



# Simulation Tasks

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# Solvers and stiff systems

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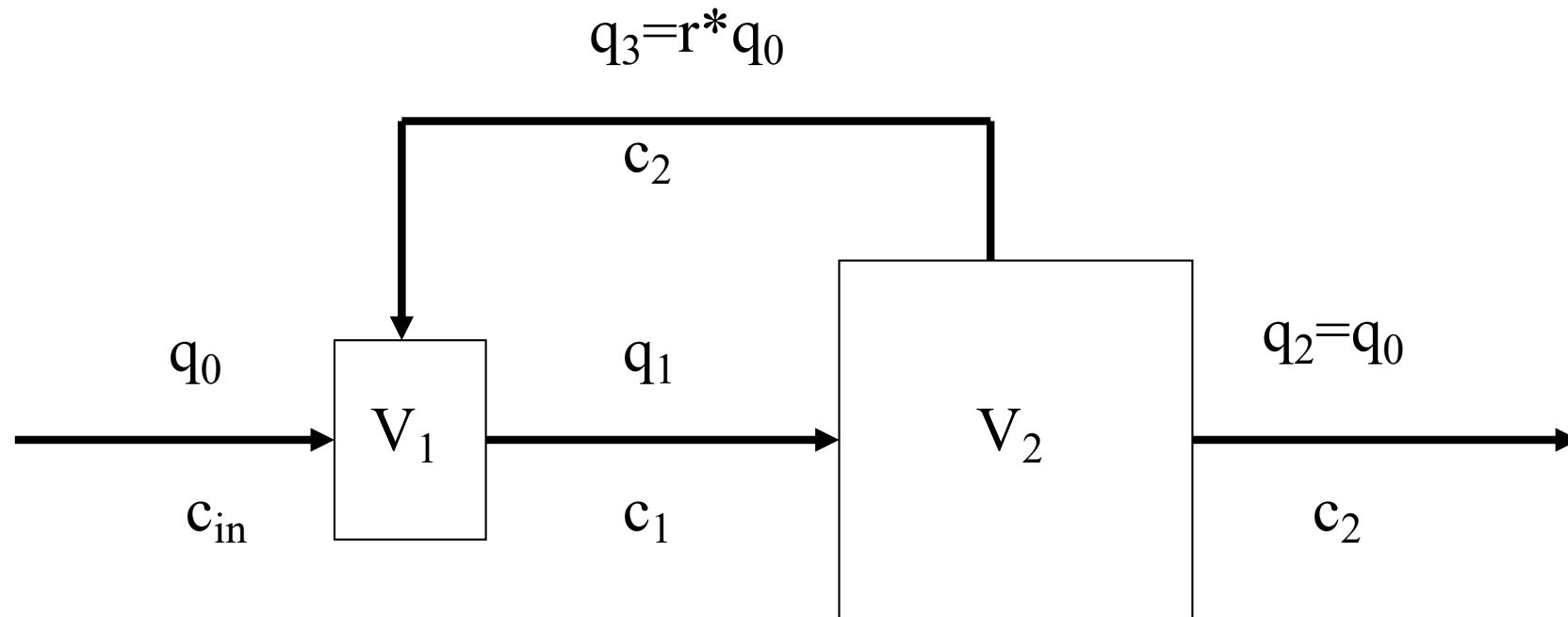
- Average time: 6.8 hours (std = 1.8 h)
- Time range: 5 to 9 hours
- Deadline was fairly well respected
- Only a short introduction to a vast field
- Results depend partly on computer, OS, 32 or 64 bit OS/Matlab and also Matlab version
- Conclusion: selection of solver and way to set up the problem is important

# Multivariate monitoring

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- Average time: 6 hours (std = 1.4 h)
- Time range: 5 to 8 hours
- Deadline was very well respected
- Only a short introduction to a huge field
- Complex methods but very powerful tool when detailed models are not available, especially for detection
- Conclusion: requires expert knowledge to actually do the fault isolation and diagnostic part

# System with recycle



# Model of recycle

$$q_1 = (1 + r) \cdot q_0$$

$$q_2 = q_0$$

$$q_3 = r \cdot q_0$$

$$V_1 \frac{dc_1}{dt} = q_0 [c_{in} - (1 + r)c_1 + rc_2]$$

$$V_2 \frac{dc_2}{dt} = q_0 [(1 + r)c_1 - rc_2 - c_2] = q_0 \cdot (1 + r)(c_1 - c_2)$$

# Model of recycle (2)

What happens when  $V_1$  becomes small?

$$\frac{dc_1}{dt} = \frac{q_0}{V_1} [c_{in} - (1+r)c_1 + rc_2]$$

Large derivative –  
fast state

$$\frac{dc_2}{dt} = \frac{q_0}{V_2} \cdot (1+r)(c_1 - c_2)$$

The range of time constants becomes wide  
and the system becomes stiff!

# Model of recycle (3)

$$\varepsilon \rightarrow 0$$

$$0 = q_0 [c_{in} - (1+r)c_1 + rc_2] \Rightarrow$$

$$c_1 = \frac{1}{1+r} (rc_2 + c_{in})$$

$$\frac{dc_2}{dt} = \frac{q_0}{V_2} \cdot (1+r)(c_1 - c_2) \Rightarrow$$

$$\frac{dc_2}{dt} = \frac{q_0}{V_2} \cdot (c_{in} - c_2)$$

# Dimension reduction

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- Neglecting small singular values
  - Reduction of the dimension
- Aim of multivariate statistics
  - e.g. principal component analysis (PCA)
- Idea:
  - extract underlying mechanism and represent them in a low dimensional space



# PCA

Measurement  
matrix

Principal  
components

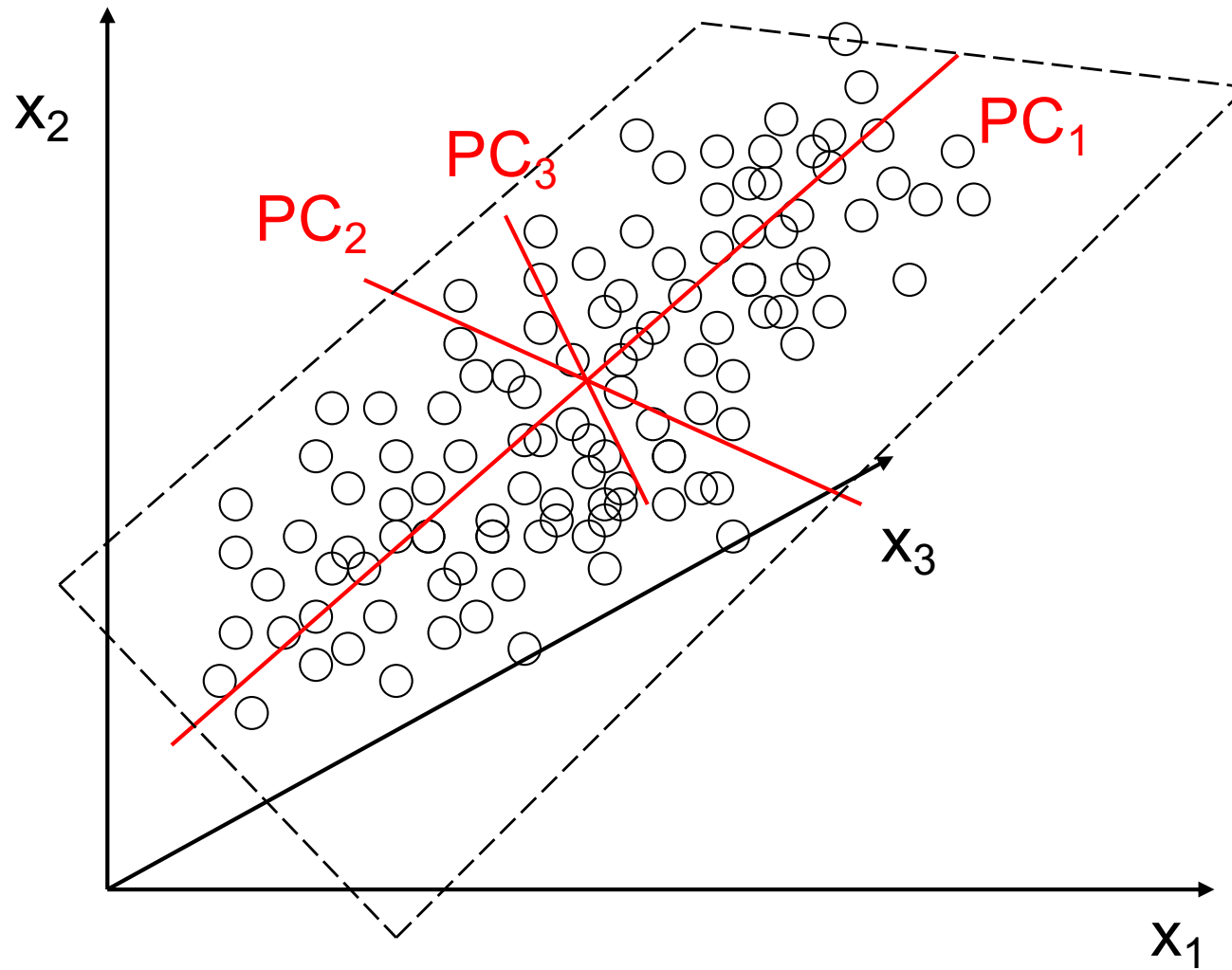
Orthogonal  
coordinate  
system

Model  
error

$$\mathbf{X} = \mathbf{T}\mathbf{P}^T + \mathbf{E}$$

$\mathbf{E} = 0$  if number of PCs = number of variables

# PCA



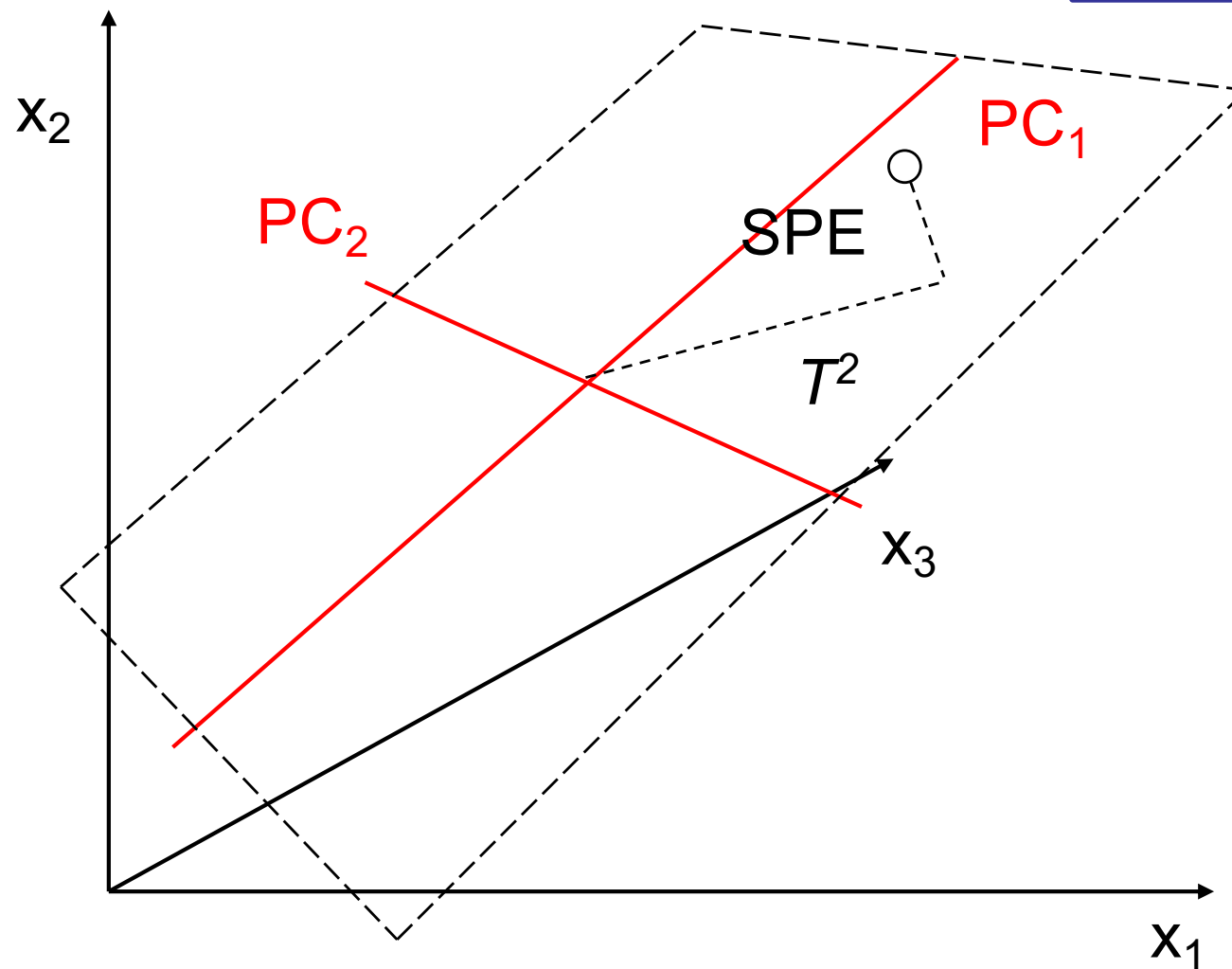
# PCA

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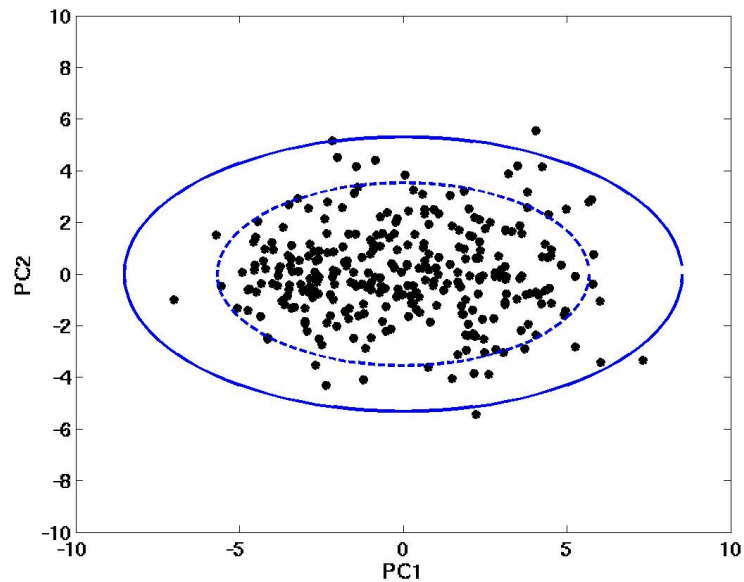
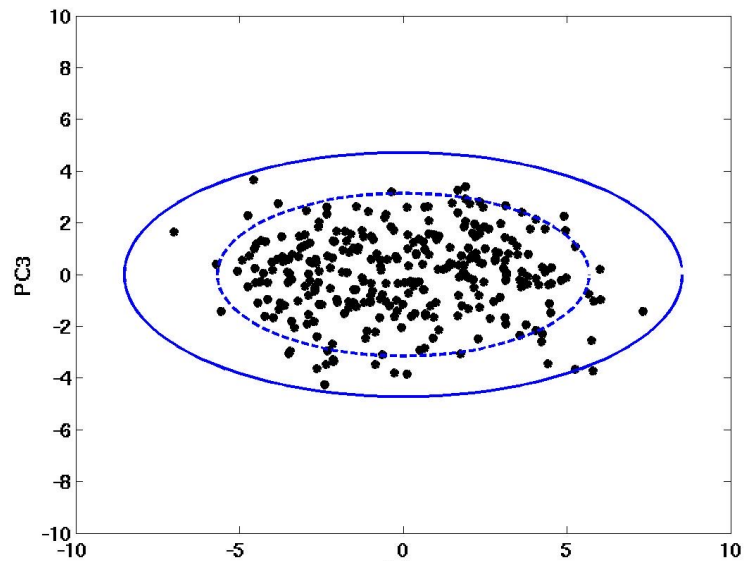
Typically for industrial data:

- 1<sup>st</sup> PC normally describes 40-60% of the variability in a process
- A few (2-4) PCs describe >80%
- **E** ideally describes noise
- Choice of number of components crucial

# SPE and $T^2$ - geometrical interpretation



# Example - score plots



Plotting the PCs:

- Compact
- Intuitive

