Computer exercise – Dynamic systems

Automation EIEN50 (former EIEF45)

Goal

The purpose of these simulations is to study some fundamental properties of dynamical systems. The first one is **linearity**. What does it mean in practice that a system is linear? You will study the consequence of the superposition principle. The other property is the relationship between poles (eigenvalues) and the transient behaviour of the linear system. In case of nonlinear systems the concept of **stability** becomes different. You will find that the stability is *not* a property of the system but a property of the solution.

File download

For this exercise you need to download some Matlab/Simulink files. The files are originally written in an old Matlab 6.5 and should work on any newer version. Download the zipped file and put it in a suitable directory on your computer (e.g. dynsys). The file is a zipped archive and once downloaded it should easily be extracted. If you run into problems, please contact Ulf Jeppsson (ulf.jeppsson@iea.lth.se).

Report

These exercises are carried out either **individually** or in **groups of two**. The report should contain answers to the questions stated including your own comments, motivations and conclusions. When "(**Provide plot**)" is indicated, plots should be included in the answer to that particular question. You can write the reports either in Swedish or English. Note: remember to put your name and email address on a cover page.

The reports can be submitted electronically (give the file a name of structure **AUT2_Familyname** (as a pdf-file)) or as a hard copy **not later than February 9, 2022** to ulf.jeppsson@iea.lth.se You have to draw some state graphs. It is OK to write them by hand and supplement the file with this paper if you do not want to draw the state graphs on your computer.

Feedback

In order to make a better course planning for the next time it would be greatly appreciated if you try to estimate the time you spend on this exercise. Indicate the number of hours you spent on this task on the cover page. Of course you can also provide other comments on the exercise. Feedback to you will be provided by email or comments on hard copy.

Help

If you need help, send me an email: ulf.jeppsson@iea.lth.se or give me a call: 046 222 9287. I will try to answer your questions as quickly as possible.

Good luck!

Ulf Jeppsson

Dynamical systems

Preparation
Read Chapter 3 in the textbook.

Simulation exercises

1. Initiation
Start Matlab and see to that you are in the directory in which you downloaded the data file. Type

>>what

to see which Matlab files are in the current directory. Note that the models used in this exercise are based on Simulink, the graphical simulation interface for Matlab. All outputs from each simulation are stored as columns in a Matlab variable "simout" in Matlab workspace and time in a variable "tout" (use to create plots).

2. Linear system
In papermaking, a large amount of water is used where it is used for various purposes. The water system in a paper machine is often referred to as the whitewater system. To minimise cost and the environmental impact of the papermaking, the water is reused through the process. The whitewater contains high concentrations of organic compounds, mostly in dissolved fractions. A simple model of how the mass of a soluble organic compound propagates through the whitewater system could look like in the figure below
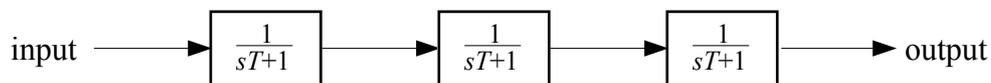
where the model basically consists of three continuously stirred tank reactors (CSTRs). Such a model is provided among the files you have downloaded.

Questions and tasks:
2.1 Write the differential equation of how the mass changes in a tank described in terms of concentration (c), volume of the tank (V) and flow rate through the tank (q). How does mass relate to concentration in the system?
2.2 How does q and V relate to T in the model in the figure above?

Load the model by typing:

>>mod1

In the model, there are three reactors, a constant input, a pulse generator (to simulate changes in the input concentration between $t = 20$ and 21) and display. To set the time constants (T) of the tank reactors and the amplitude (amp) of the pulse type:

>>T=1;amp=5;

You start the simulation by using the menu (simulation->start). The system is set up so that the initial concentrations at $t = 0$ are zero. Double-click on the display block to see the concentrations in the three reactors.

Questions and tasks:
2.3   How does the concentration (mass) propagate through the system? Describe in qualitative terms the difference in the pulse response between the reactors.
2.4   How long does it take for the system to return to steady state?
2.5   What are the top values in the three tanks, respectively?
2.6   Set `amp = 10;`. What are top values now? (**Provide plot**)
2.7   By looking at the pulse response for various values of `amp`, explain how it possible to see that the system is linear.
2.8   Change the time constant `T` to 0.5 and 2, respectively. Describe how this affects the propagation of the concentration. What is the physical interpretation of changing `T`?

To further reduce the amount of water used, whitewater is generally recycled. The recirculation efficiency may vary, but for different reasons it cannot be 100%. A simple model of such a whitewater system is shown in the figure below. Note that the 'recycling model' is extremely simplified and only returns 'pure mass' and no associated water flow. In reality, a water flow would have to be included, which would in term have an effect on `T`.
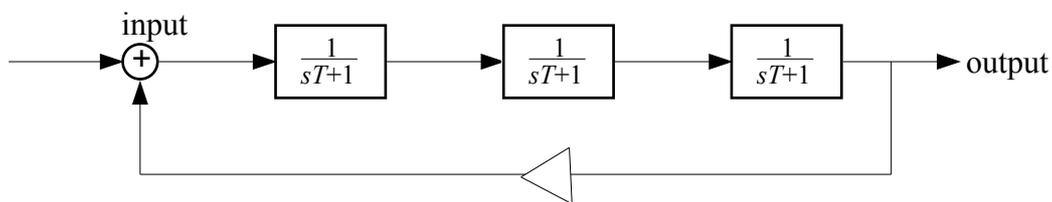
Questions and tasks:
2.9   Although not possible in practice, what will happen to the concentrations in the tanks if the 'recirculation efficiency' is 100%?
2.10   If the 'recirculation efficiency' is 50%, what will happen to the concentration in steady state?

Provided is a file with a model with recirculation of the whitewater (mass). Load a modified version of the tank model by typing:

       `>>mod1b`

and set the parameters

       `>>delay=0;rec=0.5;T=1;amp=5;`

In `mod1b` the pulse occurs between $t = 50$ and 51 to allow for the system to reach steady state (initial state values = 0). Run the model and analyse the result.

Questions and tasks:
2.11   Ignore the pulse. Is the steady-state concentration in the system what you expected? (**Provide plot**)
2.12   Describe the difference in the pulse response compared to the non-recirculation process?
2.13   Try different values for `rec, amp` and `T` and note how it affects the response and the steady state. Discuss the result in brief.

It is fair to assume that there will be a delay in the recirculation of the water due to, for instance, pumping and treatment of the whitewater (e.g. removal of particulate compounds). Set the parameter

       `>>delay = 5;`

and run the model again. The delay block will delay the recirculation (signal) by 5 time units.

Questions and tasks:
2.14 How does the response look now? (**Provide plot**)
2.15 Try to explain (in words) why the concentration is oscillating?
2.16 Give some examples of how this may make the control of the process more difficult?

3. Mechanical system

A mass and a spring with an applied force F are described by the Newton equation. Assume that the system we are interested in can be modelled as:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -(k/m)x_1 - (b/m)x_2 + F/m$$

where x_1 is the position, x_2 the velocity of the mass, k the spring constant and b the viscous damping. For simplicity we assume that the mass $m=1$. We also assume that x_1 can be measured directly.

Questions and tasks:
3.1 Express the model in state-space form and give the matrices A, B and C.
3.2 If $k = 1$, for which values of b is the system stable, oscillating and unstable. (Hint: use the `eig` function to calculate the poles of the system.)

A simple model (mod2.mdl) is among the downloaded files. Load the model into workspace by typing:

>>mod2

As you see, the model contains a block for state-space equations, a step-input block and a display block. Set the parameters:

>>F=1;k=1;b=5;

Create the A, B and C matrices as variables in Matlab (i.e. `A=…; B=…; C=…;`). Double-click on the state-space block. Type in the names of the state-space matrices you have created (let $D = 0$, initial value and tolerance: default values). Make sure you size the `A`, `B`, `C` and `D` matrices correctly.

Questions and tasks:
3.3 What are the poles of the system?
3.4 Run the model and watch the result. Describe the step-response of the system. (**Provide plot**).
3.5 Use different values for F. Does the system respond in a linear way?
3.6 Let `F=1` and `b=5` be fixed and try `k=0.5` and `k=2`. How does a reduction respectively an increase of `k` affect the response?
3.7 Let `F=1` and `k=1` be fixed. Use `b=10` and `b=1`. Describe how this affects the system. Evidently, something fundamental has happened to the system when `b` was set to 1. Explain what has happened. (**Provide plot**).
3.8 Change `b` in a few steps until the system becomes unstable. Does the value correspond to the value you calculated in 3.2?

Now, let us introduce a non-linear component in the model. The system is described by:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -(k_1/m)x_1 - (b/m)x_2 - (k_2/m)x_1^3 + F/m$$

The model for this system is loaded by typing:

>>mod2b

In this model you just have to provide the values for k_1, k_2 and b. Start with

>>k1=1;k2=0.2;b=1;F=1;

It is assumed that $m=1$.

Questions and tasks:
3.9   Run the model and compare the result to that of the linear model. Is the response different?
3.10 What is the steady state value?
3.11 If the model was *linear*, what would the steady state value be if F was set to 10?
3.12 Set $F=10$ and run the model. What is the steady state value? (**Provide plot**).
3.13 Try some different values of k2 (e.g. 10, 5, 0.01). Describe how the system differs from the linear system modelled before in terms of gain and frequency with increasing values of k2.

4. Stability of a non-linear system

Consider the non-linear system:

$$\dot{x}_1 = -x_2 + k_1 x_1^3$$
$$\dot{x}_2 = x_1 + k_2 x_2^3$$

The behaviour of this will very much depend on the values (and signs) of k_1 and k_2, respectively. By looking at the states in the phase plane (i.e. x_1 plotted as a function of x_2) some interesting behaviour can be observed.

Questions and tasks:
4.1   Try to describe what will happen in the phase plane if k_1 and k_2 are both zero and the initial state of the system is [1, 3]? (Hint: look at the poles of the system. Will the system oscillate and what is the damping of the system?)

Now, load the model:

>> mod4

and set the parameters:

>>k1=0;k2=0;amp=0;xInitial4=[1, 3];

Run the model and watch the result.

4.2   Did the phase plane trajectory behave as you expected?

4.3   Try the initial values [6, -2] (i.e. `xInitial4=[6, -2];`) and describe the result. (**Provide plot**)

The system is thus oscillating and the amplitude depends on the initial values of the system. What would happen if k_1 and k_2 are set to positive values? And what would happen if they were set to negative values? Think about it before you run the model.

4.4   Set `k1` and `k2` to –0.1, respectively and run the model. Describe what happens. Is the system stable? (**Provide plot**).

4.5   Set `xInitial4=[0.3, 0.1]` and set `k1` and `k2` to 0.1, respectively, and run the model. Describe what happens. Is the system stable?

Non-linear system may sometimes be extremely sensitive to relatively small disturbances and it is often very difficult to predict what will happen. Set
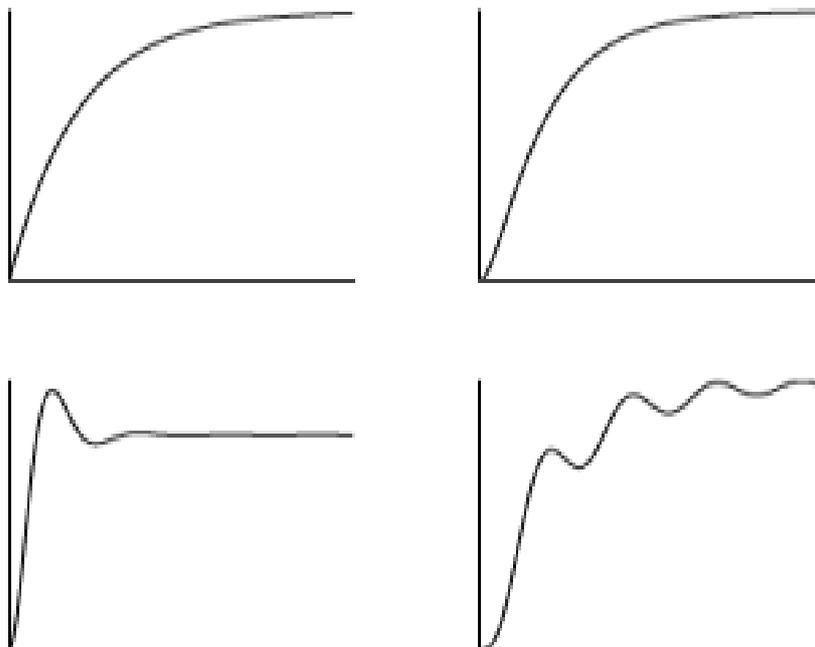
>>amp=0.16; k1=0.5;k2=-1;xInitial4=[0.3, 0.1];

`amp` is a step disturbance of the system effective from time=50 and onwards. Run the model and watch the result.

4.6   Does the system appear to be stable after the disturbance?

4.7   Now, set `amp` = 0.31. What happens to the system now? (**Provide plot**)

5. Step-responses

Look at the figure below.

Step responses of four different systems.

Questions and tasks:

5.1   Try to say something about the poles of the systems that produce the step responses in the figure. Discuss it in words or draw the principle locations of the poles in the s-plane (hint: the orders of the systems range from one to three).