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by

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Relations between size and gear ratio in spur and planetary gear trains

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Abstract

In this report equations for the minimum gear sizes necessary to drive a given load are derived. The equations are based on the Swedish standards for spur gear dimensioning: SS 1863 and SS1871. Minimum size equations for both spur gear pairs and three-wheel planetary gears are presented. Furthermore, expressions for the gear weight and inertia as function of gear ratio, load torque and gear shape are derived.

For a given load torque and gear material, it is possible to retrieve the necessary gear size, weight and inertia as function of gear ratio. This is useful for gear optimization, but also for optimization of a complete drive system, where the gear size, inertia and weight may affect the requirements on the other parts of the drive system.

The results indicate that the Hertzian flank pressure limits the gear size in most cases. The teeth root bending stress is only limiting for very hard steels. Furthermore, the necessary sizes, weights and inertias are shown to be smaller for planetary gears than for the equivalent pinion and gear configuration. Both these results are consistent with state of practice; planetary gears are commonly known to be compact and to have low inertia.

Keywords

Spur Gears, Planetary Gears, Gearhead, Servo Drive, Optimization

Contents

Nomenclature

α	Pressure angle [rad]
β	Helix angel (= 0 for spur gears)
ε_α	Transverse contact ratio
θ	Load angle [rad]
ν	Poissons number
ρ	Mass density [kg/m^3]
σ_F	Root bending stress [Pa]
σ_{Fmax}	Maximum allowed bending stress [Pa]
σ_H	Hertzian flank pressure [Pa]
σ_{Hmax}	Maximum allowed hertzian flank pressure [Pa]
ω	Angular velocity [rad/s]
b	Gear width [m]
b_c	Carrier width [m]
d	Gear reference diameter [m]
d_a	Gear tip diameter [m]
d_b	Gear base diameter [m]
E	Module of elasticity [Pa]
F	Force [N]
J	Mass moment of inertia [kgm^2]
$K_{F\alpha}$	Factor describing the division of load between teeth
$K_{F\beta}$	Load distribution factor for bending
$K_{H\alpha}$	Factor describing the division of load between teeth
$K_{H\beta}$	Load distribution factor for Hertzian pressure
k_{ro}	Relation between outer and reference diameter of ring gear
m	Module [m]
M	Mass [kg]
n	Gear ratio of a complete gear train (ω_{in}/ω_{out})
p_b	Base pitch
r	Gear reference radius [m]
S	Safety factor
T	Torque [Nm]
u	Gear ratio of a single gear pair
v	Peripheral velocity [m/s]
Y_F	Form factor for bending
Y_β	Helix angle factor for bending
$Y\varepsilon$	Contact ratio factor for bending
z	Number of teeth
Z_H	Form factor for Hertzian pressure
Z_M	Material factor for Hertzian pressure
Z_ε	Contact ratio factor for Hertzian pressure
Index 1	The small wheel (pinion) of a gear pair
Index 2	The large wheel (gear wheel) of a gear pair
Index r	The ring gear in a planetary gear train
Index s	The sun gear in a planetary gear train
Index p	The planet gears in a planetary gear train
Index c	The planet carrier in a planetary gear train

1 Introduction / Background

This work was initiated within a research project about design and optimization methods for mechatronic systems. The goal with that research project is to derive methods for optimization of mechatronic actuation modules, with respect to weight, size and/or efficiency (Roos & Wikander 2004). In order to reach that goal it is necessary to have models relating gear size and weight to gear ratio and load (torque and angle of the outgoing shaft as function of time). The load in a mechatronic system is often dynamic and therefore inertia plays a central role in the optimization of a mechatronic module. The most commonly used geartypes in mechatronic applications are spur and helical gears, planetary gears and harmonic drives.

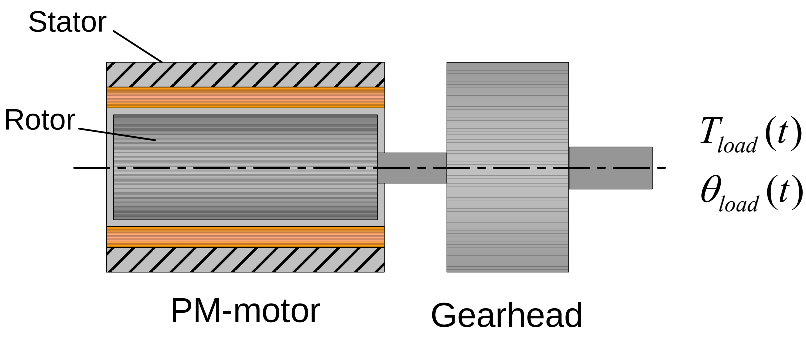

Figure 1. Electric motor and gearhead in an electromechanical servo drive

The work presented in this report treats spur gears in two different configurations, single spur geartrains and three-wheel planetary gears. All the expressions derived here are based on the two Swedish Standard documents for spur gear dimensioning, SS1863 and SS1871. The goal with the analysis is to express the gear size as function of gear ratio and output torque. From the size, it will be possible to derive the gear's mass and inertia. It is possible to derive the same relations for helical gears although it might be necessary to introduce some further simplifications.

This report is focused on gear sizes and properties that can be derived directly from the gear size and shape, e.g. inertia and weight. Other properties of a gear that are important in a mechatronic application are (Feinstein 1997):

- Configuration (inline or right angle)
- Accuracy and backlash
- Output speed
- Efficiency
- Environmental capabilities (sealing, noise, vibration)
- Cost

The number of parameters in the expressions derived in this report is large. Which parameters that are known depend on the design situation: In some cases the load is known and the gear size is to be minimized, while in others the size is known and the allowed output torque is supposed to be maximized. In this report all examples and equations are derived with the assumption that the load is known and the size, weight and inertia of gears that can drive that load are to be minimized. Moreover it is assumed that the gear material is known as well as the pressure angle.

2 Equivalent load

The load in a mechatronic application is usually a combination of inertial and friction type loads. The load torque is typically very dynamic, i.e. it changes with time. It will therefore be necessary to use an equivalent continuous load torque for the sizing of the gears. In electric motor sizing methods, the root mean square (RMS) value of the load cycle is used to calculate the equivalent continuous motor load. This is possible since it is heat that limits the continuous torque; the heat generated in the motor's winding is given by the RMS value of the motor current. Since the current is proportional to the motor torque, the RMS torque may be used for motor dimensioning.

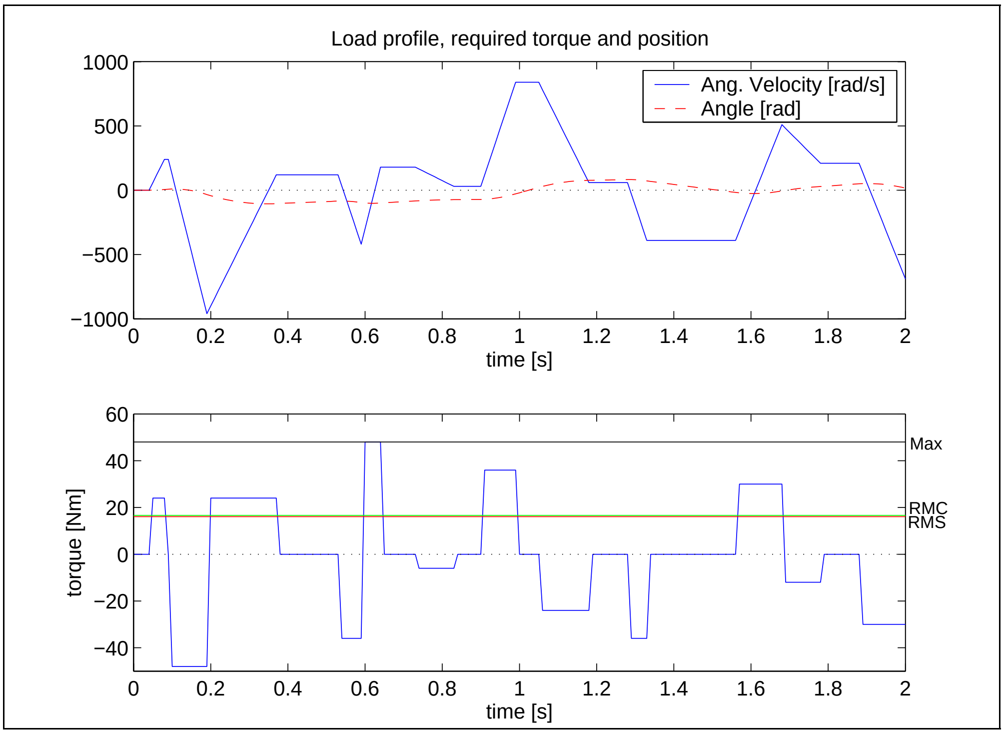

Figure 2. Example of an 'inertial' load cycle, with RMS, RMC and max norms shown.

Gear design is traditionally focused on strength of the gears. The load on the gear teeth is cyclic and therefore gear failure is most often a result of mechanical fatigue. The two classical limiting factors in gear design are surface fatigue and tooth root bending fatigue.

The combination of cyclic loading of the gear teeth when in mesh and an applied load that varies with time makes it more difficult to find an expression for the equivalent load, than in the motor case. The exponent used in the torque norms used for gear sizing is not 2 as in the RMS norm, but ranges from 3 to 50 (Anthony 2003). These expressions for equivalent load are based on the so-called linear cumulative damage rule (the Palmgren-Minor rule). It assumes that the total life of a mechanical product can be estimated by adding up the percentage of life consumed by each stress cycle. The number of stress cycles on each tooth in a gear train can be huge during a lifetime. Anthony (2003) exemplifies this with a three-wheel planetary gear in which one sun gear tooth will be exposed to almost 3 million load cycles during an 8-hour period at 2000 rpm.

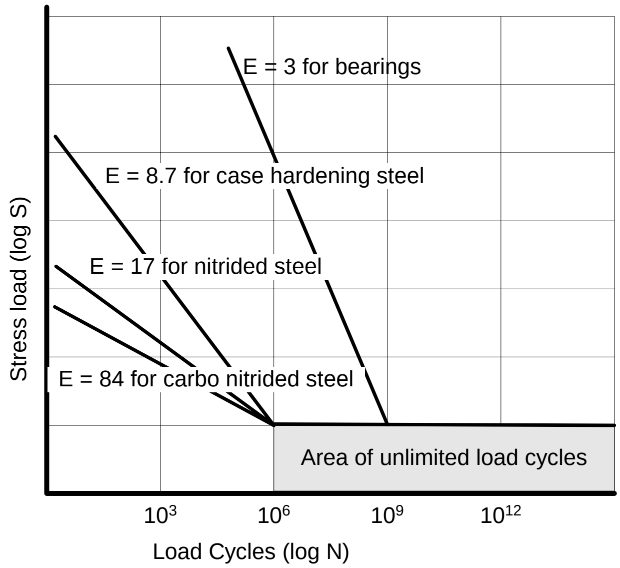

Figure 3. Whöler curves for different steels. The exponent to use in the calculation of equivalent load depends on material type, heat treatments, and loading type (Antony 2003).

It is however not obvious that the Palmgren-Minor rule can be used for infinite life design ($> 10^6$ load cycles), especially not in an application where the teeth will be subjected to the peak load more than 10^6 times. In fact only in applications where the total number of load cycles is below $2 \cdot 10^6$ is a higher load than the endurance limit load permissible (Antony 2003). This means that for infinite life dimensioning, the gears should be dimensioned with respect to the peak torque in the load cycle. Of course there are exceptions to this, for example load cycles where the peak load occurs while the gears are standing still. The calculated equivalent continuous torque, T_{cal} is hence, for unlimited life design given by:

$$T_{cal} = \|T(t)\|_{\max} \quad (1)$$

This is the approach taken in this report; it is assumed that the teeth are subjected to the peak load more than 10^6 times, and therefore is the peak torque used for dimensioning. This research area is however very complex and are not investigated further in this report. By this approach, equation (1), is at least not a too low equivalent torque used.

The sizing procedure gets even more complicated when the bearings are considered. For bearings, the Root Mean Cube (RMC) value of the load is often used as the equivalent continuous load (comp. Figure 3). This report will however only treat the actual dimensioning of the gears, not the bearings. But it should be noted that it may be the bearings that limit the maximum gear load.

3 Spur gear analysis

The analysis made here is mainly based on the formulas presented in the Swedish standard for calculation of load capacity of spur and helical gears, SS1871 and standard SS1863 for the spur gear geometry. Figure 4 shows a spur gear, to simplify the analysis only spur gears with no addendum modification are treated.

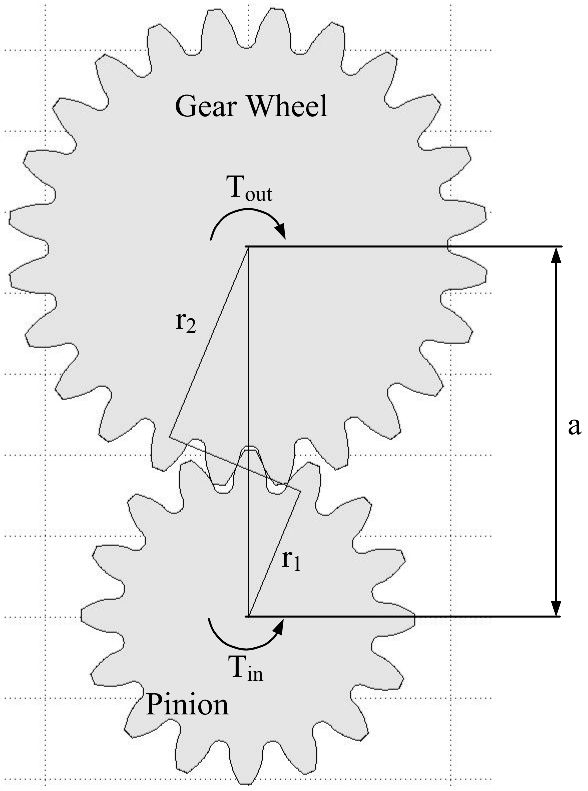

Figure 4. Spur gear.

3.1 Geometry, mass and inertia of spur gears.

3.1.1 Geometrical relationships

In order to simplify the rest of the analysis, it is useful to derive some simple geometrical relations.

The gear ratio *u* is defined as:

$$u = \frac{r_{out}}{r_{in}} = \frac{r_2}{r_1} = \frac{z_2}{z_1} = \frac{\omega_1}{\omega_2} \tag{2}$$

The center distance, *a* between the wheels is given by:

$$a = r_1 + r_2 \Rightarrow r_2 = a - r_1 \tag{3}$$

Combining equations (2) and (3) gives:

$$u = \frac{a - r_1}{r_1} \tag{4}$$

Finally, combining equations (3) and (4) gives the expressions for r_1 and r_2

$$r_1 = \frac{a}{u+1} \tag{5}$$

$$a - r_2 = \frac{a}{u+1} \Rightarrow r_2 = \frac{au}{u+1} \tag{6}$$

3.1.2 Gear pair mass

A gear wheel is here modeled as a cylinder, an approximation that is quite accurate. The mass, M of one gear is hence given by:

$$M = br^2\pi\rho \tag{7}$$

Where b is the face width, r is the reference radius and ρ is the mass density of the wheel. The total mass of a gear pair can be expressed as:

$$M_{tot} = M_1 + M_2 = \pi\rho b(r_1^2 + r_2^2) \tag{8}$$

Finally by combining equation (2), (5) and (8) the following expression for the gear pair mass is obtained:

$$M_{tot} = \pi\rho b r_1^2(1+u^2) = \pi\rho b a^2 \frac{1+u^2}{(1+u)^2} \tag{9}$$

3.1.3 Inertia

The inertia, J of a rotating cylinder is given by:

$$J_{cyl} = M_{cyl}\frac{r_{cyl}^2}{2} \tag{10}$$

The inertia reflected on the pinion shaft (axis 1) of a gear pair is hence given by:

$$J_{tot} = J_1 + \frac{J_2}{u^2} = M_1\frac{r_1^2}{2} + M_2\frac{r_2^2}{2u^2} = \pi\rho b r_1^2\frac{r_1^2}{2} + \pi\rho b r_2^2\frac{r_2^2}{2u^2} \tag{11}$$

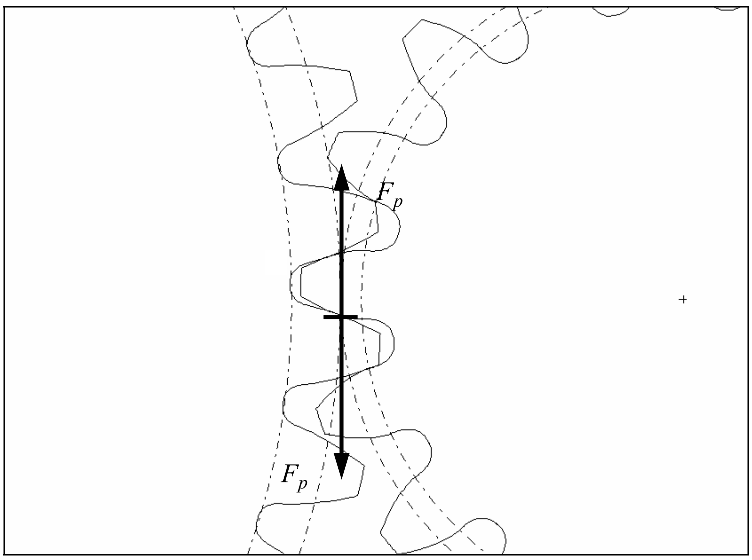

Figure 5. Gear mesh

Which, if combined with equations (5) and (6) result in the following expression of the gear pair inertia:

$$J_{tot} = \frac{\pi\rho b}{2}\left(\frac{a^4}{(u+1)^4} + \frac{a^4u^2}{(u+1)^4}\right) = \frac{\pi\rho}{2} ba^4 \frac{1+u^2}{(1+u)^4} \qquad (12)$$

3.2 Necessary gear size

According to SS 1871, the necessary gear size is determined from the teeth flank Hertzian pressure and the teeth root stress. Neglecting losses such as friction the peripheral force, see figure 5, acting on a gear tooth is given by:

$$F_p = \frac{T_{out}}{r_{out}} = \frac{T_{out}(u+1)}{au} \qquad (13)$$

For a given load the necessary gear size is determined as function of gear ratio and number of pinion teeth. Depending on material properties, gear ratio, number of teeth, etc., either the flank pressure or the root stress sets the limit on the gear size, in general both stress levels must be checked.

3.2.1 Hertzian pressure on the teeth flanks

The Hertzian pressure on a teeth flank is given by (SS1871):

$$\sigma_H = Z_H Z_M Z_\varepsilon \sqrt{\frac{F_{cal} K_{H\alpha} K_{H\beta}(u+1)}{bd_1 u}} \qquad (14)$$

For gears with no addendum modification the form factor Z_H is given by:

$$Z_H = \sqrt{\frac{2\cos\beta_b}{\sin 2\alpha_t}} \qquad (15)$$

As shown below, the transverse section pressure angle α_t is, for spur gears, the same as the normal section pressure angle, α_n. The pressure angle will therefore only be noted as α from now on.

$$\tan\alpha_t = \frac{\tan\alpha_n}{\cos\beta} \Rightarrow \{\beta = 0\} \Rightarrow \alpha_t = \alpha_n = \alpha \qquad (16)$$

Since the helix angle is zero on the pitch cylinder (β) it will be zero on the base cylinder too (β_b) :

$$\cos\beta_b = \frac{\cos\beta\cos\alpha}{\cos\alpha} \Rightarrow \{\beta = 0\} \Rightarrow \cos\beta_b = 1 \qquad (17)$$

This leads to the following expression for Z_H:

$$Z_H = \sqrt{\frac{2}{\sin 2\alpha}} \qquad (18)$$

The material factor Z_M is given by (SS1857):

$$Z_M = \sqrt{\frac{2}{\pi\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)}} \qquad (19)$$

Where E is the module of elasticity of the respective gear and v is poissons number. For spur gears the contact factor, Z_ε is according to SS 1871 given by:

$$Z_\varepsilon = \sqrt{\frac{4-\varepsilon_\alpha}{3}} \tag{20}$$

Where ε_α is the contact ratio. For an external spur gear pair it is, according to SS1863, given by:

$$\varepsilon_\alpha = \frac{1}{p_b}\left(\frac{\sqrt{d_{a1}^2 - d_{b1}^2}}{2} + \frac{\sqrt{d_{a2}^2 - d_{b2}^2}}{2} - a_w \sin\alpha_w\right) \tag{21}$$

Where a_w is the center distance between the wheels, for gears with no addendum modification, $a_w = a$. The base pitch, p_b is given by:

$$p_b = \pi m \cos\alpha \tag{22}$$

Where m is the module, which is defined as:

$$m = \frac{d_1}{z_1} = \frac{d_2}{z_2} = \frac{2a}{(u+1)z_1} \tag{23}$$

The tip and base diameters, d_a and d_b, are, for external spur gears, given by (SS1863):

$$d_a = d + 2m \qquad d_b = d\cos\alpha$$

$$d_a^2 - d_b^2 = (d+2m)^2 - d^2\cos^2\alpha = d^2 + 4m^2 + 4dm - d^2\cos^2\alpha \Rightarrow \{m = \frac{d}{z}\} \Rightarrow$$

$$\Rightarrow d_a^2 - d_b^2 = d^2(1-\cos^2\alpha + \frac{4}{z} + \frac{4}{z^2}) = d^2(\sin^2\alpha + \frac{4}{z} + \frac{4}{z^2}) \tag{24}$$

From equation (5) and (6) the following expressions for the gears diameters can be derived:

$$d_1 = \frac{2a}{u+1}, d_2 = \frac{2au}{u+1} \tag{25}$$

By combining equations (24) and (25), and inserting them into equation (21), the following expression for the contact ratio is obtained:

$$\varepsilon_\alpha = \frac{(u+1)z_1}{2a\pi\cos\alpha}\frac{1}{2}\left(\sqrt{\frac{4a^2}{(u+1)^2}(\sin^2\alpha + \frac{4}{z_1} + \frac{4}{z_1^2})} + \sqrt{\frac{4a^2u^2}{(u+1)^2}(\sin^2\alpha + \frac{4}{z_1 u} + \frac{4}{z_1^2 u^2})} - 2a\sin\alpha\right) \Rightarrow$$

$$\Rightarrow \varepsilon_\alpha = \frac{z_1}{2\pi\cos\alpha}\left(\sqrt{\sin^2\alpha + \frac{4}{z_1} + \frac{4}{z_1^2}} + u\sqrt{\sin^2\alpha + \frac{4}{z_1 u} + \frac{4}{z_1^2 u^2}} - (u+1)\sin\alpha\right) \tag{26}$$

Inserting equation (13) and (25) into equation (14) results in:

$$\sigma_H^2 = Z_H^2 Z_M^2 Z_\varepsilon^2 K_{H\alpha} K_{H\beta} \frac{T_{cal}(u+1)^3}{2ba^2u^2}$$

This equation can be rewritten as:

$$a^2 b = Z_H^2 Z_M^2 Z_\varepsilon^2 K_{H\alpha} K_{H\beta} \frac{T_{cal}(u+1)^3}{2u^2\sigma_{H\max}^2} \tag{27}$$

Where Z_H , Z_M, Z_ε, are given by equation (18), (19) and (20). $K_{H\alpha}$ and $K_{H\beta}$ are factors describing the division of load between teeth and the load distribution on each tooth respectively. Generally $K_{H\alpha}$ can be set to 1. $K_{H\beta}$ is more complicated since it only can be 1 in theory (if the gears are perfect). Here, for simplicity, it is set to 1.3, but if more exact data is available it should be used instead, see SS1871 for more information and guidelines about how to select this constant. Equation (27) gives the minimum size of the gear pair (with respect to Hertzian pressure) given a material, σ_{Hmax} , E_1, E_2, v_1, v_1, a gear ratio, u, the number of pinion teeth, z_1, the pressure angle α and the calculated torque T_{cal}.

3.2.2 Bending stress in the teeth roots

The bending stress in a tooth, σ_F can be calculated according to SS1871 as follows:

$$\sigma_F = Y_F Y_\beta Y_\varepsilon \frac{F_{cal} K_{F\alpha} K_{F\beta}}{bm} \tag{28}$$

The calculation of the form factor, Y_F is somewhat complicated the way it is done in SS1871, therefore Y_F is approximated with the following expression (Maskinelement handbok 2003):

$$Y_F \approx 2.2 + 3.1e^{-z/14} \tag{29}$$

Y_F will always be larger for the small wheel (pinion) since it decreases with z.

The helix angle factor Y_β is 1 for spur gears. Y_ε is the so called contact ratio factor, and it is according to SS1871 calculated as follows:

$$Y_\varepsilon = \frac{1}{\varepsilon_\alpha} \tag{30}$$

Where the contact ratio, ε_α is calculated as before with equation (26). By combining equation (13), (23) and (28) the following is obtained:

$$\sigma_F = Y_F Y_\varepsilon K_{F\alpha} K_{F\beta} \frac{T_{cal} z_1 (u+1)^2}{2ua^2 b} \tag{31}$$

The expression above can be rewritten as follows:

$$a^2 b = Y_F Y_\varepsilon K_{F\alpha} K_{F\beta} \frac{T_{cal} z_1 (u+1)^2}{2u\sigma_{F\max}} \tag{32}$$

Where Y_F and Y_ε is given by equation (29) and (30). $K_{F\alpha}$ and $K_{F\beta}$ are factors describing the load division between teeth and the load distribution on each tooth respectively. If no other data is available $K_{F\alpha}$ can be set to 1, and $K_{F\beta}$ to the same value as $K_{H\beta}$ (SS1871). Equation (32) can be used to calculate the minimum size of a gear pair, with respect to bending endurance.

3.2.3 Maximum allowed stress and pressure

The maximum allowed bending stress σ_{Fmax} and the maximum allowed hertzian pressure on the teeth, σ_{Hmax} is of course largely dependent on material choice and safety factor. There are a lot of factors that can be included into the calculation of the stress limits, including the number of load cycles. Here the number of load cycles is assumed to be larger than the endurance limit and therefore that factor is disregarded. The maximum allowed stress and pressure is here simply calculated as follows:

$$\sigma_{H\max} = \frac{\sigma_{H\lim}}{S_H} \qquad \sigma_{F\max} = \frac{\sigma_{F\lim}}{S_F}$$

Where σ_{Hlim} and σ_{Flim} are material properties and S_H and S_F are the safety factors. For more advanced calculations see SS1871. Values of σ_{Flim} and σ_{Hlim} can for example be retrieved from Maskinelement Handbok (2003) or from the standards.

It should be noted that if the safety factor for bending is doubled, the necessary gear size is doubled. But if the safety factor for the flank stress is doubled, it requires a four time larger gear pair, see equations (27) and (32) respectively.

3.3 Results and sizing examples

3.3.1 Necessary Size/Volume

In this section, equations (27) and (32) are applied on a (equivalent) load of 20 Nm. First, material data from an induction hardened steel with a Hertzian fatigue limit of 1200 MPa and a bending fatigue limit of 300 MPa is used. The Pressure angle is 20 degrees, and all other constants are set to the standard value (Table 1).

Material Properties		*Gear Properties*	
E	206 GPa	α	20 deg.
υ	0.3	$K_{H\alpha}$	1
ρ	7800 kg/m^3	$K_{H\beta}$	1.3
Load		$K_{F\alpha}$	1
T_{cal}	20 Nm	$K_{F\beta}$	1.3

Table 1. Values of material and gear parameters used in all examples.

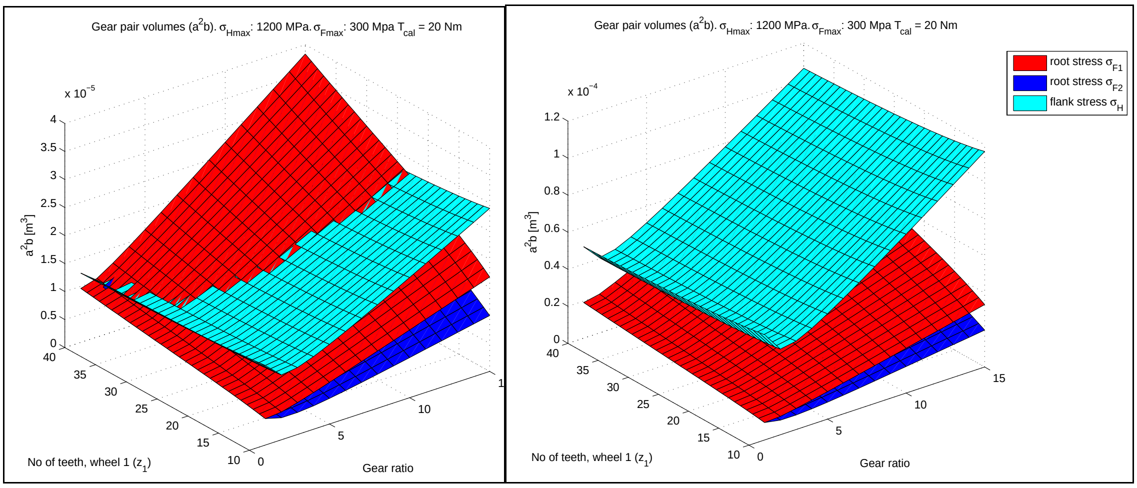

Figure 6. Gear pair volume as function of gear ratio and number of teeth. The safety factors SH and SF is 1 in the graph to the left and 2 in the graph to the right.

The following plots are obtained for a non-hardened steel with a Hertzian fatigue limit of 500 MPa and a bending fatigue limit of 200 MPa (e.g. SIS1550). The load and all other parameters are the same as in the example above.

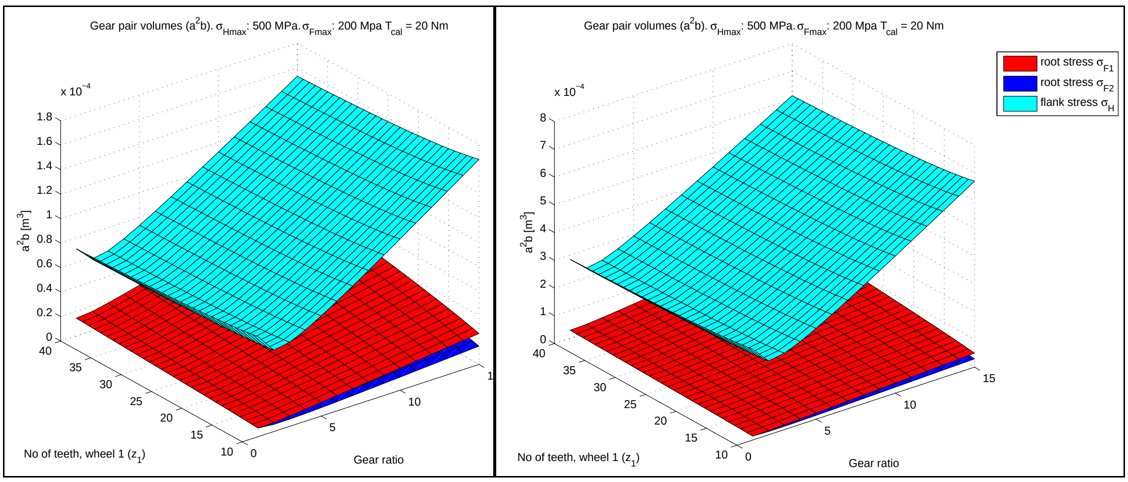

Figure 7. The same plots as in Figure 6, but for another steel. The safety factors are set to one in the plot to the left and 2 in the plot to the right.

From the plots above and a number of other examples not presented here, a couple of conclusions can be made. First of all, the Hertzian pressure is the limiting factor in the vast number of cases (it requires the largest gears). The root bending stress is the limiting factor only for steels with a large difference between the two stress limits, and only if the safety factor for Hertzian pressure is low. Of course it is possible to change a lot of constants to get a different result, but these are the results if the standard (SS1871) is applied with the constants set to the recommended values.

Furthermore the surface representing the root stress of the pinion (wheel 1) is always higher than corresponding surface for the gear wheel (wheel 2). This is consistent with the previous conclusion that the root stress always is larger for the smaller wheel. Therefore the root stress is only calculated for the pinion in the rest of this report.

The number of teeth has most influence on the root bending stress. Not surprisingly, the flank stress is in comparison almost independent of the number of teeth. By choosing a relatively small number of pinion teeth (wheel 1), the tooth flank stress will almost certainly be the limiting factor.

3.3.2 Gear pair mass, geometry and inertia

By combining equation (9) with the results shown in the left part of Figure 6 (safety factors = 1, induction hardened steel) the following graph (Figure 8) of the gear pair mass is obtained:

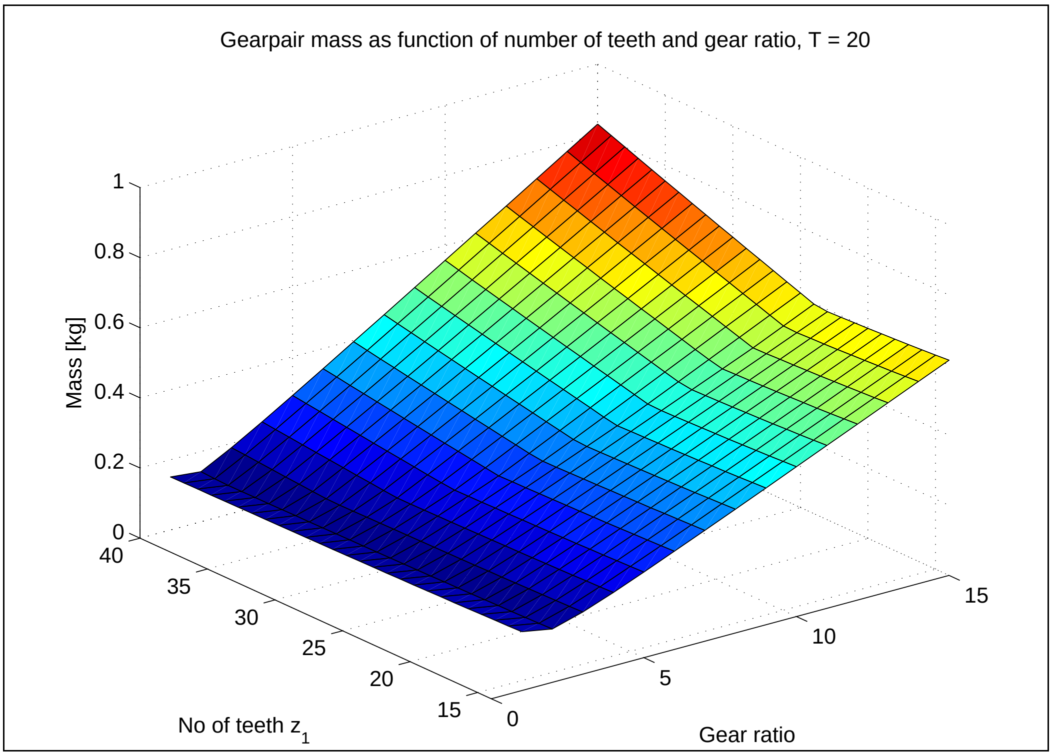

Figure 8. Gear pair weight as function of number of teeth of the pinion and gear ratio.

To continue the analysis it will be necessary to lock one of the variables in Figure 8, at least if the results are to be visualized in 3D-plots. The number of teeth of wheel 1 (pinion) seems to be the most reasonable variable to lock. In the figure below (Figure 9) plots of two different pinion teeth numbers are shown, 28 and 17. The larger choice results in, depending on the gear ratio, that booth the flank and root stress are limiting. The choice of 17 teeth of the small wheel results in, as also seen earlier, that only the flank stress is limiting the volume of the gear pair, regardless of gear ratio.

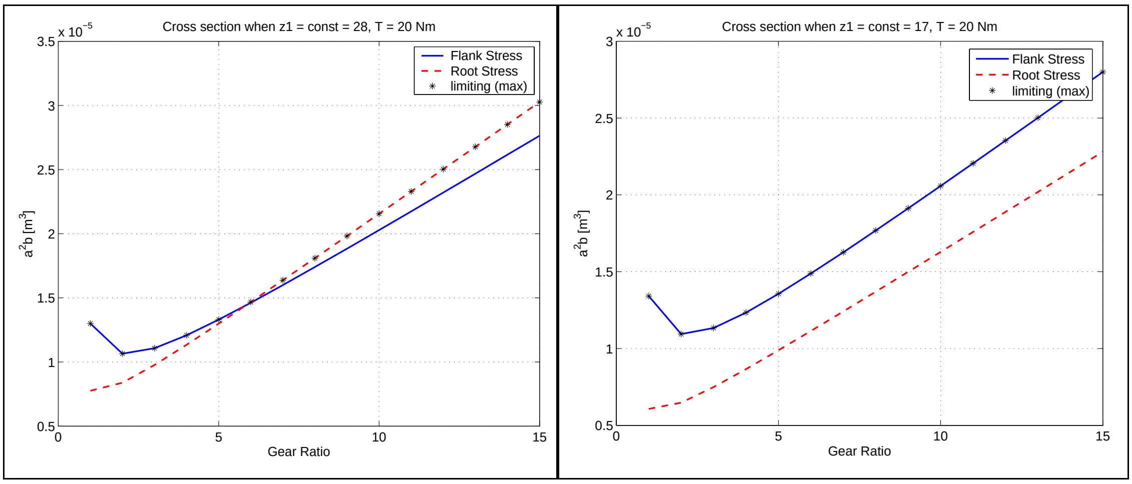

Figure 9. Cross sections of the left part of Figure 6, at 28 and 17 teeth of wheel one.

To eliminate unrealistic gear geometry the following constraints are introduced: width/diameter < 2 and diameter/width < 40.

In other words: $0.5 < \dfrac{d}{b} < 40$

From now on, only solutions that fulfill these constraints are visible in the plots.

The most obvious choice of z_1 is maybe the one that minimizes the mass and volumes of the gears. In this case the minimum occurs in the intersection between the two surfaces (root and flank stress), see Figure 6. So if the z that minimizes the gear mass (Figure 8) for each gear ratio is chosen the following plot of the center distance is obtained.

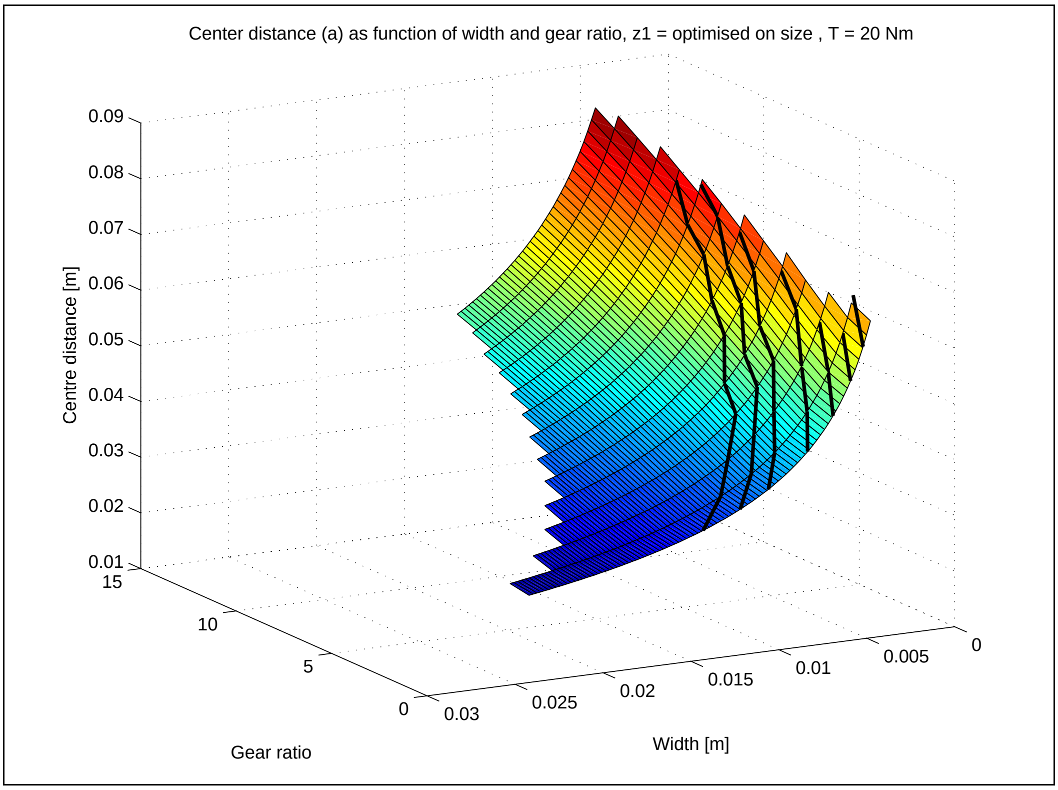

Figure 10. Required center distance when the number of pinion teeth is optimized on weight. The black lines indicate different modules, the one to the left represents module 0.5 mm and the one on the right is module 1 mm.

The black lines in Figure 10, represent different standard modules, the left one represents module 0.5 mm, and the one to the right is 1 mm. These are very small modules, 0.5 is the smallest possible standard module. This indicates that it is not always possible to select the number of teeth (*z*) so that it minimizes the mass, since it may result in unrealistically small or large teeth. In this case one can see that it is impossible to design a gear pair with a ratio above 11 by using the standard modules.

If the pinion teeth number is locked to 17 instead, the following plot is obtained:

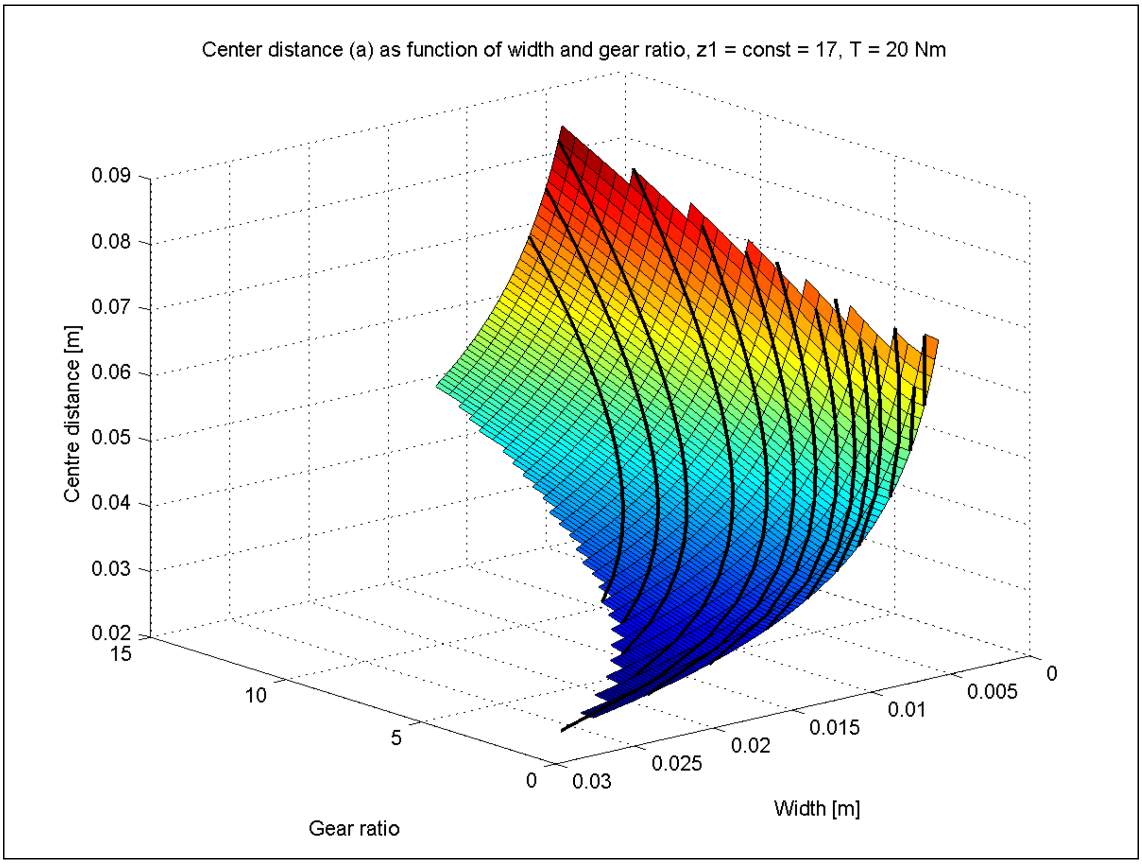

Figure 11. Center distance with the number of pinion teeth constant, 17. The black lines indicate different modules, the left one is 0.5 mm, and the most right one is 2.25 mm.

In Figure 11 the lowest black line represents module 0.5 mm, but now the one on the top right represents module 2.25 mm. It is rather intuitive that the module increase when the width decrease, since the diameter has to increase when the width decreases. At least for small loads, it is probably necessary to select a small number of teeth on the pinion to avoid very small teeth.

Not using the "optimal" number of teeth with respect to volume and weight is not a problem as long as a lower number of teeth is used. Comparing Figure 10 and Figure 11 relieves that the difference in size is very small. Choosing a larger *z* than the optimal may however have a larger impact on the gear sizes, since the root stress can become the limiting factor (see Figure 6).

Applying the equation for the inertia (equation (12)) on the data shown in Figure 11, results in the following graph:

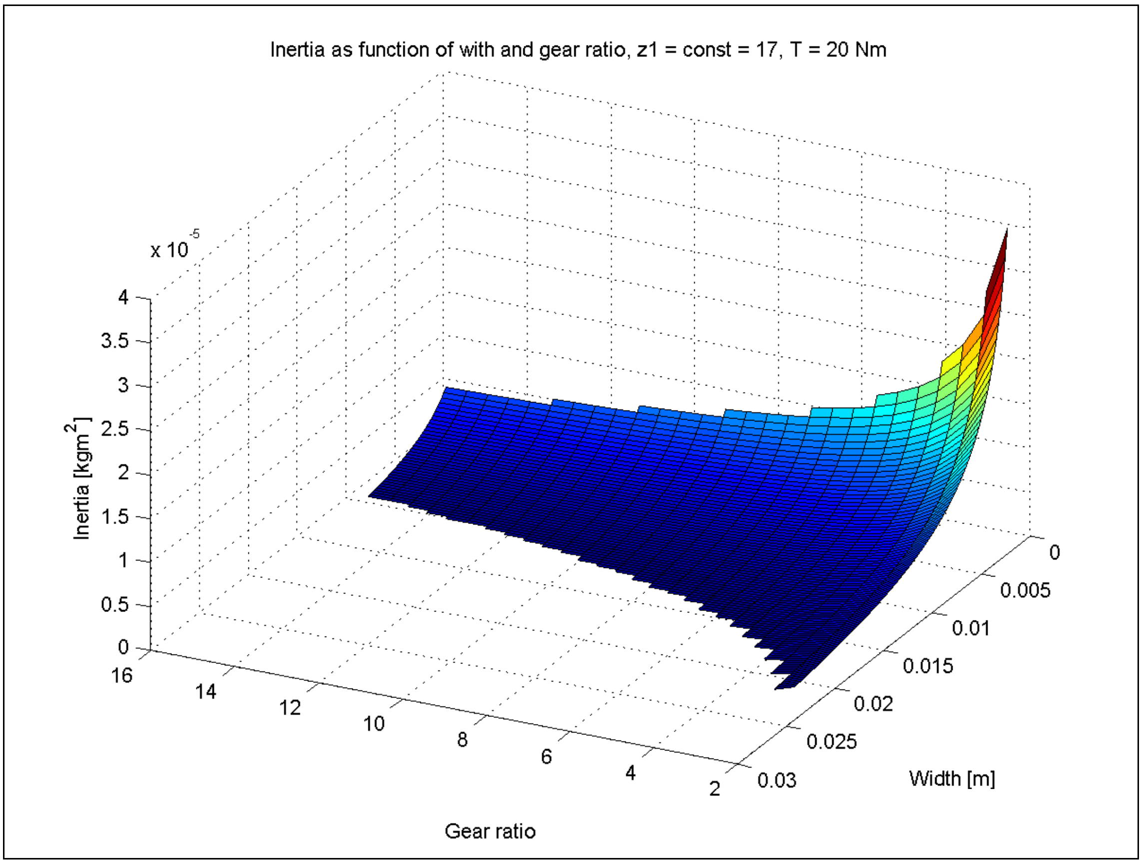

Figure 12. Gear inertia reflected on the input axis as function of gear width and gear ratio.

The inertia seems to be decreasing with gear ratio. The fact that a small gear width results in large inertia is intuitive, since it means large gear diameters.

4 Analysis of three-wheel planetary gear trains

In this section similar models for three-wheel planetary gears, as for the gear pair in the previous section, are derived. Also here the analysis only treats spur gears with no addendum modification. The sizing formulas are as before mostly based on the two Swedish standard documents SS1871 and SS1863.

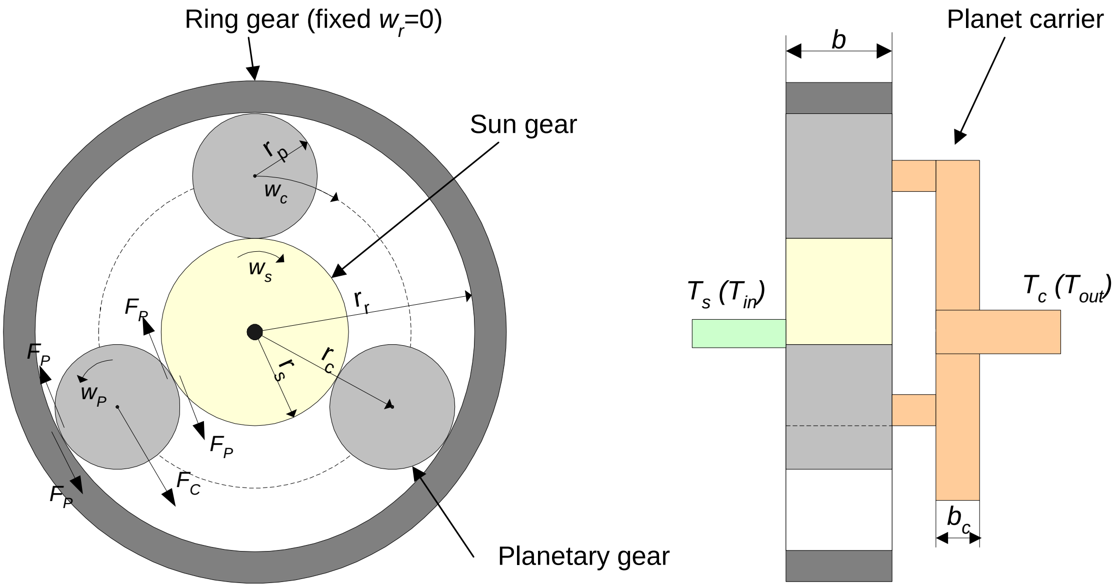

Figure 13. Simple sketch of a three wheel planetary gear train.

4.1 Gear ratio, ring radius, mass, inertia and peripheral force

4.1.1 Gear ratio and geometry

For a planetary gear train configured as in Figure 13 the 'basic' gear ratio is, according to Vedmar (2002) given by:

$$n_b = \frac{\omega_s - \omega_c}{\omega_r - \omega_c} = -\frac{z_r}{z_c}\frac{z_c}{z_s} = -\frac{z_r}{z_s} = -\frac{r_r}{r_s} \tag{33}$$

With the ring gear fixed, the 'basic' gear ratio gets:

$$n_b = \frac{\omega_s - \omega_c}{0 - \omega_c} = 1 - \frac{\omega_s}{\omega_c} \tag{34}$$

Which gives the transmission ratio from input to output, n:

$$n = \frac{\omega_{in}}{\omega_{out}} = \frac{\omega_s}{\omega_c} = 1 - n_b = 1 + \frac{r_r}{r_s} \tag{35}$$

From equation (35), the sun gear radius r_s as function of gear ratio and ring gear radius can be derived.

$$r_s = \frac{r_r}{n-1} \tag{36}$$

The radius of the planet wheels is given by:

$$r_p = \frac{r_r - r_s}{2} \Rightarrow r_p = \frac{r_r(n-2)}{2(n-1)} \tag{37}$$

The radius of the planet carrier is the radius of the sun gear plus the radius of the planet gears, i.e:

$$r_c = r_s + r_p = \frac{r_r}{n-1} + \frac{r_r(n-2)}{2(n-1)} = \frac{r_r n}{2(n-1)} \tag{38}$$

4.1.2 Weight/Mass

The total mass of the gears in an standard three-wheel planetary gear train is given by:

$$M_{tot} = M_s + M_r + 3M_p + M_c \tag{39}$$

Where M_s, M_r, M_p and M_c are the masses of the sun gear, ring gear, planet gears and planet carrier respectively. These individual masses are in turn given by:

$$M_s = r_s^2 b\pi\rho = \frac{r_r^2 b}{(n-1)^2}\pi\rho \tag{40}$$

$$M_p = r_p^2 b\pi\rho = \frac{r_r^2 b(n-2)^2}{4(n-1)^2}\pi\rho \tag{41}$$

$$M_r = (r_{ro}^2 - r_r^2)b\pi\rho = r_r^2 b\pi\rho(k_{ro} - 1) \tag{42}$$

$$M_c = r_c^2 b_c \pi\rho = \pi\rho\frac{b_c r_r^2 n^2}{4(n-1)^2} \tag{43}$$

The ring gear is approximated as a cylinder with an outer radius of k_{ro} times the reference radius. The planet carrier is modeled as a cylinder with the width b_c and the radius r_c The total mass is then given by:

$$M_{tot} = r_r^2 b\pi\rho\left(\frac{1}{(n-1)^2} + \frac{3}{4}\frac{(n-2)^2}{(n-1)^2} + (k_{ro} - 1) + \frac{b_c}{b}\frac{n^2}{4(n-1)^2}\right) \tag{44}$$

Note that only the masses of the actual gears (and the carrier) have been taken into account, shafts and bearings are for example not included.

4.1.3 Inertia

The total inertia of the planetary gear train is modeled as the sum of three components: the inertia of the sun gear, the inertia of the planet carrier (with the planet wheels) and the inertia of each planet wheel, where the two latter needs to be reflected to the input shaft.

$$J_{tot} = J_s + \frac{1}{n^2}J_c + 3\frac{1}{u_{sp}^2}J_p \tag{45}$$

Where u_{sp} is the gear ratio between the sun and planet gear pair.

$$u_{sp} = \frac{\omega_s}{\omega_p} \tag{46}$$

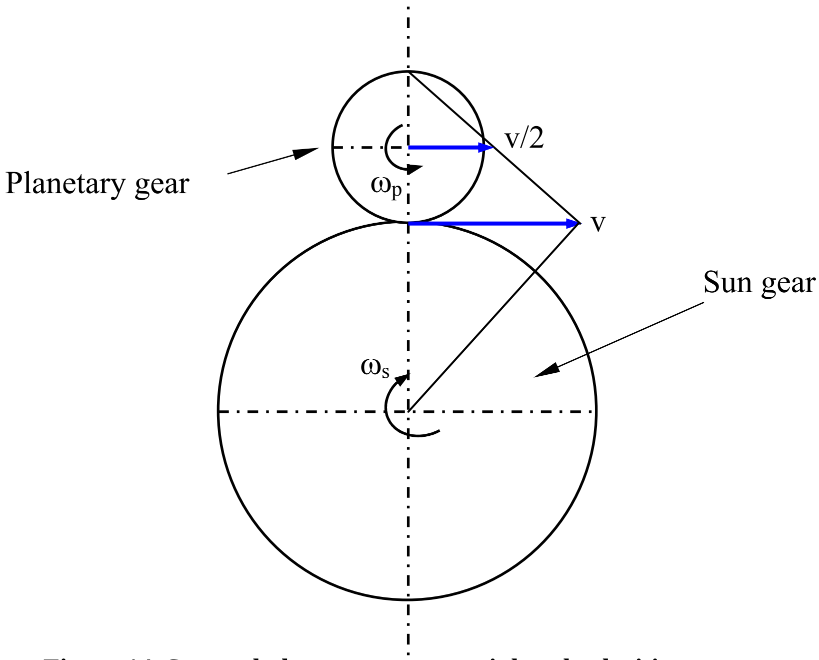

Figure 14. Sun and planetary gears peripheral velocities.

The angular velocity of the planet wheels are determined by calculating the peripheral velocity, v of the two wheels (Figure 14). Since the ring gear is fixed the peripheral velocity has to be 0 in the planet/ring mesh.

$$\omega_s = \frac{v}{r_s} \tag{47}$$

$$\omega_p = \frac{v}{2r_p} \tag{48}$$

Combining equations (36), (37),(46),(47) and (48) leads to the following expression for u_{sp}.

$$u_{sp} = \frac{2r_p}{r_s} = n-2 \tag{49}$$

The inertias are given by:

$$J_s = \frac{1}{2}M_s r_s^2 = \frac{1}{2}\frac{r_r^4}{(n-1)^4}\rho\pi b \tag{50}$$

$$J_p = \frac{1}{2}M_p r_p^2 = \frac{1}{2}\frac{r_r^4(n-2)^4}{16(n-1)^4}\rho\pi b \tag{51}$$

$$J_c = 3M_p r_c^2 + M_c\frac{r_c^2}{2} = \left(3M_p + \frac{M_c}{2}\right)\frac{r_r^2 n^2}{4(n-1)^2} = \frac{6b(n-2)^2 + b_c n^2}{32(n-1)^4} r_r^4 n^2 \pi\rho \tag{52}$$

Combining equations (45), (49), (50), (51) and (52) leads to following expression for the inertia of the planetary gear:

$$J_{tot} = \frac{1}{32}\frac{((9b+b_c)n^2 - 36bn + 52b)r_r^4}{(n-1)^4}\rho\pi r_r^4 \tag{53}$$

4.1.4 Forces and torques

The peripheral forces on the sun and planet gears are given by (friction losses neglected):

$$F_p = \frac{T_s}{r_s}\frac{1}{3} \tag{54}$$

The 'torque producing' force from one planet gear on the planet carrier is given by:

$$F_c = 2F_p = \frac{2}{3}\frac{T_s}{r_s} \tag{55}$$

To verify this, the expression for the gear ratio can be derived:

$$T_c = 3F_c r_c = \frac{2}{3}3\frac{T_s}{r_s}r_c = 2\frac{r_c}{r_s}T_s$$

$$T_s = T_{in}$$

$$T_c = T_{out}$$

$$T_{out} = 2\frac{r_c}{r_s}T_{in} \Rightarrow n = 2\frac{r_c}{r_s} \tag{56}$$

Inserting equation (38) and (36) into equation (56) above, verifies the calculations so far:

$$n = 2\frac{r_c}{r_s} = 2\frac{r_r n(n-1)}{2(n-1)r_r} = n \tag{57}$$

The peripheral force as function of T_{out} (T_c):

$$T_c = 6F_p r_c = 3F_p\frac{r_r n}{(n-1)} \Rightarrow F_p = \frac{1}{3}\frac{(n-1)}{nr_r}T_c \tag{58}$$

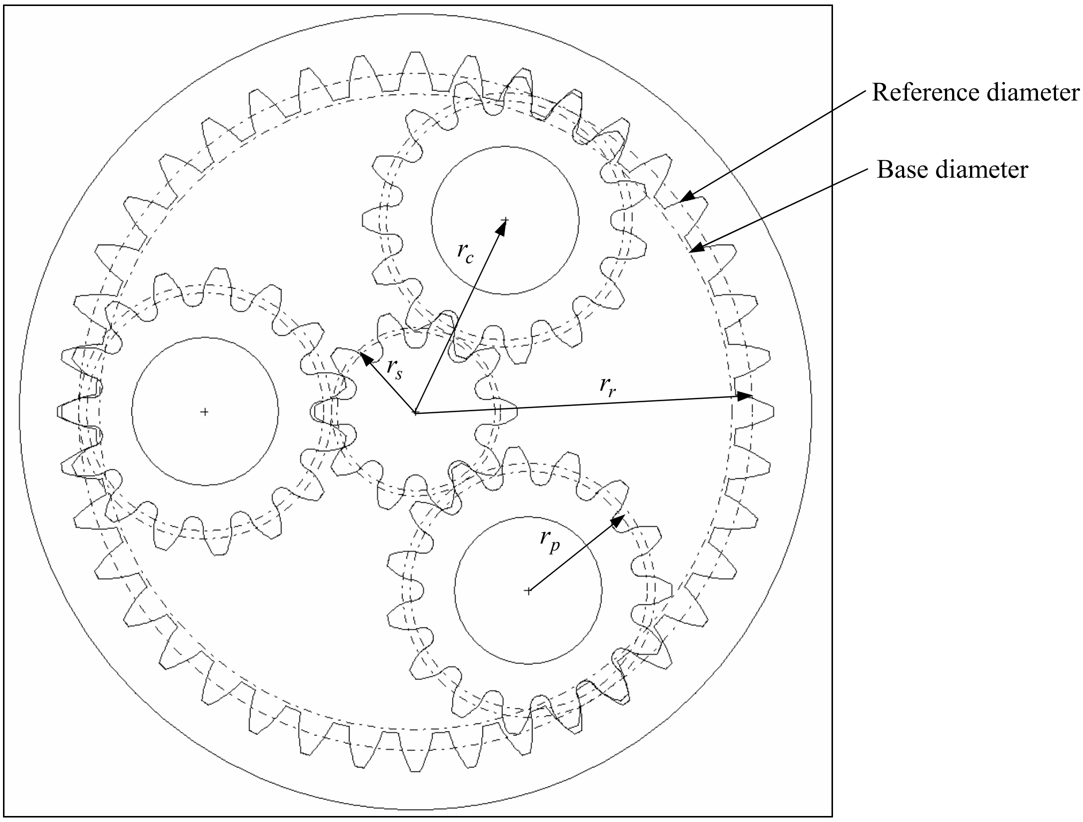

Figure 15. Three wheel planetary gear

4.2 Planetary gear sizing models based on SS1863 and SS1871

This analysis is similar to the work presented in the previous chapter on single gear pairs. The main difference from the previous analysis is that planetary gears consists of one internal gear pair (the ring and planet gears) and one external gear pair (the sun and planet gears), see Figure 15. Both has to be checked with respect to Hertzian pressure and bending fatigue. Another important difference from the previous chapter is that the forces on the planet gear teeth changes sign once every half revolution.

4.2.1 Sun planet gear pair

The sun/planet gear ratio, u is given by:

$$u = \frac{r_2}{r_1} = \frac{r_p}{r_s} \tag{59}$$

Inserting equation (36) and (37) into the expression above, yields:

$$u = \frac{n-2}{2} \tag{60}$$

Which, if inserted into equation (14) leads to:

$$\sigma_H^2 = Z_H^2 Z_M^2 Z_\varepsilon^2 K_{H\alpha} K_{H\beta} \frac{F_{cal} n(n-1)}{2br_r\,(n-2)} \tag{61}$$

Finally the insertion of equation (36) and (58) leads to:

$$\sigma_H^2 = Z_H^2 Z_M^2 Z_\varepsilon^2 K_{H\alpha} K_{H\beta} \frac{T_{cal}(n-1)^2}{6br_r^2(n-2)} \tag{62}$$

Solving the equation above with respect to $r_r^{\,2}b$ gives:

$$r_r^{\,2}b = Z_H^2 Z_M^2 Z_\varepsilon^2 K_{H\alpha} K_{H\beta} \frac{T_{cal}(n-1)^2}{6(n-2)\sigma_{H\max}^2} \tag{63}$$

Where Z_H and Z_M is given by equations (18) and (19). Z_ε is as before given by equation (20) but ε_α has to be expressed as a function of n and r_r instead. Inserting $z_1 = z_s$ and equation (60) into equation (26) leads to the following expression for ε_α:

$$\varepsilon_\alpha = \frac{z_s}{4\pi\cos\alpha}\left(2\sqrt{\sin^2\alpha + \frac{4}{z_s} + \frac{4}{z_s^2}} + (n-2)\sqrt{\sin^2\alpha + \frac{8}{z_s(n-2)} + \frac{16}{z_s^2(n-2)^2}} - n\sin\alpha\right) \tag{64}$$

Now the root stress is left, using equation (28) as before. The module can be expressed as:

$$m = \frac{2r_s}{z_s} = \frac{2r_r}{(n-1)z_s} \tag{65}$$

Inserting this expression and equation (58) into equation (28) results in:

$$\sigma_F = Y_F Y_\beta Y_\varepsilon K_{F\alpha} K_{F\beta} \frac{1}{3}\frac{T_{cal}(n-1)^2 z_s}{2nr_r^2 b} \tag{66}$$

Solving this equation with respect to $r_r^{\,2}b$:

$$r_r^2 b = Y_F Y_\beta Y_\varepsilon K_{F\alpha} K_{F\beta} \frac{1}{3}\frac{T_{cal}(n-1)^2 z_s}{2n\sigma_{F\max}} \tag{67}$$

Where Y_F is approximated as before with equation (29), Y_β is 1 and Y_ε is calculated with equation (30) with ε_α from equation (64).

4.2.2 Planet and ring gear pair

This is an internal gear pair (Figure 16), which means a couple of changes in the formulas derived earlier. The equation for the bending fatigue is the same, with the exception of the contact ratio, ε_α, that has to be calculated for an internal gear pair instead. The equation for the Hertzian pressure is similar to the one used before, but not exactly the same. For an internal gear, the following apply for the Hertzian pressure (SS1871):

$$\sigma_H = Z_H Z_M Z_\varepsilon \sqrt{\frac{F_{cal} K_{H\alpha} K_{H\beta}(u-1)}{bd_1 u}} \tag{68}$$

By using equation (37) the following expression is derived:

$$u = \frac{r_r}{r_p} = \frac{2r_r(n-1)}{r_r(n-2)} = 2\frac{n-1}{n-2} \quad \text{and} \quad d_1 = d_p = 2r_p = \frac{r_r(n-2)}{(n-1)}$$

Inserting the two expressions above and equation (58) into equation (68) yields:

$$\sigma_H^2 = Z_H^2 Z_M^2 Z_\varepsilon^2 K_{H\alpha} K_{H\beta} \frac{T_{cal}(n-1)}{6br_r^2(n-2)}$$

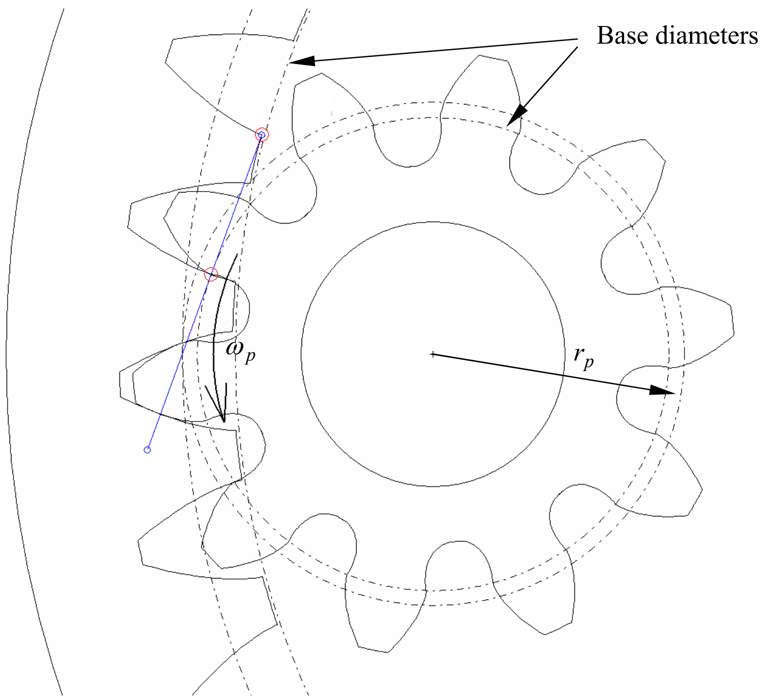

Figure 16. Planetary/ring gear mesh.

$$r_r^2 b = Z_H^2 Z_M^2 Z_\varepsilon^2 K_{H\alpha} K_{H\beta} \frac{T_{cal}(n-1)}{6\sigma_{H\max}^2 (n-2)} \tag{69}$$

Where Z_H is calculated as before with equation (18), Z_M is calculated with equation (19) and Z_ε is derived with equation (20), but ε_α is now calculated as follows:

The equation below gives the formula for the contact ratio of an internal spur gear (SS1863):

$$\varepsilon_\alpha = \frac{1}{p_b}\left(\frac{\sqrt{d_{a1}^2 - d_{b1}^2}}{2} - \frac{\sqrt{d_{a2}^2 - d_{b2}^2}}{2} + a_w \sin\alpha\right) \tag{70}$$

For an internal spur gear pair, the following apply according to SS1863:

$$d_a = d - 2m$$
$$d_b = d\cos\alpha$$
$$a = r_2 - r_1$$

Index 1 denotes the planet gear (external) and index 2 denotes the ring gear (internal). For the planet gear it is possible to use the results from equation (24). For the ring gear the following apply:

$$\sqrt{d_{ar}^2 - d_{br}^2} = \sqrt{d_r^2 + 4m^2 - 4md_r - d_r^2\cos^2\alpha} = \sqrt{d_r^2\sin^2\alpha + 4m^2 - 4md_r}$$

Combing the equation above with equation (65) gives:

$$\sqrt{d_{ar}^2 - d_{br}^2} = \sqrt{d_r^2\sin^2\alpha + \frac{4d_r^2}{(n.-1)^2 z_s^2} - \frac{4d_r^2}{(n-1)z_s}} = d_r\sqrt{\sin^2\alpha + \frac{4}{(n-1)^2 z_s^2} - \frac{4}{(n-1)z_s}}$$

For the planet ring gear pair, equation (70) becomes:

$$\varepsilon_\alpha = \frac{1}{p_b}\left(r_p\sqrt{\sin^2\alpha + \frac{4}{z_p} + \frac{4}{z_p^2}} - r_r\sqrt{\sin^2\alpha + \frac{4}{(n-1)^2 z_s^2} - \frac{4}{(n-1)z_s}} + (r_r - r_{p1})\sin\alpha \right) =$$

$$= \frac{1}{p_b}\left(\frac{r_r(n-2)}{2(n-1)}\sqrt{\sin^2\alpha + \frac{8}{(n-2)z_s} + \frac{16}{(n-2)^2 z_s^2}} - r_r\sqrt{\sin^2\alpha + \frac{4}{(n-1)^2 z_s^2} - \frac{4}{(n-1)z_s}} + \frac{nr_r}{2(n-1)}\sin\alpha \right) =$$

$$= \frac{z_s}{2\pi\cos\alpha}\left(\frac{1}{2}\sqrt{(n-2)^2\sin^2\alpha + \frac{8(n-2)}{z_s} + \frac{16}{z_s^2}} - \sqrt{(n-1)^2\sin^2\alpha + \frac{4}{z_s^2} - \frac{4(n-1)}{z_s}} + \frac{n}{2}\sin\alpha \right) \qquad (71)$$

The contact ratio, equation (71), gets complex if the tip diameter becomes smaller than the base diameter. From equation 71 this happens if the number of teeth on the ring gear is less than 34. This condition is normally refered to as involute interference. Buckingham (1963) describes involute interference and identifies a maximum possible module to avoid interference. Since a maximum possible module corresponds to a minimum possible number of teeth the result obtained here is consistent with Buckingham (1963). It is of course possible to make ring gears with less than 34 teeth, but it will be necessary to reduce the addendum. This is however not investigated further in this report, no ring gears with less than 34 teeth will be allowed here.

Comparing equations (63) and (69) leads to the conclusion that Hertzian pressure always is larger for the sun/planet mesh than the planet/ring mesh. This means that it is unnecessary to use equation (69), since it always results in a smaller gear than equation (63).

For the bending fatigue in the planet/ring gear mesh, the same expression as for the planet and sun gear pair can be used (equation (67)). The contact ratio must however be calculated with equation (71) and the form factor Y_F can be approximated to 2.06 if internal gears with basic rack according to SMS 296 or SMS 1861 are used (SS1871).

4.2.3 Maximum allowed stress and pressure

Assuming the same material in all wheels within the planetary gear train, the maximum allowed stresses may be calculated as in section 3.2.3. There is however one exception to this, the maximum root bending stress of the planet wheels. Since the peripheral (load) force changes sign every second contact, it is necessary to reduce the allowed bending stress with 30% (Maskinelement handbok 2003), i.e.

$$\sigma_{Fp\max} = \frac{\sigma_{F\lim}}{S_F}\cdot 0.7$$

4.3 Results and sizing examples

In all these examples the same induction hardened steel that was used in the previous examples for gear pairs are used.

σ_{Hlim}	1200 MPa	α	20 deg
σ_{Flim}	300 MPa	$K_{H\alpha}$	1
ρ	7800 kg/m^3	$K_{H\beta}$	1.3
υ	0.3	$K_{F\alpha}$	1
E	206 GPa	$K_{F\beta}$	1.3
T_{cal}	20 Nm		

Table 2. Parameter values used in the following examples.

4.3.1 Necessary size/volumes

If the same 20 Nm load, as in the gear pair examples, is inserted in the equations for planetary gears, the following graphs are obtained:

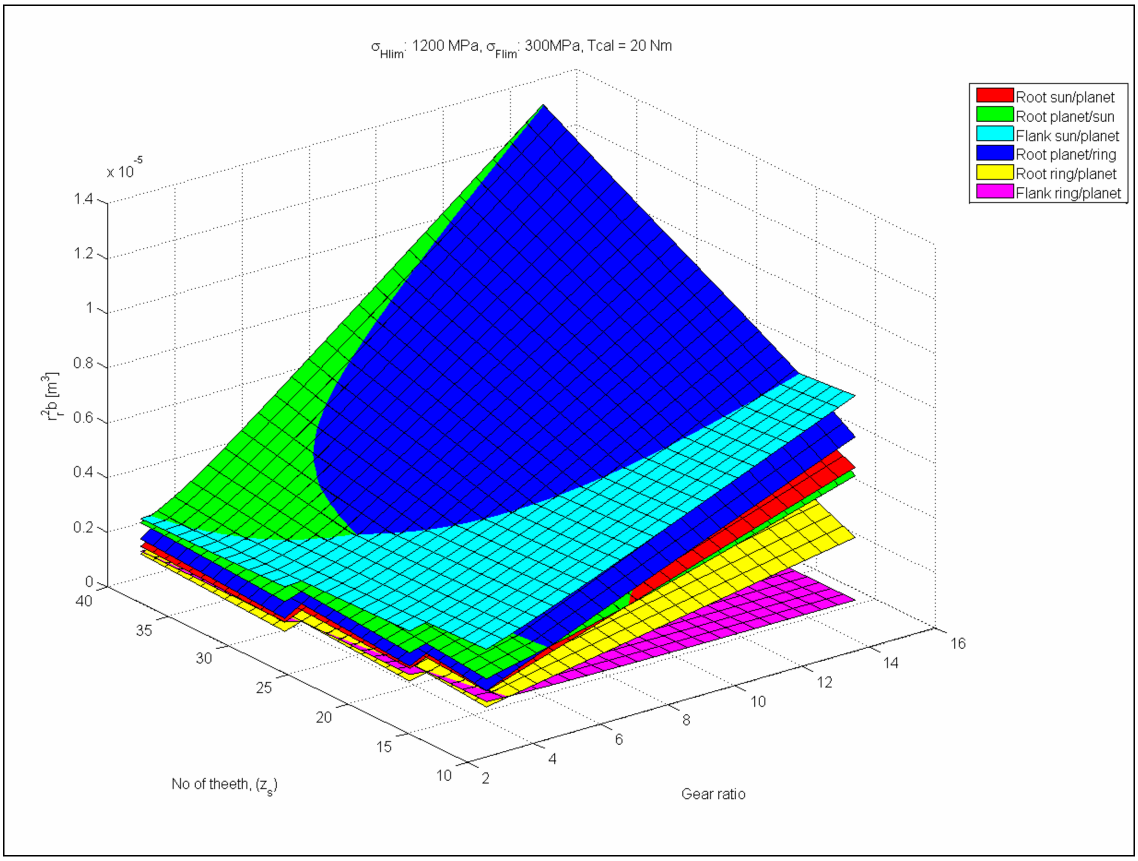

Figure 17. Induction hardened steel. The minimum sizes (r^{2}b) for root and flank stresses. Safety factors are set to one.

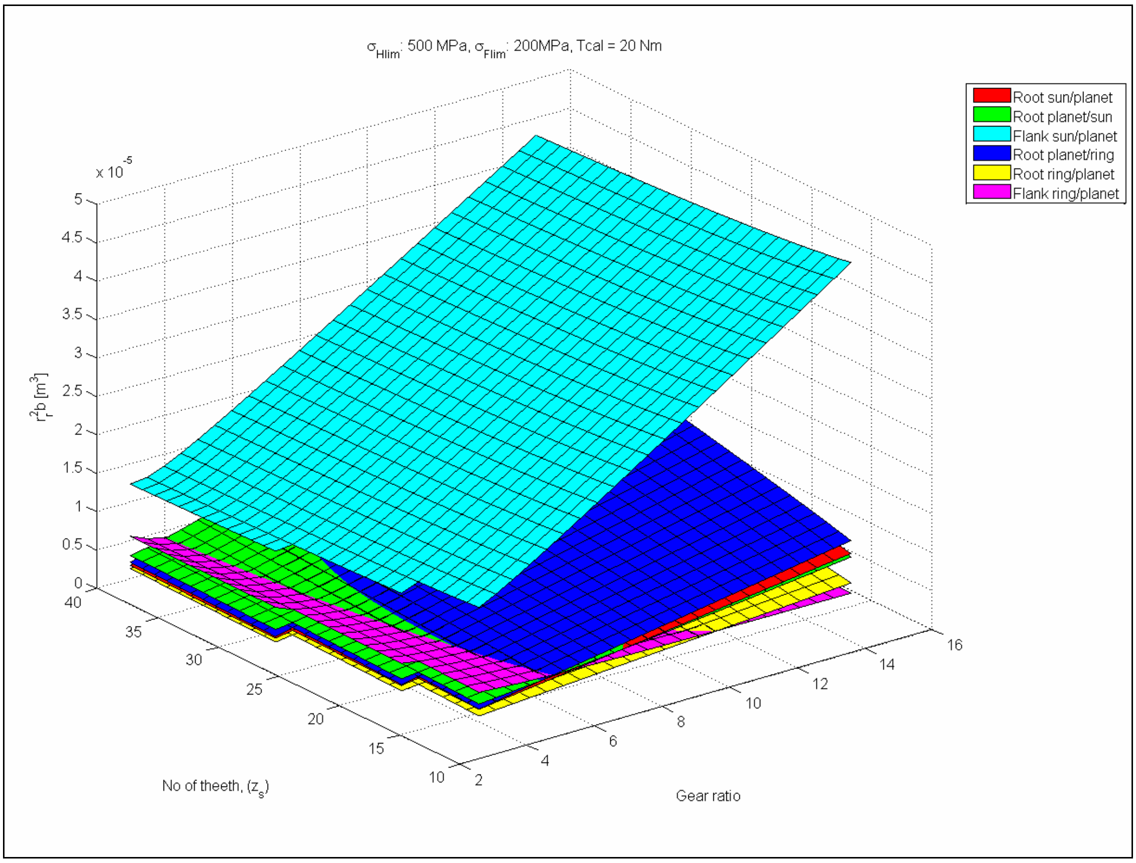

Figure 18. The minimum sizes (r^2b) for root and flank stresses for a non-hardened steel. Safety factors are set to one.

Also here it seems like the flank stress sets the limit on the gear size in the majority of cases. It is only for hardened steels with a large difference between the two fatigue limits that the bending stress can set the limit. However, to be sure, both have to be checked. Further the results support the earlier conclusion that the Hertzian pressure always is higher in the sun/planet mesh than in the planet/ring mesh.

4.3.2 Weight, radius and inertia.

The expression for weight and inertia (equations (53), (44)) of the planetary gears are applied here. As before it is necessary to 'lock' one of the variables to be able to visualize the radius and inertia as function of width and gear ratio. The same induction hardened steel as in Figure 17 is used. The weight is calculated with the k_{ro} constant set to 1.2, i.e. the outer radius of the ring gear is approximated to be 1.2 times the reference radius.

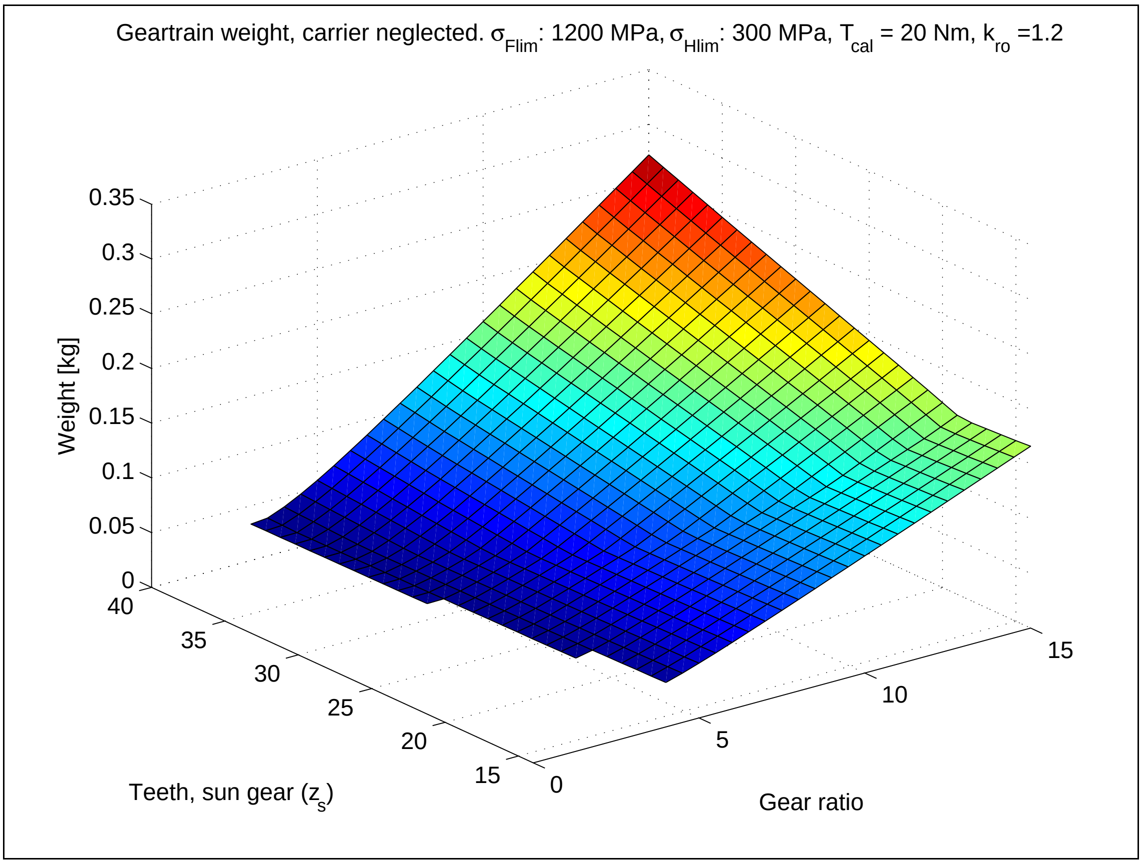

Figure 19. Weight as function of gear ratio and sun gear teeth.

The weight in Figure 19 only represents the weight of the gears, the carrier is not included (b_c=0). In order to include the carrier weight it is necessary to first determine the gear geometry, and in order to do that it is necessary to select the sun gear teeth number.

If the number of teeth that minimizes the volume and mass is selected (Figure 20), it results in very small modules, as it did in the gear pair case. Therefore the radius is calculated with a fixed number of sun gear teeth.

As in the single gear pair case, a number of restrictions on the gear shape are introduced:

$0.25 \le \frac{r}{b} \le 40$ and $z_r \ge 34$. From now on only solutions that fulfill these constraints are shown.

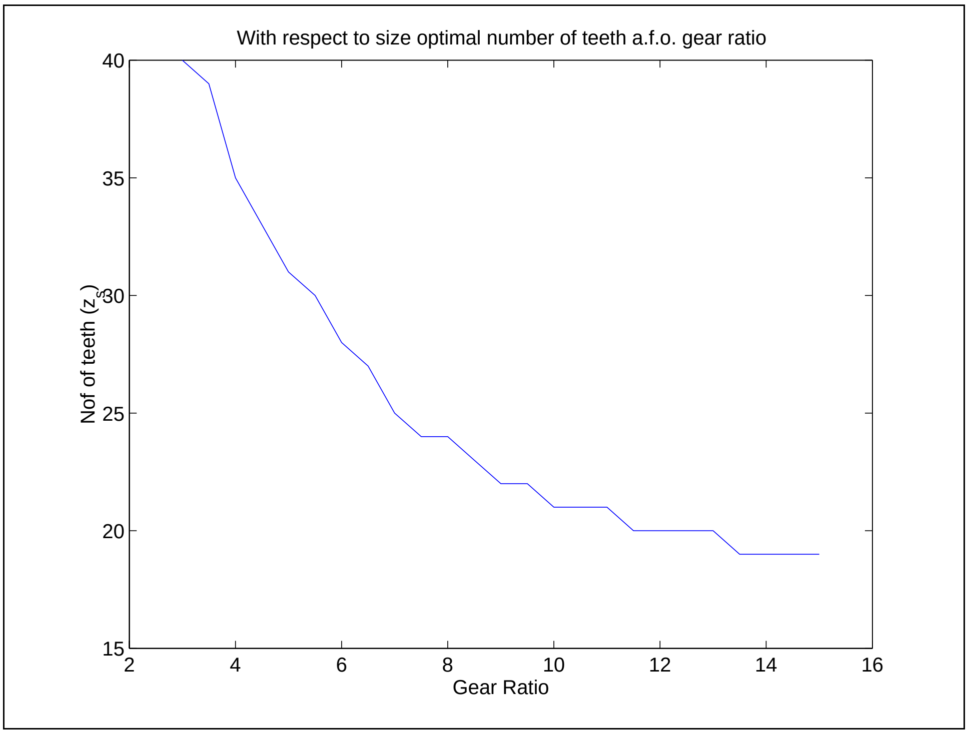

Figure 20. The sun gear teeth number that minimizes the gear size as function of gear ratio.

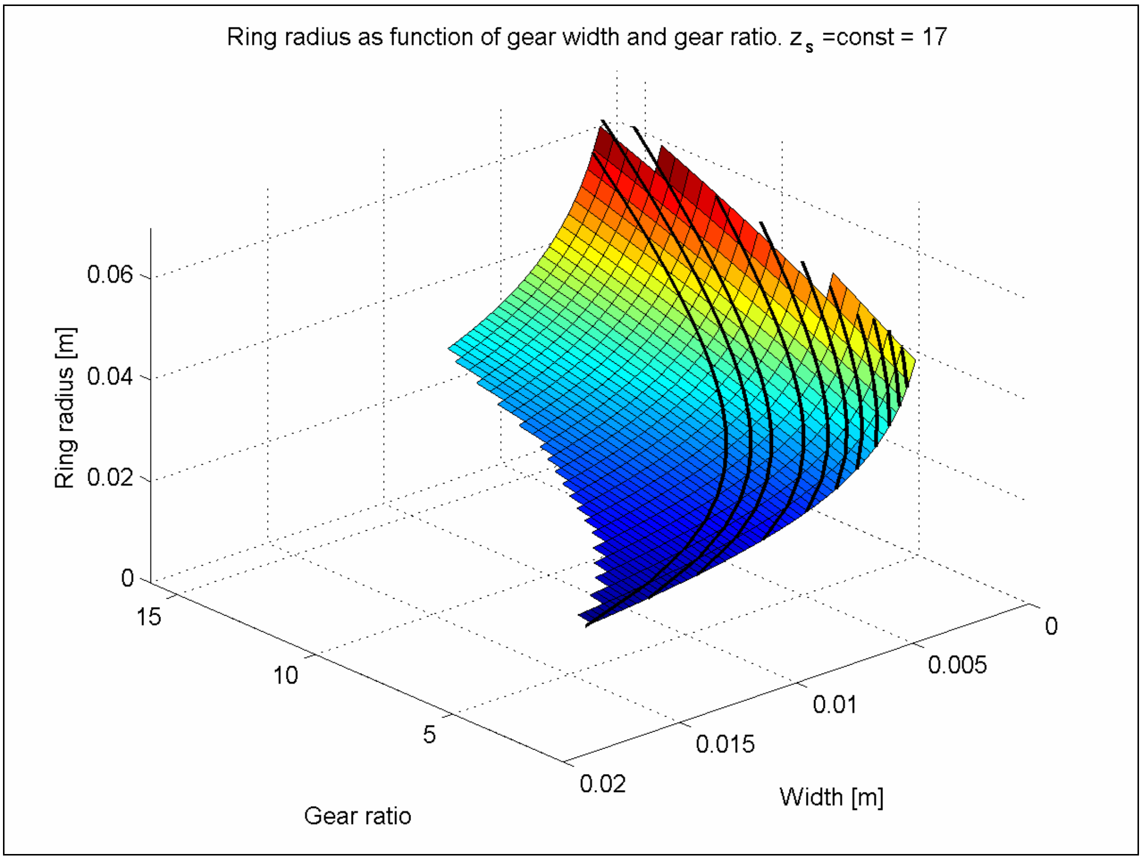

Figure 21. Ring radius. The black lines indicate different standard modules, the left one is 0.5 mm and the right one represent module 1.5 mm.

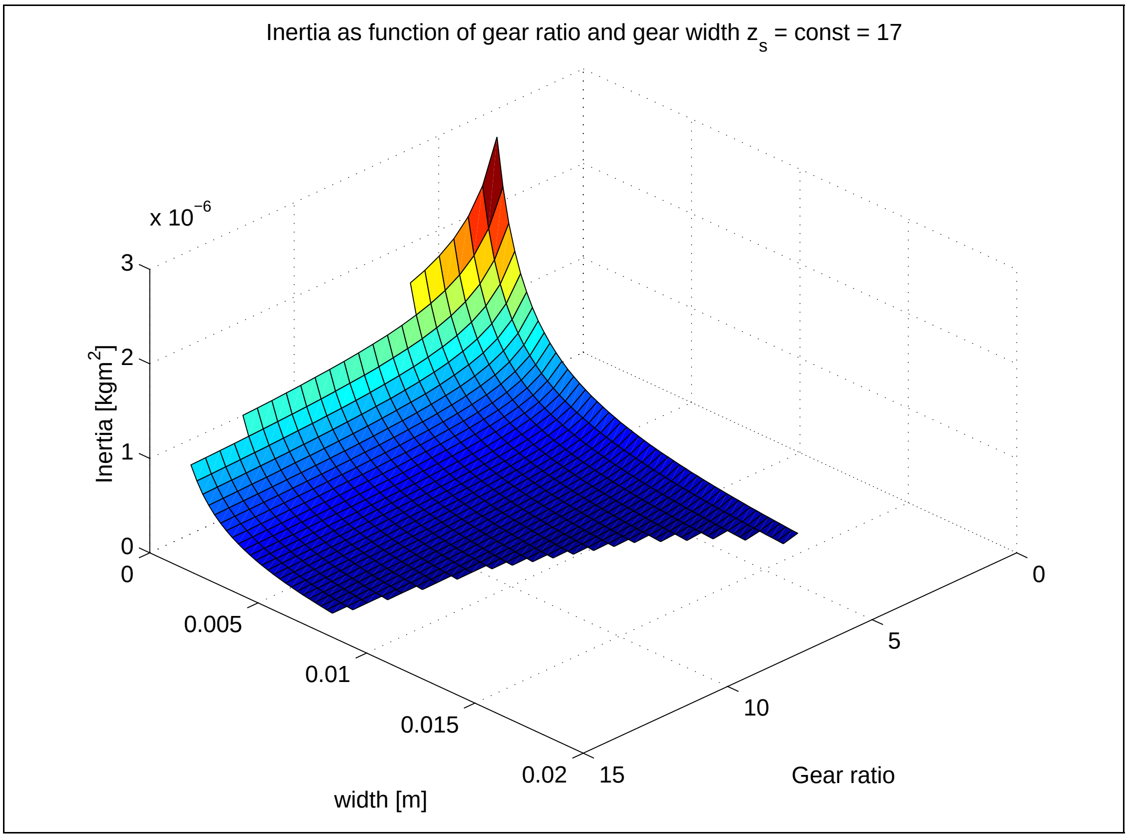

Figure 22. Planetary gear inertia.

In Figure 23 the gear train weight with carrier is shown (equation (44) $b_c=b$). If compared with Figure 19 it can be concluded that the carrier weight is rather small.

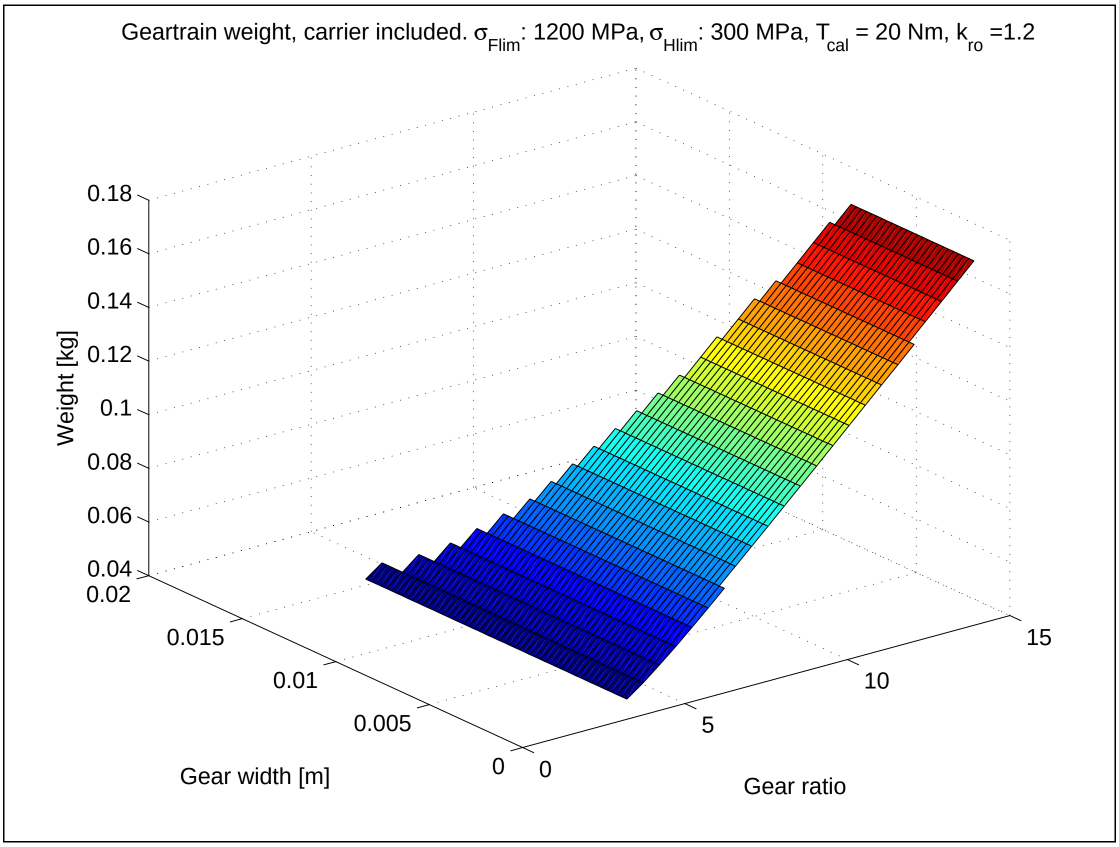

Figure 23. Weight with carrier. Compare to Figure 19.

5 Comparison between planetary and spur gear trains

Planetary gears are, in comparison with conventional gear pairs, known to have compact size and low inertia. This is here verified by comparing the results from the previous two chapters. The same induction hardened steel is used in these examples as before, the load is still 20 Nm and all other parameters are the same as in the previous sections. First the gears weight is plotted:

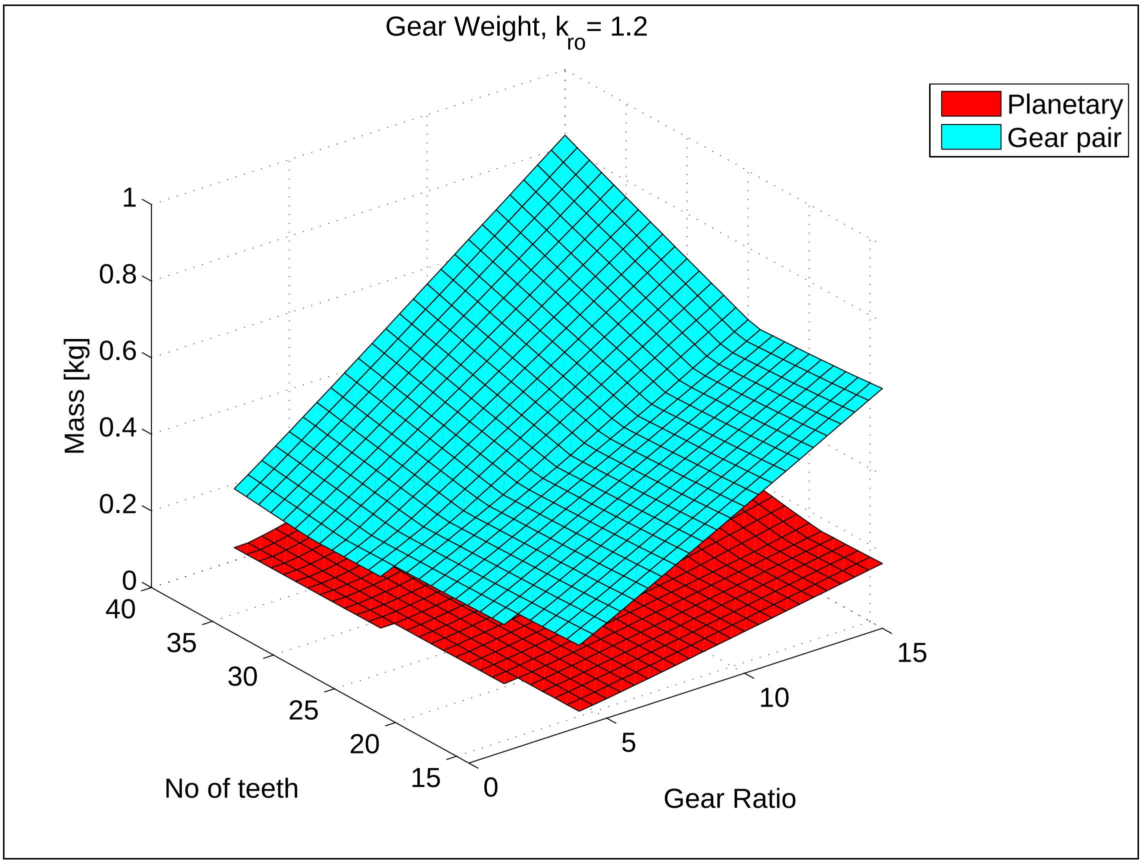

Figure 24. Comparison of the weight of a planetary gear train (carrier not included) with the weight of a single gear pair. It is the number of sun gear teeth that is used for the planetary gear, and the number of pinion teeth for the gear pair.

The planetary gear mass does not include carrier, but as concluded in the previous section, the weight of the carrier is rather small. As seen in Figure 24, a planetary gear has lower weight than the corresponding gear pair. But, maybe the comparison is somewhat unfair; it might be possible to make the gear pair lighter by not using a solid cylinder as gear wheel. Furthermore, the carrier mass must be added to the weight of the planetary gear. However the weight of the planetary gear will probably still be smaller.

In the next graph (Figure 25) the ring gear radius of the planetary gear is compared with gear pair's center distance. As seen, the equations predict that a planetary gear is more compact than a conventional gear pair, which seems to be inline with reality.

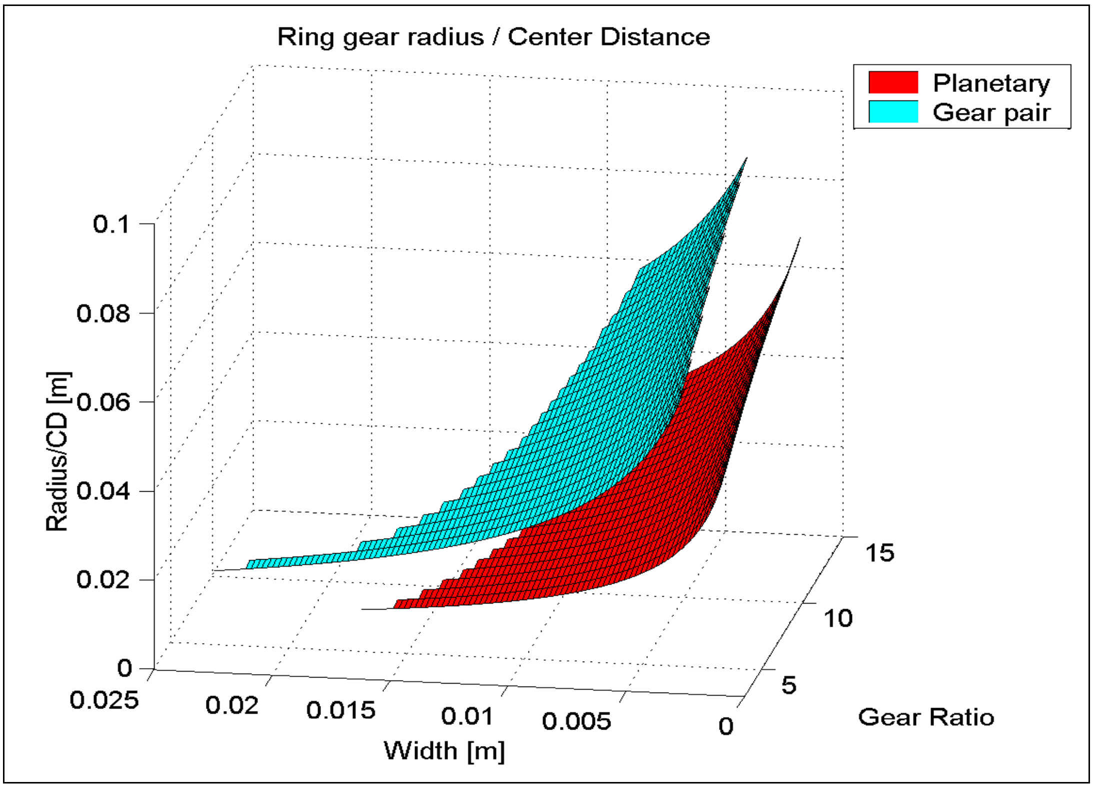

Figure 25. Center distance of gear pairs compared with the ring radius of planetary gears. The number pinion teeth and sun gear respectively, has been 'locked' to 15

Last the inertia of the two gear types is compared, as seen in the figure below, the inertia of a planetary gear train is much lower than the inertia of a single gear pair. This result also appears to be correct, since planetary gears often are used when low inertia is required, e.g. in robotic applications.

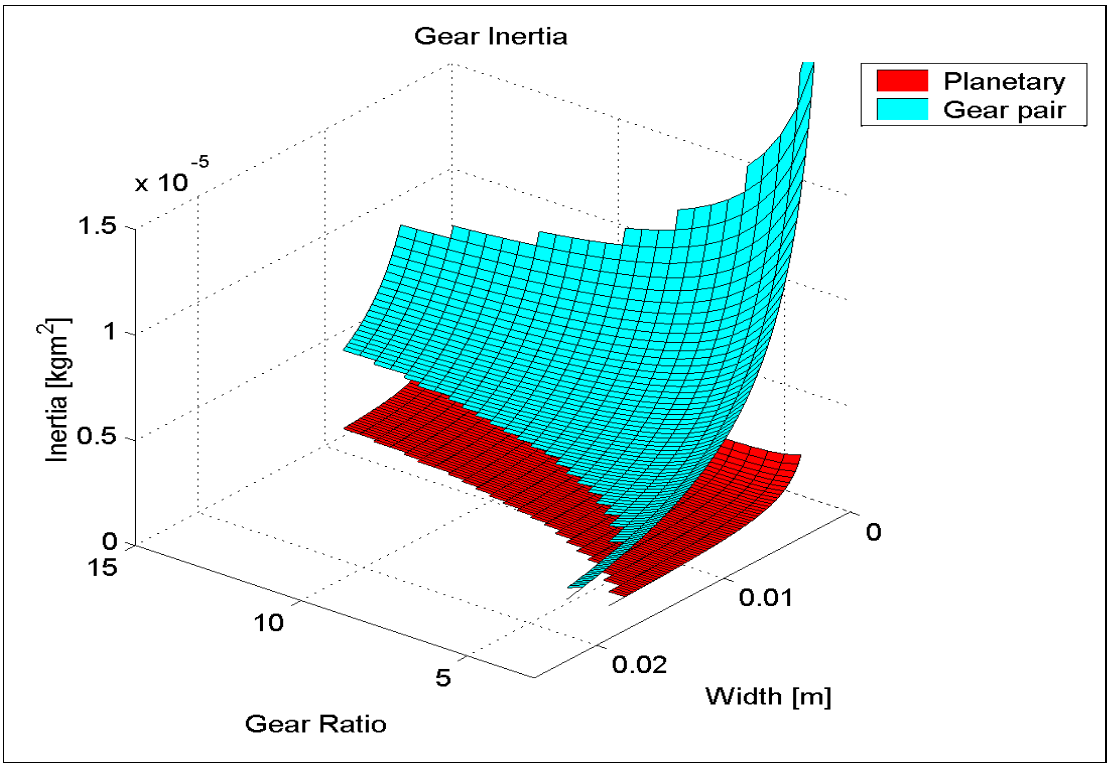

Figure 26. Comparison of gear inertia, for the gear shapes in the graph above. (z=15)

6 Conclusions

The basis for all expressions derived in this report is the Swedish standard for dimensioning of spur gears. From the standard, four central expressions have been derived that all further analysis is based on. These equations are summarized in Table 3 and express the smallest possible size of a single gear pair and a three-wheel planetary gearhead respectively.

	Root bending fatigue	*Hertzian pressure*
Gear pair	$a^2 b = Y_F Y_\varepsilon K_{F\alpha} K_{F\beta} \dfrac{T_{cal} z_1 (u+1)^2}{2u\sigma_{F\max}}$	$a^2 b = Z_H^2 Z_M^2 Z_\varepsilon^2 K_{H\alpha} K_{H\beta} \dfrac{T_{cal}(u+1)^3}{2u^2 \sigma_{H\max}^2}$
Planetary	$r_r^2 b = Y_F Y_\beta Y_\varepsilon K_{F\alpha} K_{F\beta} \dfrac{1}{3} \dfrac{T_{cal}(n-1)^2 z_s}{2n\sigma_{F\max}}$	$r_r{}^2 b = Z_H^2 Z_M^2 Z_\varepsilon^2 K_{H\alpha} K_{H\beta} \dfrac{T_{cal}(n-1)^2}{6(n-2)\sigma_{H\max}^2}$

Table 3. Summary of minimum gear size expressions.

For both gear types, it is the larger of the two values of the required ‘volume’ for Hertzian pressure and bending fatigue respectively that sets the constraint on the gear size. In the planetary case the bending fatigue has to be checked for both the sun/planetary mesh as well as the planetary/ring mesh.

At least three conclusions can be drawn from the contents of this report:

- For a changing torque load (dynamic load) it is necessary to use an equivalent constant load for the gear sizing. For infinite life dimensioning (> 2 10^6 cycles), the peak load of the load cycle is here used as equivalent load.
- It is the Hertzian pressure limit that sets the constraint on the gear sizes in the majority of cases. The root bending fatigue is only limiting the gear size when very hard steels are used in combination with low safety factors.
- Three-wheel planetary gears are lighter, smaller and have lower inertia than spur gear pairs. This is not new, but it is nice that the theory is inline with practice.

However, the main benefits of the expressions derived in this report are that they can be used for gear optimization and optimization of complete drive systems. Combined with a graphical user interface this can result in a nice tool for gear dimensioning.

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