Markov chain

states

M1: broken (0) / operating (1)
M2: — " —
B: empty (0) / full (1)

} 8 states

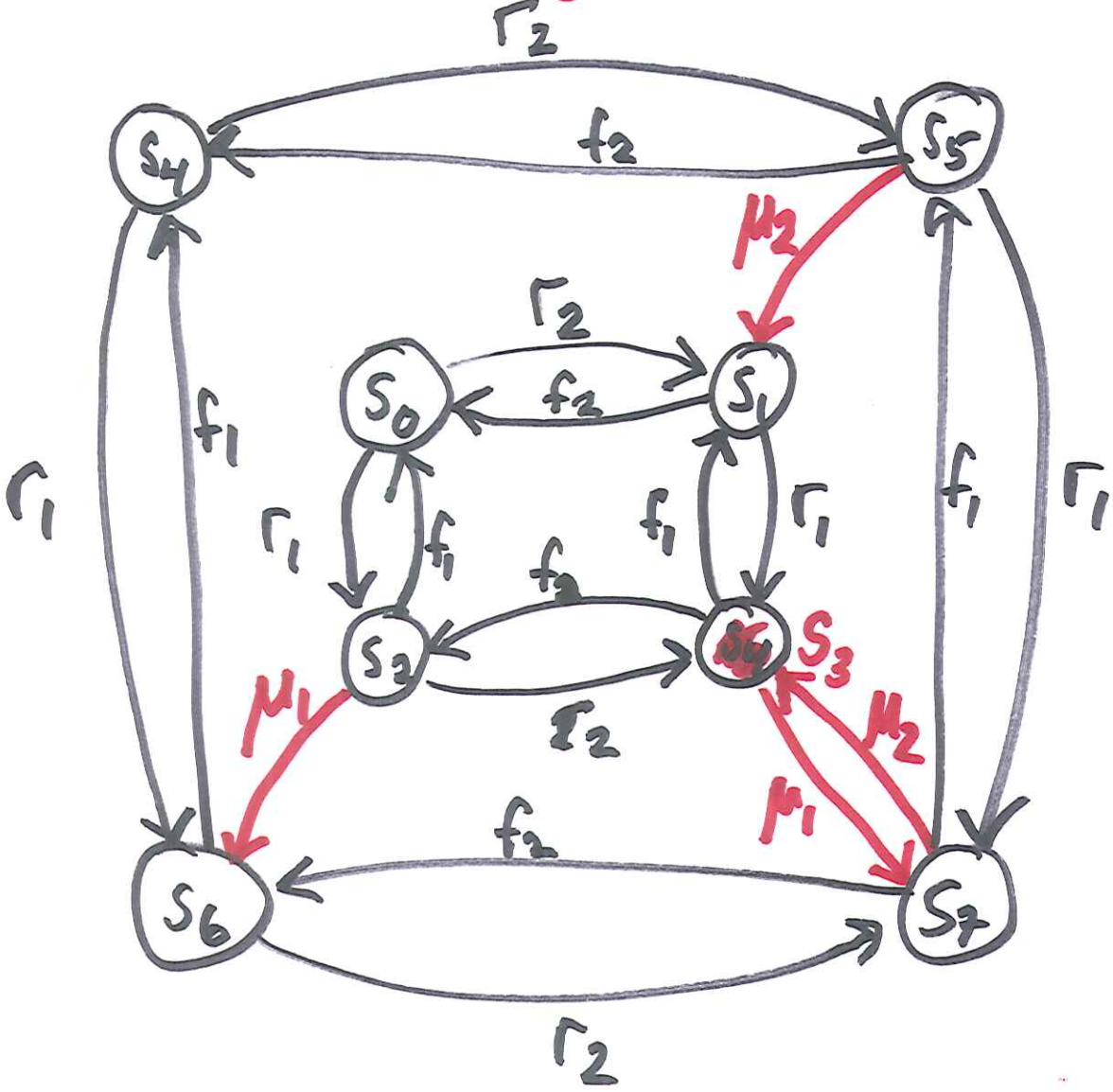
parameters

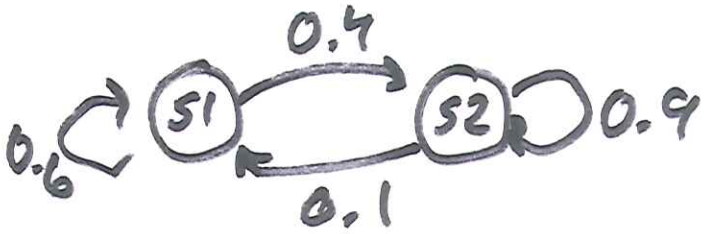
repair prob/rates: r_1, r_2
failure prob/rates: f_1, f_2
production rates: μ_1, μ_2

state name	B	M1	M2
S_0	0	0	0
S_1	0	0	1
M1 prod. S_2	0	1	0
M1 prod. S_3	0	1	1
S_4	1	0	0
S_5	1	0	1
M2 prod. S_6	1	1	0
M2 prod. S_7	1	1	1

$\Rightarrow$ state graph

State graph


r2
f2
S4
S5
μ2
r2
f2
S0
S1
f1
r1
r1
f1
f1
r1
f1
r1
f2
S3
S2
S3
μ1
r2
μ2
μ1
f2
S6
S7
r2



$$P = \begin{pmatrix} 0.6 & 0.4 \\ 0.1 & 0.9 \end{pmatrix}$$

$p(k+1) = p(k) \cdot P$

assume $p(0) = (0\ \ 1)$

$p(0)$ row sum $= 1$

$p(0) = (0.5\ \ 0.5)$

$p(0) = (0.5\ \ 0.7) \neq 1$ wrong

$P_{stat}(I-P) = 0$

$P_{stat} = (p_1\ \ p_2)$

$$(I-P) = \begin{pmatrix} 0.4 & -0.4 \\ -0.1 & 0.1 \end{pmatrix}$$

$$\begin{cases} 0.4p_1 - 0.1p_2 = 0 \\ -0.4p_1 + 0.1p_2 = 0 \end{cases}$$

$\Longrightarrow 0 = 0$

need info

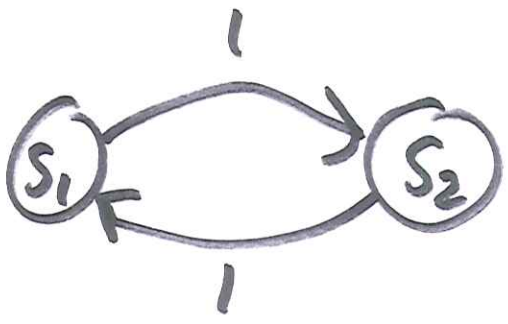
$p_1 + p_2 = 1$

then solve

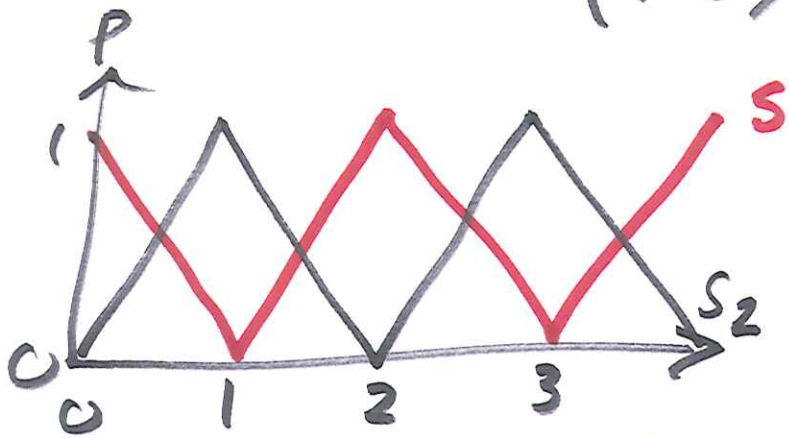
(3)

Periodic Markov Chain

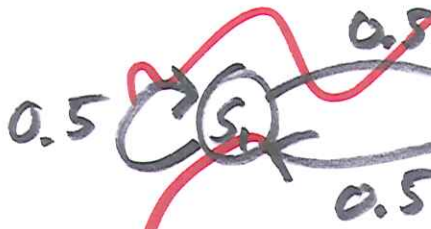
$$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



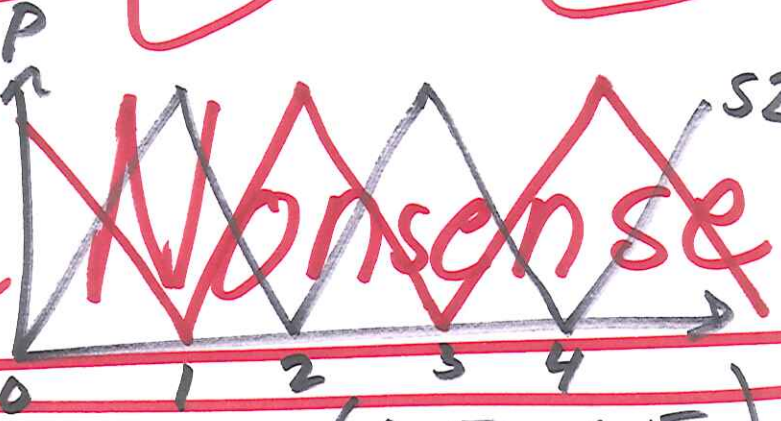
periodic



$eig(P) = -1$



$P_{stat} = (0.5 \quad 0.5)$



Complete Nonsense!!

$eig(P) = -1$

$$P = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$$

also periodic according to our defintion