As it has been calculated in the book, the probability that the machine works after time t can be obtained as below.

$$\frac{dp_2}{dt} = -f \cdot p_2 \Rightarrow p_2(t) = p_2(0) \cdot e^{-ft}$$

If we assume that the machine is working at $t = 0$, we will have,

$$p_2(t) = e^{-ft} \tag{1}$$

Equation 1 shows the probability of zero failures after t units of time. In mathematical point of view would be $p(T > t)$ where the random variable for the waiting time until the first faliure.
We know that "*Mean Time To First Filure* (MTTFF)" can be obtained from **Equation** 2, where $f(t)$ is "*Probability Density Function (PDF)*" the time between the failures.

$$MTTFF = \int_0^\infty t \cdot f(t)\,dt \tag{2}$$

For obtaining PDF, we must first derive the "*Cumulative Distribution Function (CDF)*" which is the probability that the the first failure would occur at time $T \leqslant t$.

$$CDF = p(T \leqslant t) = 1 - P(T > t) = 1 - e^{-ft} \tag{3}$$

$$PDF = \frac{d}{dt}(CDF) = \frac{d}{dt}(1 - e^{-ft})$$

$$PDF = fe^{-ft} \tag{4}$$

Thus, the mean MTTFF would be,

$$MTTFF = \int_0^\infty f \underbrace{t}_{u} \underbrace{e^{-ft}\,dt}_{dv} = uv - \int_0^\infty v\,du$$

$$u = t \Rightarrow du = dt \qquad dv = e^{-ft}\,dt \Rightarrow v = -\frac{1}{f}e^{-ft}$$

$$MTTFF = -te^{-ft}\Big|_0^\infty - \int_0^\infty -e^{-ft}\,dt$$

$$MTTFF = \lim_{t\to\infty} -te^{-ft} - \cancelto{0}{\lim_{t\to\infty} \frac{1}{f}e^{-ft}} + \frac{1}{f}$$

$$MTTFF = -\lim_{t\to\infty} \frac{1}{te^{ft}} + \frac{1}{f} \Rightarrow MTTFF = \frac{1}{f}$$

Corrected by Mr. Parsa Rostami, Automation student 2021.
Great thanks to Parsa!
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