Automation – EIEF45

Exam Monday April 20, 2020

You may bring the course book and the hardcopy of the chapter on HMI, but *not* the solutions to problems nor your own solutions to simulation tasks, lecture slides, home works or other personal notes. A calculator (memory cleared), TEFYMA, formula sheet from the Automatic Control department and a dictionary are permitted. You may answer in **Swedish** or in **English**.

Grading: There are 30 points all together. The following grades will apply:
Grade 3: at least 15 points
Grade 4: at least 20 points
Grade 5: at least 25 points

Note that all answers should be complete and well motivated. Your line of thought should be easy to follow and hand calculations should be provided for all mathematical problems. Corrections will be completed not later than *Tuesday **May 19,** 2020*.

As this is a remote exam (due to the Corona pandemic) there is an **extra need** that you provide complete analytical/well formulated/well written solutions to all your answers. By taking this exam you hereby confirm that you have read all the information and instructions regarding the exam sent by email (also mostly available at the home page of the course) by the IEA division, Dr Ulf Jeppsson (major emails: April 2, April 14, April 17, April 19) and you have agreed to follow all those instructions to the letter. If NOT, you should stop the examination NOW and come back for the next exam opportunity on August 28, 2020.

Also note that this written exam may be complemented by an individual oral exam before a final grade is issued to any student (as described in the emailed instructions).

Good Luck!

Problem 1 (4 points)
A non-linear third-order dynamic system is defined as:

$$\begin{cases} \dfrac{d^3 z}{dt^3} + z^2 \dfrac{dz}{dt} + a \cdot z^3 = u \\ y = \begin{cases} z \\ \dfrac{dz}{dt} \\ \dfrac{d^2 z}{dt^2} + u \end{cases} \end{cases}$$

where z is the state variable, u is a scalar input of the system, a is a real constant model parameter and $\boldsymbol{y}$ is the measurement vector.

a) Rewrite the above system as three differential equations using the state variable substitution $x_1 = z$, $x_2 = dz/dt$ and $x_3 = d^2z/dt^2$. (**1 p**)
b) Linearize the system at the point $x_1 = 1$, $x_2 = 1$, $x_3 = 1$, $u = 1$ and provide the complete state-space representation (**A**, **B**, **C**, **D** matrices). (**1 p**)
c) Assume a=–4/3 for the linearized system. Is it stable? (**1 p**)
d) Discuss advantages and disadvantages of using the linearized system rather than the original non-linear one. (**1 p**)

Solution:

a) Using the suggested variable substitution yields:

$$\begin{cases} \dfrac{dx_1}{dt} = x_2 \\ \dfrac{dx_2}{dt} = x_3 \\ \dfrac{dx_3}{dt} + x_1^2 \cdot x_2 + a \cdot x_1^3 = u \quad \Leftrightarrow \quad \dfrac{dx_3}{dt} = -x_1^2 \cdot x_2 - a \cdot x_1^3 + u \end{cases}$$

$$y = \begin{cases} x_1 \\ x_2 \\ x_3 + u \end{cases}$$

b) The principle of how to linearize a system is given in the book on page 69. Clearly the selected operating point is not a good choice since it is not a steady state. But it provides some values so we can determine the state space matrices. In accordance with equations 3.38, 3.40, 3.41 and 3.42 in the book we get:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2x_{1,0} \cdot x_{2,0} - 3a \cdot x_{1,0}^2 & -x_{1,0}^2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2-3a & -1 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

c) One can determine the eigenvalues of **A** and check for stability or not, but that requires solving a third-order equation. In this case there is, however, a much easier way. With a=–4/3 the row sum of all rows of the **A**-matrix are equal to 1. Based on knowledge from Markov chains we know that if all row sums are equal to 1 then one eigenvalue *must* be equal to 1. A continuous system with the real part of an eigenvalue > 0 proves that the system in unstable and therefore the linearized system in (b) is unstable.

d) The possible mathematical tools that can be used to directly analyse and draw conclusions about the linear system are enormous and we can basically say everything about how the system will behave in any situation. It is also much easier to, for example, design a controller for the linear system. However, the main drawback is that it is only a perfect representation of the non-linear system at the operating point at which it was linearized and the further away we move the bigger will the differences be between the systems. Consequently, the behavior of the two systems may be very different just a short distance from the selected operating point and everything we thought we knew about the linear system may turn out to be very different for the actual non-linear one. The non-linear system may better represent reality but requires numerical tools/software and computers to analyse (e.g. sensitivity analysis, Monte Carlo simulations) and it is difficult say when we have really analysed all aspects of the system completely.

Problem 2 (2 points)
A 3rd order linear dynamic continuous system is defined by its transfer function

$$G(s)=\frac{2}{s^3+4s^2+9s+10}$$

a) What are the characteristics of this system in terms of stability and possible oscillations)? Motivate. (hint: one eigenvalue = –2) (**1 p**)
b) Transform the system into its state-space representation and provide the **A**, **B**, **C** and **D** matrices. (hint: assume **y**=**x** in the time domain, *u(t)* is scalar and use $dx_1/dt=x_2$ and $dx_2/dt=x_3$) (**1 p**)

Solution

a) The poles of a transfer function are identical to the system's eigenvalues. We know one eigenvalue = -2 and can therefore rewrite the denominator as

$$s^3+4s^2+9s+10=(s+2)(s^2+2s+5)$$

Setting the above equation = 0 gives the eigenvalues: -2, -1+2i and -1-2i. All eigenvalues have a negative real part, i.e. the system is *stable*. The complex conjugated pair of eigenvalues proofs that the system will demonstrate some *oscillatory behaviour*.

b) Inverse Laplace transformation yields:

$$\frac{d^3\mathbf{y}(t)}{dt^3}+4\frac{d^2\mathbf{y}(t)}{dt^2}+9\frac{d\mathbf{y}(t)}{dt}+10\mathbf{y}(t)=2u(t)$$

Based on the information in the task we can rewrite the equation as:

$$\begin{cases}\frac{dx_1}{dt}=x_2\\ \frac{dx_2}{dt}=x_3\\ \frac{dx_3}{dt}=-4x_3-9x_2-10x_1+2u\end{cases}$$

and from that identify the **A** and **B** matrices as (**C** and **D** obvious since **y**=**x**):

$$\mathbf{A}=\begin{bmatrix}0&1&0\\0&0&1\\-10&-9&-4\end{bmatrix}\quad \mathbf{B}=\begin{bmatrix}0\\0\\2\end{bmatrix}\quad \mathbf{C}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\quad \mathbf{D}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

Problem 3 (1 point)
It is essential that raw material is delivered to a manufacturing process in time. Assume that the delivery can be described as a Poisson process with the average delivery rate of 3 parts per hour and that the delivery process is initiated two hours before the process work should start at the beginning of the working day. What is then the probability that there is at least 3 parts available at the beginning of the working day?

Solution
This is a traditional birth process. We define $p_k(t)$ as the probability that there are k jobs in the system at time t. Here $\lambda = 3$ per hour. At least three parts means the same as (1 – no parts – one part – two parts). The equation from the book, page 198, gives:
The probability that there are exactly 0 jobs in the system at time $t = 2$ is:

$p_0(2) = e^{-3\cdot 2} = e^{-6} \approx 0.00248$, i.e. about 0.248%.

The probability that there are exactly 1 job in the system at time $t = 2$ is:

$p_1(2) = (3\cdot 2)\cdot e^{-3\cdot 2} = 6\cdot e^{-6} \approx 0.0149$, i.e. about 1.49%.

The probability that there are exactly 2 jobs in the system at time $t = 2$ is:

$p_2(2) = (3\cdot 2)^2/(2!)\cdot e^{-3\cdot 2} = 36/2\cdot e^{-6} \approx 0.0446$, i.e. about 4.46%.

So the probability of there being at least 3 jobs in the buffer at time t=2 is $1 - p_0(2) - p_1(2) - p_2(2) \approx 0.938$, i.e. about 93.8%.

Problem 4 (2 points)
A transfer line consists of three machines (M1, M2 and M3) in series with a buffer, B1, between M1 and M2 and another buffer, B2, between M2 and M3. The system is described as a Markov process. The machines can be either operating or broken, B1 can hold 0, 1, 2 or 3 items, B2 can hold 0, 1, 2, 3 or 4 items.

a) How many states are required to represent the complete system? (**0.5 p**)
b) Define a mathematical expression for the probability that M3 is starved? (**0.5 p**)
c) Define a mathematical expression for the average buffer size of B1? (**1 p**)

(Clearly explain any symbols you use in (b) and (c).)

Solution:

a) 2*2*2*4*5 = 160 states
b) M3 is starved when the B2 buffer is empty and M3 is operating. The states of M1, M2 and B1 do not matter. If we use the formulation p(α1, α2, α3, B1n, B2n) to represent the probability of a specific state, where α1 is the state of M1 (0 or 1), α2 is the state of M2 (0 or 1), α3 is the state of M3 (0 or 1), B1n is the state of B1 (0, 1, 2 or 3) and B2n is the state of B2 (0, 1, 2, 3 or 4) we can give the general expression as the sum of 16 states when M3 is starved:

$$p(starved) = \sum_{\alpha 1=0}^{1} \sum_{\alpha 2=0}^{1} \sum_{B1n=0}^{3} p(\alpha 1, \alpha 2, 1, B1n, 0)$$

c) The average buffer size of B1 can be expressed as the sum of 120 states*B1 size:

$$\text{average buffer size of B1} = \sum_{\alpha 1=0}^{1} \sum_{\alpha 2=0}^{1} \sum_{\alpha 2=0}^{1} \sum_{B1n=1}^{3} \sum_{B2n=0}^{4} \left(B1n \cdot p(\alpha 1, \alpha 2, \alpha 3, B1n, B2n)\right)$$

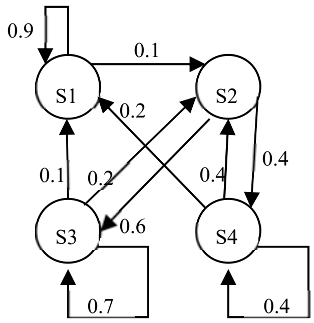
where the unimportant 40 states related to the case when B1n=0 are not included above.

Problem 5 (4 points)
A machine can be described as being in four different states: (1) under repair, (2) waiting for a new job, (3) working mode 1, and (4) working mode 2. While the machine is working in mode 1 the probability to break down is 0.1 and the probability to get finished (go to waiting) is 0.2. While the machine is working in mode 2 the probability to break is 0.2 and the probability to get finished (go to waiting) is 0.4. If the machine is under repair there is a 0.1 probability to get repaired, and then the machine will become waiting. A broken machine is never brought directly (in one step) to operation. If the machine is waiting there is a 0.6 probability to get into operation mode 1 and a 0.4 probability to get into operation mode 2. A waiting machine does not break and the machine does not change directly from one operating mode to the other.

a) Describe the system as a Markov chain, define the transition probability matrix and make a state diagram. (**1 p**)
b) Assume that the machine is broken at time 0. What is the probability
   a. to be operating in mode 1 at time=2? (**0.5 p**)
   b. to be waiting at time=3? (**0.5 p**)
c) What is the probability to be in working mode 2 after a long time? Complete solution required. (**2 p)**

Solution

a) S1 = under repair; S2 = waiting; S3 = working mode 1; S4 = working mode 2



The transition probability matrix is **P** =

$$\begin{bmatrix} 0.9 & 0.1 & 0 & 0 \\ 0 & 0 & 0.6 & 0.4 \\ 0.1 & 0.2 & 0.7 & 0 \\ 0.2 & 0.4 & 0 & 0.4 \end{bmatrix}$$

b) **p**(2) = **p**(1)***P** = **p**(0)***P**^{2}. Gives **p**(2) =
   $[0.81 \quad 0.09 \quad 0.06 \quad 0.04]$, i.e. the probability that the machine is in working mode 1 (i.e. in state 3) at time = 2 is consequently 0.06 = 6%.
   p(3) = **p**(2)***P**. Gives **p**(3) =
   $[0.743 \quad 0.109 \quad 0.096 \quad 0.052]$, i.e. the probability that the machine is waiting (i.e. in state 2) at time = 3 is consequently 0.109 = 10.9%.

c) Use e.g. Gauss elimination to solve the equation system (p1 p2 p3 p4)*(**I** – **P**) = 0 together with the extra condition p1 + p2 + p3 + p4 = 1. This gives:

$$\begin{cases} 0.1 \cdot p1 - 0.1 \cdot p3 - 0.2 \cdot p4 = 0 \\ -0.1 \cdot p1 + p2 - 0.2 \cdot p3 - 0.4 \cdot p4 = 0 \\ -0.6 \cdot p2 + 0.3 \cdot p3 = 0 \\ p1 + p2 + p3 + p4 = 1 \end{cases}$$

Above the fourth equation has already been removed and replaced by the normality condition. Some further calculations (not shown here but must be provided) yield p1 = 0.4762, p2 = 0.1429 and p3 = 0.2857 and p4 = 0.0952. So the stationary probability to be in working mode 2 (state 4) is p4(∞) = 0.0952 = 9.5%.

Problem 6 (2 points)
To save energy in a supermarket during the cold season the setpoint for the indoor temperature in the heating system is very low at night. This saves energy in two ways. The heating cost is lower and the energy consumption for the refrigerators is lower. However,

when the setpoint automatically is changed in the morning it takes quite some time to increase the indoor temperature and then, after that, it becomes too hot for 1-2 hours. The system contains only heating (no cooling) and is controlled by a PID controller.

a) Suggest a functionality that is probably not implemented in the controller. **(1 p)**
b) Assume the original system again. What is likely to happen if a much higher available heating power is installed? **(1 p)**

Solution

a) Since it takes a long time to reach the setpoint it can be assumed that the output has reached the limit for maximum power and consequently the integral part will create a windup problem. This is furthermore confirmed by the overshoot in the temperature. The controller probably has no anti-windup function.
b) When the output signal is unlimited there is no windup problem. Thus, a higher available heating power will most probably reduce the problem since the time with a saturated output will decrease. The setpoint temperature is reached faster and there is less overshoot.

Problem 7 (3 points)

a) Give a definition of what a real time program is. (**1 p**)
b) Mutual exclusion is an essential function in multitasking systems. Explain the basic functionalities that a tool must have for creating this function. (Note! It is not the complete tool that is asked for but the functionalities.) (**1 p**)
c) What is done by a real time scheduler during a context switch? (**1 p**)

Solution

a) "A program where the result depends upon when it was executed and not only the set of input data." This implies an interaction with I/O and/or real time clocks and the definition could also be formulated based on this interaction.
b) The basic functionality can be described by the "test-and-set" instruction. To avoid that two processes operate on a resource simultaneously it has to be possible to test a variable and based on the result change the value in an indivisible instruction.
c) The context of the running process bust be stored. Based on the state of the processes and their priority the next process to be run is selected. The context of this process is loaded and it is started at the address given by the stored context.

Problem 8 (2 points)

Why is the industry still using fieldbuses although Ethernet and TCP/IP is cheaper, faster and more available? Give as many reasons you can think of.

Solution

The fieldbuses are designed to operate in an industrial environment. Their physical and electrical specifications makes them more suitable. Real time aspects are also important since deterministic data transfer might be required. However, modifications to make Ethernet components like cables, connectors and switches more suitable for industry today often makes it an alternative today.

Problem 9 (3 point)
A production facility includes 4 machines. Two different product types are manufactured. The required processing sequence and time in each machine for the two product types are:

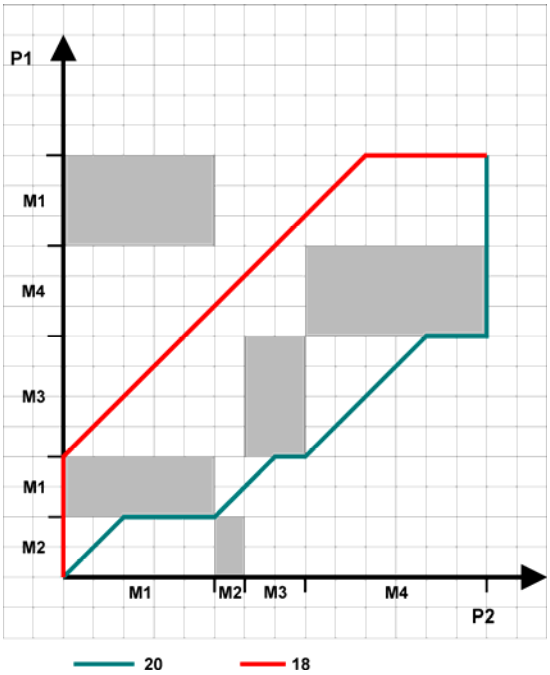
P1: M2(2), M1(2), M3(4), M4(3), M1(3)
P2: M1(5), M2(1), M3(2), M4(6)

(Machine M1 is used twice for product P1.) P1 and P2 are subparts and should be produced in equal quantities so next production cycle is not started until both are ready. Make a graphical solution showing the production flow for one cycle.

a) What is the shortest possible cycle time? (**2 p**)
b) An installation of a second machine of the same type as M1 is considered. Then one machine would be available for each product. How will this affect the production cycle time? (**1 p**)

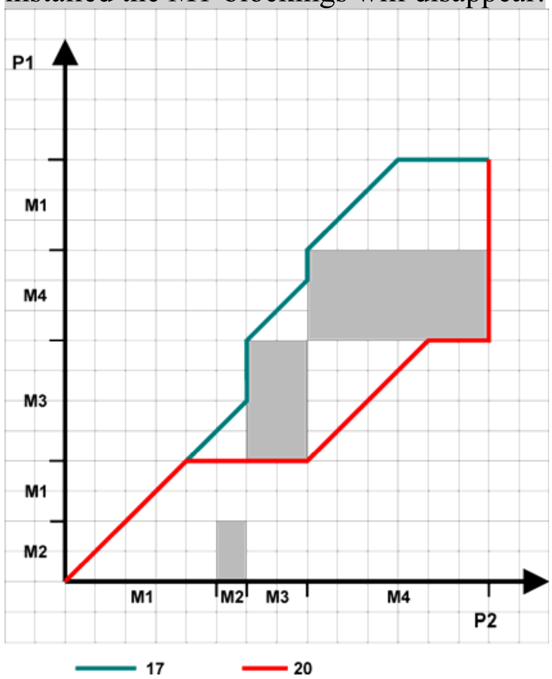
Solution

a)



The shortest path is the red one that consumes 18 timeunits.

b) If a second machine of the type M1 is installed the M1-blockings will disappear:



Problem 10 (1 point)
A 200 Hz sinusoidal measurement signal (constant frequency) is sampled with 1 kHz. However, when analysing the signal in the computer there seems to be another signal with a frequency of 400 Hz in addition to the measurement signal. No anti-alias filter has been installed. Provide a general expression of *all* input frequencies *above* the Nyquist frequency that could give rise to this ghost frequency.

Solution

Using the graph in Figure 10.3 in the book and the value from the task $f_0/f_s = 4/10 = 0.4$ gives the answer. At each instant (except the first crossing, which represents that the sampled frequency would have been a true frequency = 400 Hz) a horizontal line from $f_o/f_s = 0.4$ intersects the graph, that measurement frequency will give rise to the same alias frequency, i.e. when $f/f_s = 0.6, 1.4, 1.6, 2.4, 2.6$ etc. This means that the actual disturbance frequency could be 600, 1400, 1600, 2400, 2600 etc. Hz, all producing an alias frequency of 400 Hz. A general expression can therefore be given as $n*f_s \pm f_o$, where $f_o = 400$ Hz, $f_s = 1000$ Hz where n is a positive integer value larger than 0.

Problem 11 (1 point)

Digital filtering is often used to treat raw measurements before used in control or stored in data bases. Non-linear filtering compared to linear filtering is much more potent for this purpose.

a) Why? (**0.5 p**)
b) Give an example of a non-linear filter. (**0.5 p**)

Solution:

a) We can design the filter any way we need and add on logical rules, make the filters adaptive etc. and consequently produce a much better filter output for its intended purpose. With linear filter we are restricted to the general formulation of the ARMA filter and nothing else.
b) One of the simplest possible non-linear filter is a median-filter (calculates the median value of a number of previous data points as the filter output rather than the average value, as a linear MA-filter does).

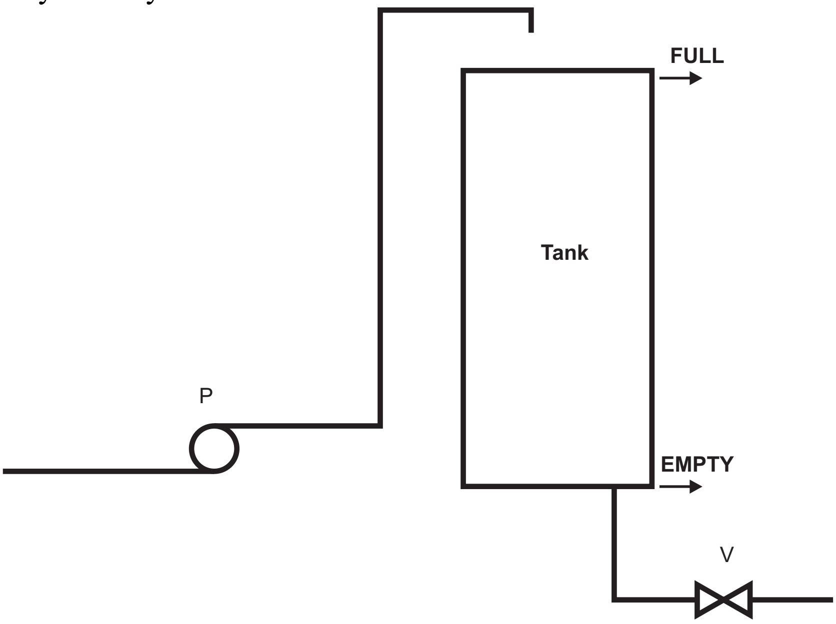
Problem 12 (2 points)

a) Explain why the number of choices should be kept low in a selection situation in a good HMI system? (**1 p**)
b) The operator's previous skills and experience are important when designing HMI systems. Why? What is achieved if they can be utilized? (**1 p**)

Solution:

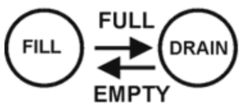
a) The 7+/-2 rule states that a human only can keep a limited number of "chunks" in the short term memory. In the menu selection situation we use this memory and it is important for the overview to remember the previous options when we scan through. This can be done if only a limited number of options are available at each level.
b) Human actions can be performed on three main levels. These levels differ in speed and security. The fastest level is the one where we almost don't notice the decision. It is so well learned that it does not require any attention. E.g. we move the hand away from a hot surface, we open a door handle or we ride a bike. The second level is rule based so that we associate the situation with something that we know how it works either from experience or learning. On the third level we have to analyse the problem and synthesise a solution and act according to that. This is of course the slowest level. To achieve fast and correct actions in an HMI system one should try to reach to the lowest level. This can be done by training but if there is a contradiction between the HMI design and the operator's previous experience, this is difficult. On the other hand, if the user can associate the design with previous experience the learning has already taken place.

Problem 13 (3 points)
A tank is filled by the pump, P. When the tank is full the signal FULL is given. The pump should then stop and the tank should be emptied by the valve V. When the tank is empty the signal EMPTY is given. The valve should close and the pump should start again. This should be repeated in a cyclic way.



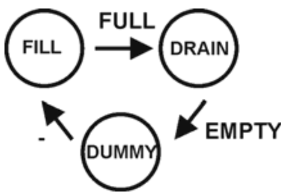
Draw a minimal state graph for the control of this system and write an output signal table.
Explain why this graph can't be directly implemented in a ladder program.
Make the necessary modifications and write a working ladder program for the control.

Solution:
The simple straightforward solution is:

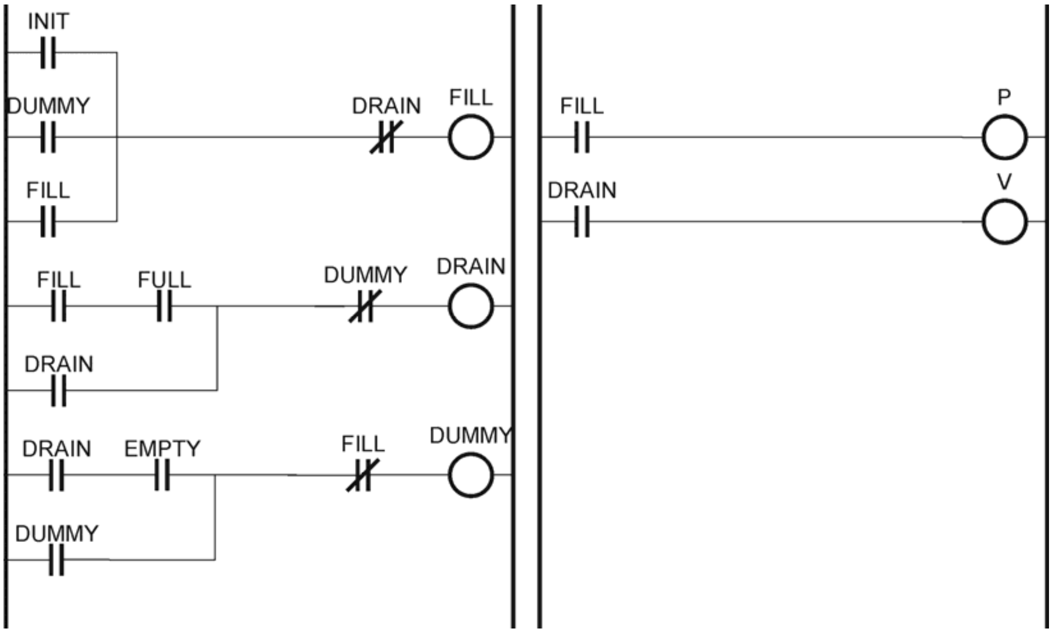


State	Output
FILL	P
DRAIN	V

To implement this in ladder directly is not possible with our normal method. Since the state transition is made safe by breaking/leaving a state upon the arrival to the next state a problem occurs. If the next and the previous states are the same state it means that the entry condition and the exit condition will contain the same state but in invers in the exit-part. A signal and it's invers in series can never be true.
The solution to this problem is the introduction of a dummy state that the sequence just passes through.



In this case the working ladder-code will be:


INIT
DUMMY
FILL
DRAIN
FILL
FILL
FULL
DUMMY
DRAIN
DRAIN
DRAIN
EMPTY
FILL
DUMMY
DUMMY
FILL
P
DRAIN
V