling, EDM, or abrasive grinding when machining.  |
| Clean room environments   | Surface treatment recommended  | Surface treatment not required   |
| Vacuum applications   | Needs to be coated, with nickel or IVD recommended (metallic - do not outgas).<br>Polymer coatings can be used, but not in an ultra high vacuum or high temperatures in a vacuum, due to outgassing.   | Surface treatment is not required. However, fairly porous; parts may outgas for a limited duration before high vacuums can be achieved   |
| High field requirements   | Higher fields achievable, higher energy products (up to 440kJ/m <sup>3</sup> )   | Highest energy product is 240kJ/m <sup>3</sup> , difficult to obtain higher.   |
| Cryogenic temperatures  | Only special formulation grades  | Operates well in cryogenic temperatures.   |
| Aerospace applications  | Gaining use in aerospace applications, optionally with surface treatments.<br>Used in aircrafts, missiles, and satellites.   | Established in aerospace, military, and defence applications.  |
| Salt, open seas, and salt spray environments  | Surface treatment essential, limited life  | Stable in this environment   |
| Acid and alkaline environments  | Surface treatment is necessary, limited life   | Stable in this environment, however, qualification tests recommended   |
| Thin walled, thin cross-section applications (a dimension below 1mm)                | Mechanically stable, parts as thin as 1/8mm  | Poor in thin cross-sections.<br>Under 1/2mm is not recommended   |
| Single piece large parts  | Better than SmCo, larger blocks can be sintered  | Larger blocks (over 75mm in any dimension) are challenging   |
| Nickel plating as per military specifications                                       | Not available – proprietary plating to specifications, only electrolytic nickel  | Electroless and electrolytic nickel plating  |
| Plating as per military specifications  | Not available.<br>Plating as per proprietary specifications.   | Not typically used.  |
| Radial ring (for true radially oriented field)                                      | Possible.  | Not available  |
| Sensitivity of flux density, $B_r$ , and coercivity, $H_c$ , to temperature changes | Temperature coefficient of $B_r$ ranges from negative, 0.11%/°C to 0.13%/°C.<br>Higher coercivity materials (>160kA/m) are closer to negative, 0.11%/°C<br><br>Temperature coefficient of $H_c$ ranges from negative, 1/2%/°C to 3/4%/°C.<br>The higher the intrinsic coercivity, the lower the temperature coefficient of $H_c$ | Temperature coefficient of $B_r$ ranges from negative, 0.03%/°C to 0.04%/°C.<br>Series Sm <sub>2</sub> Co <sub>17</sub> less sensitive to temperature changes (about negative 0.03%/°C) than SmCo <sub>5</sub> (about negative 0.04%/°C)<br>Temperature coefficient of $H_c$ ranges from negative, 0.15%/°C to 1/4%/°C.<br>Series Sm <sub>2</sub> Co <sub>17</sub> less sensitive to temperature changes (about negative 0.15) than SmCo <sub>5</sub> (about negative 1/4) |



Based on table 27.7, in summary, do **NOT** use Neodymium Iron Boron magnets under the following conditions:

- In an acidic, alkaline, or organic solvent (unless the magnet is hermetically sealed)
- In water or oil (unless hermetically sealed, or a limited life results)
- In an electrically conductive liquid, such as electrolyte containing water
- In a hydrogen-containing atmosphere, especially at elevated temperatures. Hydrogenation, a process where the  $H_2$  molecule reacts with the NdFeB, causing rapidly deteriorate
- Environments containing corrosive gasses, such as  $Cl_2$ ,  $NH_3$ ,  $NO_x$ , etc.
- In the path of radioactive rays

#### 27.15 Appendix: Magnetic Axioms

1. Flux lines, like electrical currents, follow the path of least resistance. In magnetic terms, this means that flux lines will follow the path of greatest permeance (lowest reluctance). Flux lines will always follow the shortest path through any medium. They therefore can only travel in straight lines or curved paths, and they can never take true right-angle turns.
2. Flux lines repel each other, hence never cross, if their direction of flow is the same.
3. For unsaturated ferromagnetic materials, flux lines will always leave and enter at right angles.
4. All ferromagnetic materials have a limited ability to carry flux. When saturated, they behave as though they do not exist (like air, aluminium and so on). Below the level of saturation, a ferromagnetic material will substantially contain the flux lines passing through it. As saturation is approached, because of axioms one [1] and two [2], the flux lines may travel as readily through the air as through the material (which appear like air when saturated).
5. Flux lines will always travel from the nearest north pole to the nearest south pole in a path that forms a closed loop. They need not travel to their own opposite pole; although they ultimately do if poles of another magnet are closer and/or there is a path of lower reluctance (greater permeance) between them.
6. Magnetic poles are not unit poles. In a magnetic circuit, any two points equidistant from the neutral axis function as poles, so that flux will flow between them (assuring that they meet the other conditions stated above).

**Table 27.8 The relationship between magnetic parameters in cgs and SI units.**

| Quantity                   | symbol     | Gaussian     | Gaussian (cgs units)    | S.I.                 | S.I. Units          | Conversion factor (cgs to S.I.) |
|----------------------------|------------|--------------|-------------------------|----------------------|---------------------|---------------------------------|
| Magnetic Induction         | $B$        | gauss        | G                       |                      | Tesla, T            | $1G = 10^{-4} T$                |
| Applied Field strength     | $H$        | Oersted      | Oe                      |                      | $Am^{-1}$           | $10^3 / 4\pi$                   |
| Magnetisation              | $M$        |              | $emu\ cm^{-3}$          |                      | $Am^{-1}$           | $10^3$                          |
| Magnetisation              | $4\pi M$   |              | G                       |                      | -                   | -                               |
| Magnetic Polarisation      | $J$        |              | -                       |                      | T                   | -                               |
| Specific Magnetisation     | $s$        |              | $emu\ g^{-1}$           |                      | $JT^{-1}\ kg^{-1}$  | 1                               |
| Induction in free space    |            | $B=H$        | G                       | $B=\mu_0 H$          | T                   | $1G = 10^{-4} T$                |
| Induction in medium        |            | $B=H+4\pi M$ | G                       | $B=B_0 + \mu_0 M$    | $B = \mu_0 (H + M)$ | $1G = 10^{-4} T$                |
| Permeability of free space | $\mu_0$    | unity        | Dimensionless           | $4\pi \cdot 10^{-7}$ | $H\ m^{-1}$         | $4\pi \cdot 10^{-7}$            |
| Relative Permeability      | $\mu_r$    |              | -                       |                      | Dimensionless       | -                               |
| Susceptibility             | $\chi$     |              | $emu\ cm^{-3}\ Oe^{-1}$ |                      | Dimensionless       | $4\pi$                          |
| Maximum Energy Product     | $BH_{max}$ |              | M G Oe                  |                      | $kJ\ m^{-3}$        | $1\ MGOe = 10^2 / 4\pi$         |

G = Gauss, Oe = Oersted, T = Tesla

#### Reading list

[http://www.magnets.bham.ac.uk/magnetic\\_materials/](http://www.magnets.bham.ac.uk/magnetic_materials/)  
<http://www.mceproducts.com/>  
<http://www.dextermag.com/Permanent-Magnet-Materials.aspx>  
<http://www.arnoldmagnetics.com/>  
[http://www.intemag.com/magnet\\_materials.html](http://www.intemag.com/magnet_materials.html)  
<http://www.magnetweb.com/>  
<http://www.femm.info/wiki/PMEnergy>