Exercises on Dynamic Systems and Markov Chains/Processes

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Reading instructions

Chapter 3: Section 3.6 not included

Chapter 6: Section 6.6 not included.

All questions in chapter 6.9 except 6.21c, 6.26 – 6.29 can be solved. Solutions available at the course web site.

Start working on the simulation exercises as soon as possible.

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Example 1: Markov chain

A machine can be described as being in three different states: (1) under repair, (2) waiting for a new job, (3) working. While the machine is working the probability to break is 0.05 and the probability to get finished (go to waiting) is 0.1. If the machine is under repair there is a 0.1 probability to get repaired, and then the machine will become waiting. A broken machine is never brought directly (in one step) to operation. If the machine is waiting there is a 0.9 probability to get into operation. A waiting machine does not break.

a) Describe the system as a Markov chain and define the transition probability matrix.
b) Make a state diagram
c) Assume that the machine is waiting at time 0. What is the probability
   a. to be operating at time=1?
   b. to be under repair at time=1?
d) What is the probability to be operating after a long time?
e) If the machine is under repair (after a long time), how long time does it take, in average, to return to the repair state again?

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Example 2: Markov process

Consider the same machine as in the previous problem, still being described as being in three different states: (1) under repair, (2) waiting for a new job, (3) working. We will now analyze the machine behavior in *continuous* time.
The repair rate is now *r*, and you have to determine *r*.
The rate from waiting to operating is 3 per hour, the rate from operating to waiting is only 0.1 per hour, while the failure rate is 0.02 per hour (which is to say that the MTBF is 50 hours).
a) Describe this problem as a Markov process, and define the generator matrix
b) Draw the state diagram
c) Determine the repair rate *r*, so that the machine in steady state will be in the repair state less than 3% of the time.
d) What is the machine availability after a long time (using *r* from (c) and defining availability = working)?

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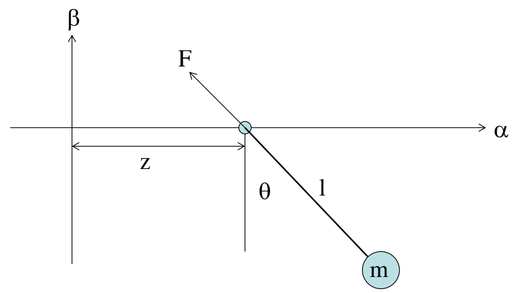
Dynamic system: the pendulum



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Example 3: Dynamic system

The pendulum on the previous slide represents a classical non-linear control problem. The pendulum has mass *m* and length *l* in a homogenous gravity field (*g*). The 'pivot point' of the pendulum is free to move along a straight line perpendicular to the gravity field (*z*). The control variable *u* is the acceleration of the pivot point (let $u = -d^2z/dt^2$) and the angular deviation $\theta = y$ is the measured variable.

a) Set up a general time-domain model for the pendulum and note that you have a fixed coordinate system(α,β) to use as a reference. Motivate any assumptions you make (Newton's law F=m·a).
b) Determine the linearised system at the equilibrium $\theta = 0$.
c) Determine the linearised system at the equilibrium $\theta = \pi$.
d) Based on the linearisations discuss the stability of the system at $\theta = 0$ and $\theta = \pi$.
e) Determine the transfer function for the system in (b).

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Matlab stuff

Eigenvalues of **A**: eig(A)
A raised to the power of n: A^n
Inverse of **A**: inv(A)
Transponate of **A**: A'
Determinant of **A**: det(A) always use Matlab help!!!

To solve an equation system **A**·**x** = **B**: B\A

For Markov an extra condition is needed to solve $\mathbf{p}_{stat}\cdot(\mathbf{I} - \mathbf{P}) = 0$ or $\mathbf{p}_{stat}\cdot\mathbf{A} = 0$ based on the sum(**p**) = 1. Example for Markov chains:

```
m = size(P,1);
a = (eye(m,m) - P);  %create (I-P)
a = a';              %transpone (I-P)
a(m,:) = 1;          %replace last (could be any) row with 1:s
b = zeros(m,1);      %create a zero column vector
b(m)=1;              %the sum of probabilities = 1
c = a\b;             %solve the system using \ (Gauss elimination)
```

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