Automation in Complex Systems – EIEN35

Exam Tuesday June 3, 2020

You may bring the course book and the reprints (defined in the course requirements), *but not* the solution to problems or your own solutions to simulation tasks and home works or other personal notes. A calculator is also permitted (memory cleared). You may answer in **Swedish** or in **English**.

Grading: There are 30 points all together. The following grades will apply:

Grade 3:	at least 15 points
Grade 4:	at least 20 points
Grade 5:	at least 25 points

Note that all answers should be complete and well motivated. Your line of thought should be easy to follow and hand calculations should be provided for all mathematical problems. Corrections will be completed not later than *Wednesday **June 24,** 2020.*

As this is a remote exam (due to the Corona pandemic) there is an **extra need** that you provide complete analytical/well formulated/well written solutions to all your answers. By taking this exam you hereby confirm that you have read all the information and instructions regarding the exam sent by email 2020-05-28 and you have agreed to follow all those instructions to the letter. If NOT, you should stop the examination NOW and come back for the next exam opportunity on August 26, 2020.

Also note that this written exam may be complemented by an individual oral exam before a final grade is issued to any student (as described in the emailed instructions).

Good Luck!

Problem 1 (3 points)
Consider a system described as a birth-death process with an arrival rate of 5 jobs per hour and a production rate of 10 jobs per hour. What is the minimum number of states required to represent the system if we can allow a maximum rejection probability of 10% on an average? Also provide the complete probability vector in stationarity for your selected number of states. (Hint: the number of needed states is less than 10)

Solution:
This is a classical birth-death process with x states.
There can be x-1 'jobs' in the system, each having the probability p0, p1, p2, etc. respectively.
The departure rate is all the time $\mu = 10$ per hour. The arrival rate is all the time $\lambda = 5$ per hour.
The following condition must always hold: p0 + p1 + p2 + p3 … + p(x-1) = 1.
The general steady state condition for this type of system is:

$p_{k+1} = \frac{\lambda_k}{\mu_{k+1}} p_k$ which gives us

p1 = 5/10*p0; p2 = 5/10*p1 = $(5/10)^2$*p0; p3 = $(5/10)^3$*p0; etc. So the full probability vector as a function of p0 is:

p = p0 * [1 5/10 $(5/10)^2$ …] = p0 * [1 0.5 0.25 0.125 0.0625 …]. If we normalize this vector (since p0 + p1 + p2 + p3 + … = 1) we know the values within [ ] will become smaller than the values given above (since the sum of all the values is larger than 1) and we can conclude that certainly no more than 5 states will be required (0.0625 < 0.1). If we write the normalised **p** vector based on five states we get [0.5161 0.2581 0.1290 0.0645 0.0323]. But then we see that also p3 < 10% and it must therefore be enough to use less than 5 states to represent the system. Normalising the vector based on the first four states instead gives [0.5333 0.2667 0.1333 0.0666]. p2 is now larger than 10% and will only become larger if we normalise the vector based on only the first 3 states. Consequently we need four states to represent this birth-death process to meet the defined criteria in terms of rejection probability and the stationary probability vector is **p** = [0.5333 0.2667 0.1333 0.0666].

Problem 2 (4 points)

A machine has insufficient production rate in relation to the demand. Incoming jobs can be described as a Poisson process with an arrival rate of one job per 10 minutes (λ). The existing machine needs 12 minutes to complete a job (exponentially distributed). No input buffer size limitation exists. The service discipline is FIFO. There are two alternatives for improvement:

i) to replace the existing machine with one new machine with double capacity;

ii) to install a second machine (identical to the existing one) in parallel with the existing machine. The two machines would have one common queue.

The main goal is to improve (reduce) the waiting time in the system. Which alternative is the best? Demonstrate by calculating:

a) the average queue lengths in the three systems (**1 p**)
b) the total waiting times in the two new systems. Motivate in words why there is a difference. (**1 p**)
c) Assume we make two separate queues (of unlimited size) for alternative (ii), each with the arrival rate of $\lambda/2$. Will this result is a shorter or a longer waiting time compared to the case with one common queue? (**1 p**)
d) How big is the probability to have more than 3 jobs in the queue in the alternative with one fast machine (alternative (i))? (**1 p**)

Note: complete step-by-step calculations must be handed in!

Solution:

a) For the original system the arrival rate is 6 per hour and the production rate is 5 per hour (i.e. for the existing system the buffer size grows towards infinity). System (i) means increasing the production rate to 10 per hour in an M/M/1 system. For such a system

$$L_q = \frac{\rho^2}{1-\rho} = \frac{0.6^2}{1-0.6} = 0.9 \text{ items}$$

For system (ii) we have an M/M/2 system where

$L_q = \frac{m^m \rho^{m+1}}{m!(1-\rho)^2} p_0$ and $p_0 = \left[\frac{m^m \rho^{m+1}}{m!(1-\rho)} + \sum_{n=0}^{m} \frac{(m\rho)^n}{n!} \right]^{-1}$. The utilization rate is 0.6 and p_0 is therefore equal to 0.25. This gives L_q = 0.6749 items. So system (ii) has on an average fewer items in the queue but on the other hand it has two items in the machines whereas

(i) only has one item in the machine.

b) The total waiting time for system (i) is $W = \frac{1}{\mu-\lambda} = \frac{1}{10-6} = 0.25$ hours and for system (ii) $W = W_q + \frac{1}{\mu} = \frac{L_q}{\lambda} + \frac{1}{\mu} = 0.6749/6 + 1/5 = 0.3125$ hours. The waiting time in the queue is actually shorter for (ii) compared to (i), 0.1125 hours versus 0.15 hours. But when the job goes into the machine in (ii) it does not help that much that there are two machines in parallel, the individual job must go through just one machine, and that machine is only working at half the speed of the machine in (i). The time in one machine in (ii) is 1/5 = 0.2 hours and in (i) 1/10 = 0.1 hours. Therefore the total waiting time in the system will be larger in (ii) than in (i).

c) System (ii) is now modified into two independent M/M/1 systems working in parallel. W for each of the subsystems is then 1/(5-3) = 0.5 hours, i.e. the total waiting time in the system is increased by 60%.

d) The probability to have more than 3 jobs in the queue is equal to the probability of having more than 4 jobs in the system (1 item is in the machine) = $p_5+p_6+p_7+p_8\ldots = 1-p_0-p_1-p_2-p_3-p_4 = 1-0.4-0.24-0.144-0.0864-0.5184 = 0.07776$ where $p_0 = 1-\rho$ and $p_n = \rho^n(1-\rho)$.

Problem 3 (2 points)
Consider a system where three identical machine works in parallel, each with the average capacity 10 jobs per hour. The machine outputs are exponentially distributed. Likewise, jobs arrive to the machines stochastically (one common queue) with 24 jobs per hour at an average. Also the job arrival time is exponentially distributed. There is one common queue and the service discipline is FIFO.
Assume that a maximum of 3 jobs is allowed *in the queue*.

- Calculate the average waiting time in the queue (**1 p**)
- Calculate the rejection rate, in other words, the number of jobs per hour that are never admitted to the system (**1 p**)

Note: complete step-by-step calculations must be handed in!

Solution:
This is a traditional M/M/3/6 system. The utilization rate is 24/30 = 0.8. The probability vector is calculated as

$$p_0 = \left[\frac{m^m \rho^{m+1}\left(1-\rho^{K-m}\right)}{m!(1-\rho)} + \sum_{n=0}^{m} \frac{(m\rho)^n}{n!}\right]^{-1}$$

$$p_n = \frac{(m\rho)^n}{n!} p_0 \quad \text{(n=1,2,3)}$$ which gives (after some calculations)

$$p_n = \frac{m^m \rho^n}{m!} p_0 \quad \text{(n=4,5,6)}$$

p = [0.0764 0.1835 0.2202 0.1761 0.1409 0.1127 0.0902]. In this case we only need p_0 and p_6.

$\bar{L}_q = \frac{m^m \rho^{m+1}}{m!(1-\rho)^2}\left[1-\rho^{K-m}-(1-\rho)(K-m)\rho^{K-m}\right]p_0$ which gives $L_q = 0.6369$ jobs. From here we calculate W_q as $W_q = \frac{\overline{L}_q}{\overline{\lambda}}$ where $\overline{\lambda} = \lambda(1 - p_6)$= 21.84 h^{-1} and W_q = 0.0292 hours. The rejection rate is $\lambda - \overline{\lambda}$ = 2.165 jobs per hour.

Problem 4 (4 points)
Numerical stiffness can be a major problem when simulating dynamic systems.

a) Specific numerical solvers exist, which are designed especially to deal with numerically stiff systems. Discuss the principles of how such solvers work, their advantages and what their potential problems are. (**2 p**)
b) If you do not have access to a specialized stiff solver how can you work around the problem and still use a traditional explicit solver to simulate your system. (**1 p**)
c) A simple differential equation is given by d$x(t)$/dt = $4x(t)$ + 1. 'Simulate' by hand calculations this equation forward in time using both Euler backward and forward approximations (5 time steps is enough). Use h = 1 and $x(0)$ = 0. (**1 p**)

Solution

a) Implicit solvers (e.g. Gear) take into account also future values x(n+1) when determining in which direction to move. Therefore at least one algebraic equation must be solved for each integration step, and this may require some kind of built in iterative solution method for non-linear systems (e.g. Newton-Raphson). On the other hand the implicit solver can take long integration steps to compensate for this extra computational burden and thereby maintain high simulation speed since the implicit solvers are much less prone to running into instability issues due to integration step lengt compared to explicit solvers. In particular all stiff solvers are based on implicit solvers. Implicit solvers sometimes use a built-in explicit solver to calculate a good initial value for the next iteration, so called predictor-corrector methods. The more sophisticated explicit AND implicit solvers used today are variable step-size solvers that adjust the integration step according to the selected tolerances and the system dynamics on-line.
A problem with stiff solvers is that they do not handle transient and random conditions well. Typical examples of these are: poor initial conditions, noise, (highly) dynamic input, discrete events, mixture of continuous and discrete systems. Stiff solvers can also get completely stuck.
b) In such a case the fast dynamics of the system may be rewritten as a DAE system (thereby eliminating the stiffness) and an explicit solver used instead. If the system of DAEs is non-linear we will need an iterative solver to solve the system at every integration step. We could also also assume the slow dynamics to be constant values and remove the stiffness that way. In both cases we should be aware of that the full dynamics of the original system is not maintained. We have made a simplification.
c) Euler backwards –> dx/dt≈(x(t)–x(t-h))/h gives after rewriting: x(t) = (x(t-h) + h)/(1-4h). Using the values in the text: x(0)=0; x(1)=–1/3; x(2)=-2/9; x(3)=–7/27; x(4)=–20/81; x(5)=–61/243 ≈ -0.2510.
Euler forwards –> dx/dt≈(x(t+h)–x(t))/h gives after rewriting: x(t) = (1+4h)·x(t-h) + h. Using the values in the text: x(0)=0; x(1)=1; x(2)=6; x(3)=31; x(4)=156; x(5)=781. It may appear Euler backwards is finding the correct solution whereas the Euler forward goes towards infinity as a result of using a too large integration step. But this is actually the correct solution. We can see the potential problem for the Euler backwards: if h=1/4 the denominator is zero and if h is smaller than 1/4 the expression can never produce negative numbers if we start in x(0)=0. Not the solutions from Euler forward is very good either but it moves in the correct direction. The analytical solution to the equation

is $x(t) = (e^{4t} - 1)/4$, i.e. goes towards positive infinity.

Problem 5 (3 points)

a) Explain why many industrial systems are sufficiently described by only a few independent variables (e.g. principal components) although many, many more measurement variables are generally available. Describe at least two reasons! (**2 p**)

b) Give at least two reasons why multivariate methods are generally better than traditional SPC for monitoring of complex processes. (**1 p**)

Solution

a) Redundant measurements for safety reasons, many variables are coupled (e.g. pressure in a pipe at different locations), the measured variables are often indirect measures of a few underlying main mechanisms driving the process and methods like PCA 'reshapes' the system description to describe those more fundamental mechanisms. This means that normally many sensors will react to the same type of process disturbance.

b) In multivariate monitoring we take advantage of how the different measurements impact one another and how they are correlated. Thereby much more information can be gathered from the data and still presented to an operator in a fairly simple way. However, the underlying mathematics and statistics used are much more complicated. Modern industrial processes involve many hundreds or thousands of signals. Operators cannot handle information from all individual measurements and it may be difficult to define which are the most essential ones in order to cover all possible types of process disturbances. For most complex processes signals are coupled and not independent. But those synergetic effects cannot be detected by individual signal analysis used in univariate monitoring. Using multivarite monitoring algorithms the thousands of signals can often be reduced into only a few key components that describe the main mechanisms or variability in the process by using the fact that the signals are correlated and dependent.

Problem 6 (3 points)

Use the framework presented by T. Gillblad to describe the type of complexity, and explain why it occurs, in the following examples:

a) Give a correct weather forecast for the next 10 days. (**1,5 p**)

b) Provide drinking water of good quality in densely populated cities in comparably dry regions. **(1,5 p)**

More than one type of complexity may be involved but the explanation should connect the type and the origin of the complexity.

Solution

a) If we look upon the global weather as a system with the current weather situation as the input and try to predict using this model one could argue that this is a functional complexity since the relation between input and output is very complicated. However, this is only a part of the difficulty. If we instead look upon a local forecast one could say that we see operational complexity since the forecast can be accurate for a short period of time but when the conditions provided by the neighbouring regions change the forecast becomes uncertain if it is made for a longer time period. With this view one could also look upon the global weather as built up from many sub-regions. We could then probably for an isolated small region make a good forecast but when all the regions interact we have integrative complexity.

b) In this case the operational complexity is obvious. Amount of available "raw" water and the consumption of the drinking water are affected by the temperature and the weather. The recycling of water has also in many areas created recursive complexity. Pollutions from industrial processes and pharmaceuticals are sometimes difficult to remove. Consequently the concentration of these pollutants increase every time the water is recycled.

Problem 7 (3 points)

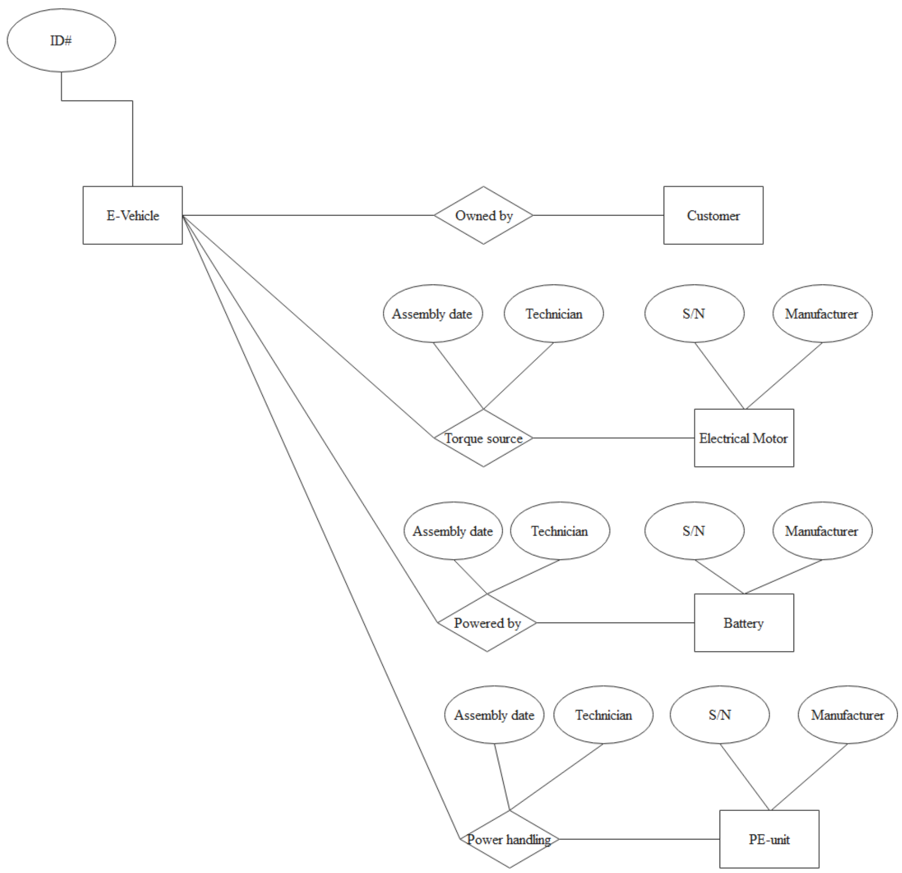
a) Why are industrial databases "special" so that they sometimes require a special version of the DBMS (Database Management System). (**1 p**)

b) A company builds (assembles) electric vehicles. The driveline consist of two electric motors, a power electronic unit and three battery units. For quality control, the company wants to be able to trace all these components by manufacturer, serial number, assembling date, assembling technician, vehicle ID and customer. Suggest an ER diagram that describes the schema for a usable database for this purpose. **(2 p)**

Solution

a) In industry the data is produced by machines. In a production line a large number of systems produce data at a very high rate. Consequently, the industrial DBMS has to cope with large amount of data produced at high speed. There are also other aspects regarding the use of the data e.g. where it is used for direct process control.

b)



Problem 8 (2 points)
Industrial Interne of things, IIoT, is an important factor in what we call Industry 4.0. It is also a huge data producing technology. Describe the use of these data in different aspects. You should try to make a complete picture of the possible utilization.

Solution

IIoT can in many cases make it possible to measure and collect more data from production processes. This enables the use of new more statistical or complex control methods. By collecting the data, a better control of the process is possible and it is also possible to have a better system for diagnosis and maintenance of the production equipment. A higher degree of autonomous production can be developed in this way.
In the case of more complex products the IIoT solution may be a part of the final product. This means that when the product is bought, leased or in any other way used by the customer, the manufacturer can still provide essential instructions and support for the product. The customer becomes dependant on the manufacturer but the quality of the product functionality is probably increased a lot.

Problem 9 (3 points)
Climate change is one of the most difficult challenges for mankind for the next decades. From an energy point of view the global warming has several consequences:

a) How is energy generation influenced by climate change? **(2 p)**
b) What do we mean that solar photovoltaic systems are scalable? **(1 p)**

Solution

a) (i) Higher temperatures: lower energy costs for heating but higher energy costs for air condition and cooling. (ii) Many areas become more arid: lack of cooling water for thermal power plants. Also, higher temperatures mean too warm cooling water. (iii) Hydro plants: evaporation will increase. Lack of water in rivers leading to the hydro dam may also lower the level of the dams. This means less hydropower generated.
b) We can use the same system structure for 1, 10, or 1000 households.

Problem 10 (3 points)

a) Describe briefly the need for "plant wide control" although we have excellent tools from control theory. **(1 p)**
b) Many industrial processes have storage tanks. Explain the difference between dynamic degrees of freedom (DOF) and steady state DOF around these tanks. **(1 p)**
c) Why is the number of actuators not necessarily equal to the dynamic DOF? **(1 p)**

Solution

a) In a large production system many processes are connected. These processes may operate with different timeconstants, different known and unknown input variations and individual constraints. The interconnection between the state variables in itself very often leads to a combinatorial explosion if everything should be included in one system description. This calls for an approach with a layered control structure like the one used in PWC.
b) If we assume one controlled inflow (valve/pump) to the tank and one controlled outflow from the tank we have two dynamic degrees of freedom. However, since everything that is filled into the tank must come out in the long run the one and only steady state DOF

is the flow through the tank.

c) Actuators may be dependent. E.g. a valve and a pump in series at the same pipe cannot be operated independently.