Power Electronics (EIEN25)
Exercises with Solutions

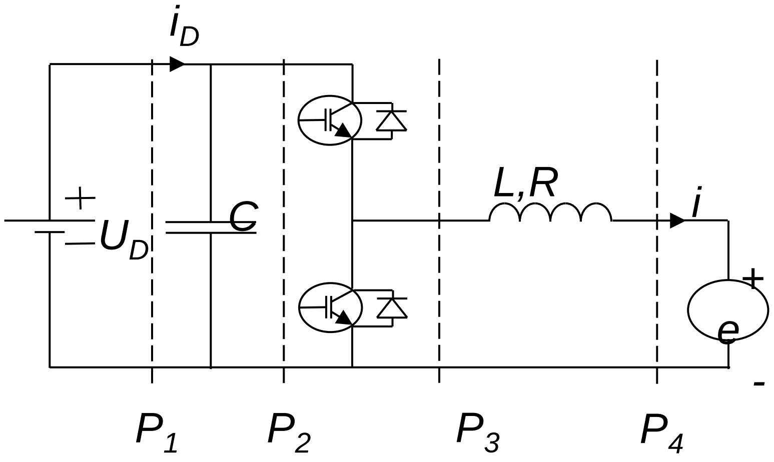
1

Exercises on Modulation

Exercise 1.1 2QC Buck converter no resistance

- Determine
  - *Phase voltages incl graphs*
  - *Dclink current incl graphs*
  - *Power at P1, P2, P3 and P4*

Data for the Buck converter	
U_D	300 V
e	100 V
L	2 mH
R	0 ohm
F_s(switch- freq)	3.33 kHz
i_{avg} (constant)	10 A



Solution 1.1

- **Calculation steps**
  1. ***Duty cycle***
  2. ***Phase current ripple, at positive or negative current slope. Max and min current***
  3. ***Phase current graph, phase voltage graph and dclink current graph***
  4. ***Average current and average voltage at p1, p2, p3 and p4***
  5. ***Power at p1, p2, p3 and p4***

$$1)\ U_{phase\_avg} = e + R \cdot i_{avg} = 100 + 0 = 100V$$

$$Duty_{cycle} = \frac{U_{phase\_avg}}{U_d} = \frac{100}{300} = 0.33$$

$$2)\ Current\ ripple, \max\ and\ \min\ current$$

$$I_{ripple} = \frac{(U_d - e)}{L} \cdot T_{per} \cdot Duty_{cycle} = \frac{(300-100)}{0.002} \cdot 0.0003 \cdot 0.33 = 10\,A$$

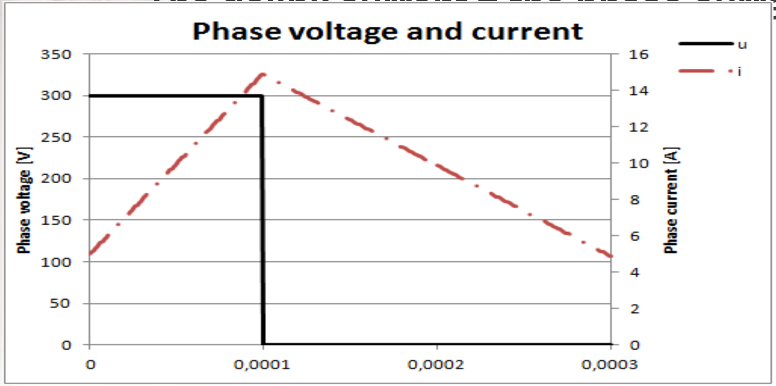
$$I_{\min} = I_{avg} - 0.5 \cdot I_{ripple} = 5\,A$$

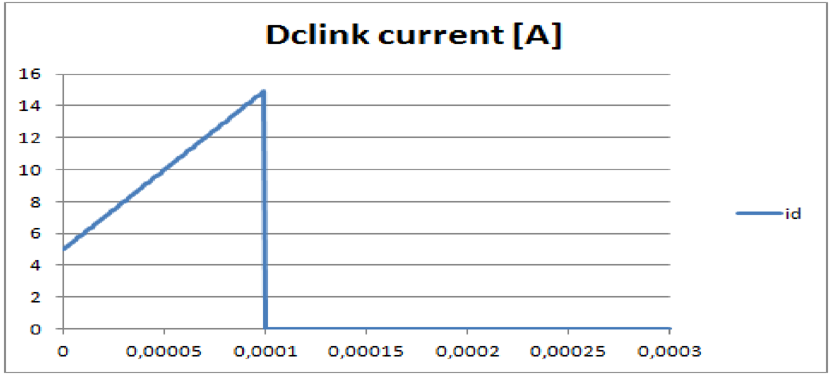
$$I_{\max} = I_{avg} + 0.5 \cdot I_{ripple} = 15\,A$$

Solution 1.1

3. *The modulation =1 during 33% of the period time(the duty cycle), and =0 the rest of the time.*
   The phase voltage = the modulation times the UD.
   The phase current increases from 5 A to 15 A while the modulation =1, and returns from 15 A back to 5 A when the modulation =0.

   The dclink current = the phase current





Solution 1.1

4. ***Average current and average voltage at p1, p2, p3 and p4***

$$At\ P_1 \begin{cases} Cons\tan t\ current\ \ I_{const1} = I_{avg\_dclink\_current} = duty\ \ cycle \cdot I_{avg\_phase\_current} = 3.33\,A \\ Cons\tan t\ voltage\ U_D = 300V \end{cases}$$

$$At\ P_2 \begin{cases} Average\ dc\ link\ current\ \ I_{avg\,2} = I_{avg\_dclink\_current} = duty\ \ cycle \cdot I_{avg\_phase\_current} = 3.33\,A \\ Cons\tan t\ voltage\ U_D = 300V \end{cases}$$

$$At\ P_3 \begin{cases} Average\ dc\ link\ current\ \ I_{avg\,3} = I_{avg\_phase\_current} = 10\,A \\ Average\ voltage\ U_{avg\,3} = duty\ \ cycle \cdot U_D = 100V \end{cases}$$

$$At\ P_4 \begin{cases} Average\ dc\ link\ current\ \ I_{avg\,4} = I_{avg\_phase\_current} = 10\,A \\ Cons\tan t\ voltage\ \ e = 100V \end{cases}$$

5. ***Power at P1, P2, P3 and P4***

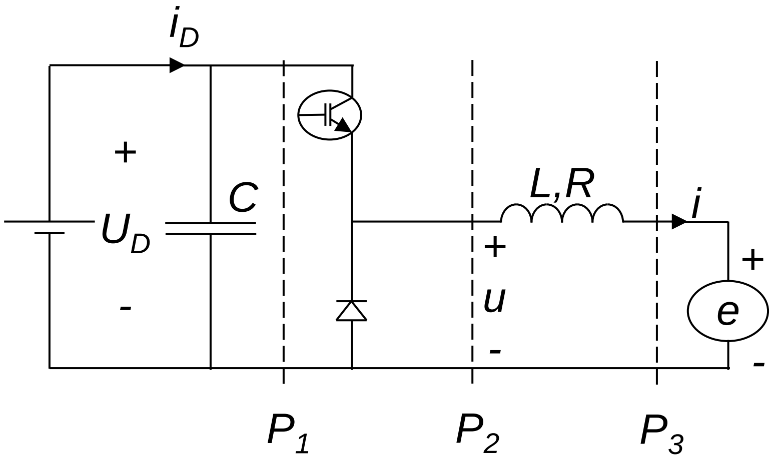
$$\begin{cases} Power\ P_1 = U_d \cdot I_{const1} = 1kW \\ Power\ P_2 = U_d \cdot I_{avg\,2} = 1kW \\ Power\ P_3 = U_{avg\,3} \cdot I_{avg\,3} = 1kW \\ Power\ P_4 = e \cdot I_{avg\,4} = 1kW \end{cases}$$

Exercise 1.2 1QC Buck converter with resistance

- **Determine**
  - ***Phase voltages incl graphs***
  - ***DC-link current incl graphs***
  - ***Power at P1, P2, and P3***

Data for the Buck converter

U_D	300 V
e	100 V
L	very large
R	1 ohm
F_s(switch- freq)	3.33 kHz
i_{avg} (constant)	10 A



Solution 1.2

Calculation steps

1. Avg phase voltage

2. Duty cycle

3. Phase current. Ripple and min and max current

4. Phase current graph, phase voltage graph and dclink current graph

5. Average current and average voltage at $p_{1,}\, p_2$ *and* p_3

6. Power at $p_{1,}\, p_2$ *and* p_3

1) Avg phase voltage

$$U_{phase\_avg} = e + R \cdot i_{avg} = 100 + 1 \cdot 10 = 110V$$

2) Duty cycle

$$Duty_{cycle} = \frac{U_{phase\_avg}}{U_d} = \frac{110}{300} = 0.37$$

Solution 1.2

3) Phase current. Ripple and min and max current

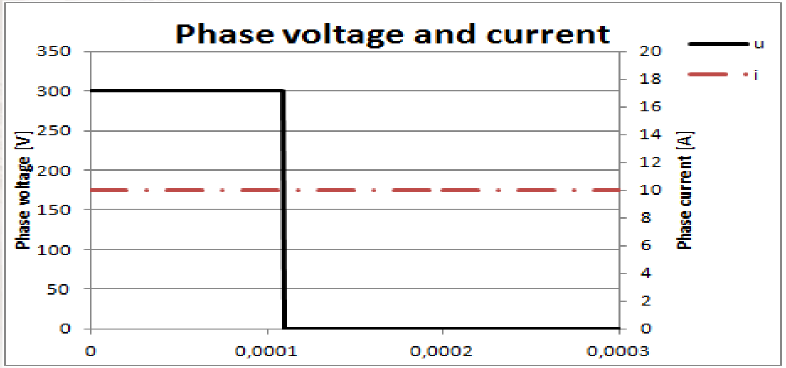
$$I_{ripple} = \frac{(U_d - R \cdot i_{avg} - e)}{\infty} \cdot \frac{1}{F_s} \cdot Duty_{cycle} = 0.0A$$

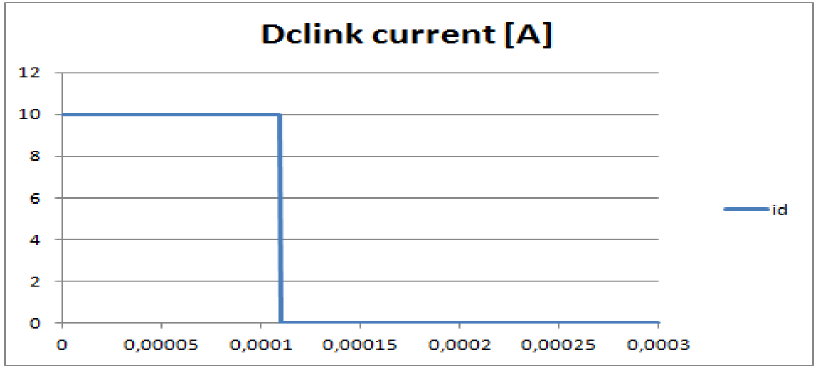
$$I_{\min} = I_{avg} = 10\,A$$

$$I_{\max} = I_{avg} = 10\,A$$

4) Phase current graph, phase voltage graph and dclink current graph

$$I_{phase}(t) = \{L \text{ is high, a straight line}\} = 10\,A$$





Solution 1.2

5) Average current and average voltage at p_1, p_2 and p_3

$$\begin{cases} I_{avg\_P_1} = duty\,cycle \cdot 10A = 3.7A \\ I_{avg\_P_2} = 10A \\ I_{avg\_P_3} = 10A \end{cases} \quad \begin{cases} U_{avg\_P_1} = 300\ V \\ U_{avg\_P_2} = 110\ V \\ U_{avg\_P_3} = 100\ V \end{cases}$$

6) Power at p_1, p_2 and p_3

$$\begin{cases} P_{p1} = U_d \cdot I_{avg} = 300 \cdot 3.7 = 1.1kW \\ P_{p2} = U_{phase\_avg} \cdot I_{avg} = 110 \cdot 10 = 1.1kW \\ P_{p3} = e \cdot I_{avg} = 100 \cdot 10 = 1kW \end{cases}$$

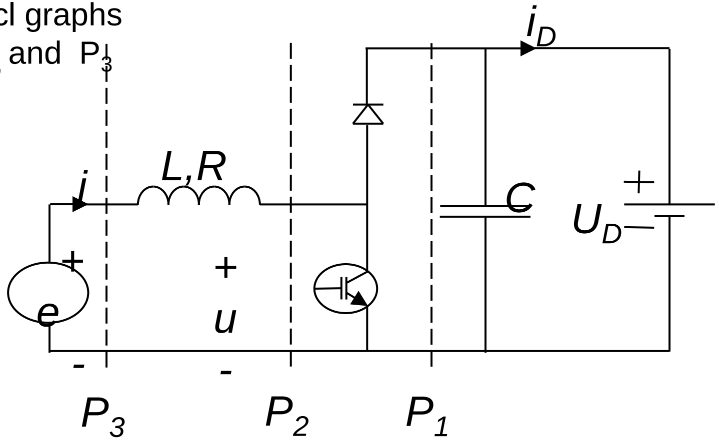
Exercise 1.3 1QC Boost converter with resistance

- **Determine**
  - *Phase voltages incl graphs*
  - *DC-link current incl graphs*
  - *Power at P1, P2, and P3*

Data for the Boost converter

U_D	**300 V**
e	**100 V**
L	**very large**
R	**1 ohm**
F_s(switch- freq)	**3.33 kHz**
i_{avg} (constant)	**10 A**

Determine
Phase voltages incl graphs
Dclink current incl graphs
Power at $P_{1,}$ $P_{2,}$ and P_3



Solution 1.3

Calculation Steps

1. Avg phase voltage
2. Duty cycle
3. Phase current. Ripple and min and max current
4. Phase current graph, phase voltage graph and dclink current graph
5. Average current and average voltage at $p_{1,}\, p_2$ *and* p_3
6. Power at $p_{1,}\, p_2$ *and* p_3

$$1)\; U_{phase\_avg} = e - R \cdot i_{avg} = 100 - 1 \cdot 10 = 90V$$

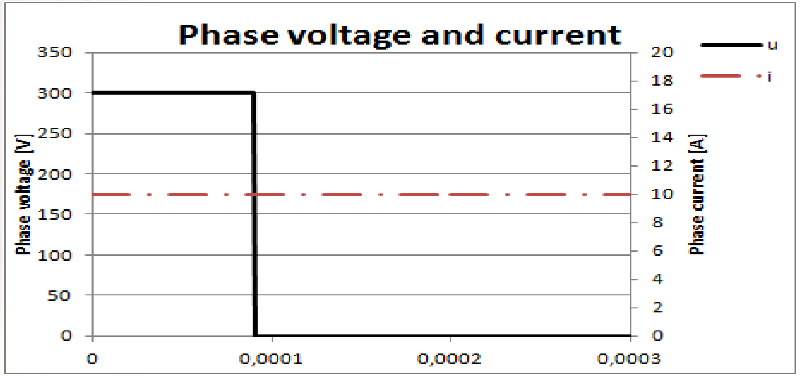
$$2)\; Duty_{cycle} = \frac{U_{phase\_avg}}{U_d} = \frac{90}{300} = 0.30$$

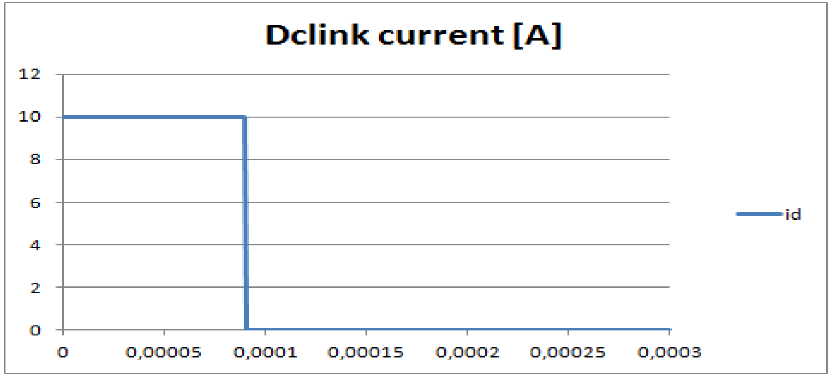
Solution 1.3

3) Phase current. Ripple and min and max current

$$\begin{cases} I_{ripple} = \dfrac{(U_d + R \cdot i_{avg} - e)}{\infty} \cdot \dfrac{1}{F_s} \cdot Duty_{cycle} = 0.0\,A \\ I_{\min} = I_{avg} - 0.5 \cdot I_{ripple} = 10\,A \\ I_{\max} = I_{avg} + 0.5 \cdot I_{ripple} = 10\,A \end{cases}$$

4) Phase current graph, phase voltage graph and dclink current graph





Solution 1.3

5) Average current and average voltage at $p_{1,} p_2$ and p_3

$$\begin{cases} I_{avg\_P_1} = duty\,cycle \cdot 10A = 3.0A \\ I_{avg\_P_2} = 10A \\ I_{avg\_P_3} = 10A \end{cases} \begin{cases} U_{avg\_P_1} = 300\ V \\ U_{avg\_P_2} = 90\ V \\ U_{avg\_P_3} = 100\ V \end{cases}$$

6) Power at $p_{1,} p_2$ and p_3

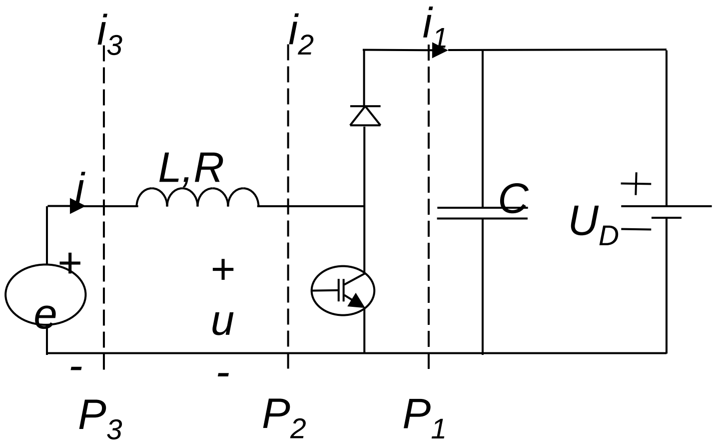
$$\begin{cases} P_{p1} = U_d \cdot I_{avg} \cdot Duty_{cycle} = 300 \cdot 10 \cdot 0.3 = 0.9\,kW \\ P_{p2} = U_{avg} \cdot I_{avg} = 90 \cdot 10 = 0.9\,kW \\ P_{p3} = e \cdot I_{avg} = 100 \cdot 10 = 1\,kW \end{cases}$$

Exercise 1.4 1QC Boost converter no resistance

- **Determine**
  - *Phase voltages incl graphs*
  - *DC-link current incl graphs*
  - *Power at $P_{1,}$ $P_{2,}$ and P_3*

<u>Data for the Boost converter</u>

U_D	300 V
e	100 V
L	2 mH
R	0 ohm
F_s(switch- freq)	3.33 kHz
i_{avg} (constant)	5 A



Solution 1.4

Calculation steps

1. *Duty cycle*
2. *Phase current ripple, at positive or negative current slope.*
3. *Medium, max and min current*
4. *Phase current graph, phase voltage graph and dclink current graph*
5. *Average current voltage and power at $p_{1,}\, p_{2,}\, p_3$*

1)

$$\overline{u} = e - R \cdot i_{avg} = 100 + 0 = 100V$$

$$DutyCycle = \frac{\overline{u}}{U_d} = \frac{100}{300} = 0.33$$

2)

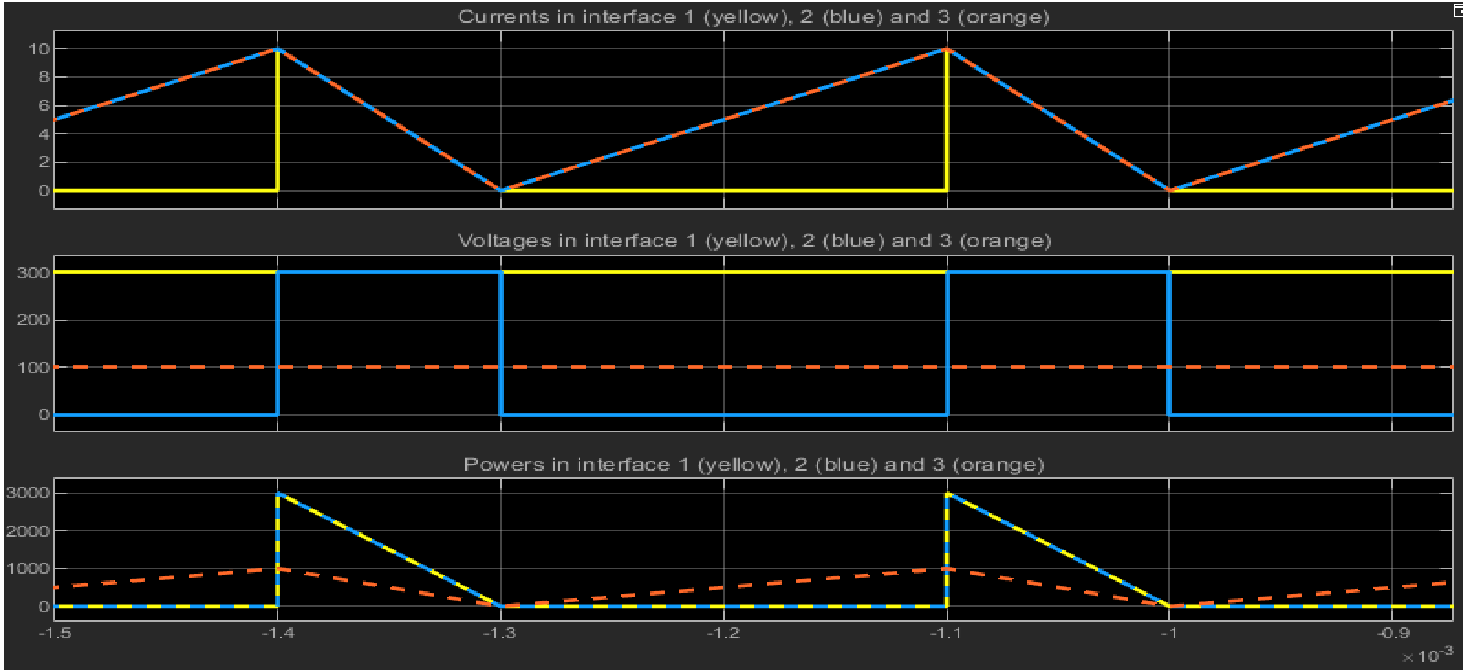
$$\frac{di}{dt} = \begin{cases} \dfrac{e - U_d}{L} & \text{if Transistor OFF} \\ \dfrac{e}{L} & \text{if Transistor ON} \end{cases}$$

$$\Delta i = \begin{cases} \dfrac{e - U_d}{L} \cdot \Delta t = \dfrac{e - U_d}{L} \cdot DutyCycle \cdot T_{sw} = \dfrac{100 - 300}{0.002} \cdot \dfrac{1}{3} \cdot 300 \cdot 10^{-6} = -10 \\ \dfrac{e}{L} \cdot \Delta t = \dfrac{e}{L} \cdot (1 - DutyCycle) \cdot T_{sw} = \dfrac{100}{0.002} \cdot \dfrac{2}{3} \cdot 300 \cdot 10^{-6} = 10 \end{cases}$$

Solution 1.4

3) *The current ripples between 0 and 10 A, with an average value of 5 A.*

4) *Phase current, Phase voltage and dclink current graph*



Exercise 1.5 1QC Buck converter no resistance

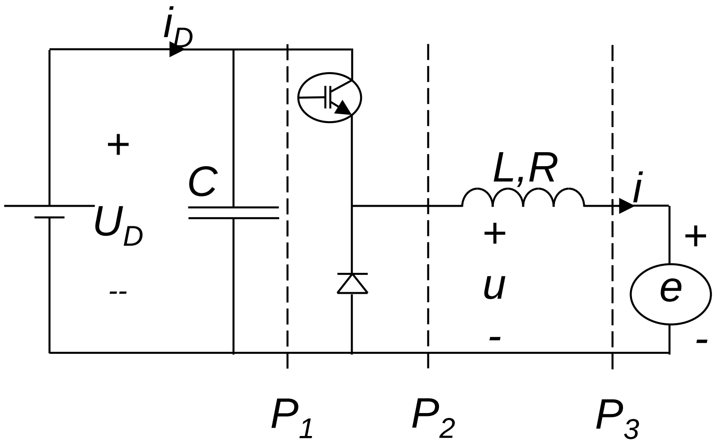
- **Determine**
  - *Phase voltages incl graphs*
  - *DC-clink current incl graphs*
  - *Power at $P_{1,}$ $P_{2,}$ and P_3*

Calculation steps

1. *Time for current to rise from 0 to 5 A*
2. *Time for current to fall from 5 back to 0 A*
3. *Phase voltage when transistor is off and current =0*
4. *Phase current, Phase voltage and dclink current graph*
5. *Average current and average voltage at $p_{1,}$ p_2 and p_3*
6. *Power at $p_{1,}$ p_2 and p_3*

Data for the Buck converter

U_D	**300 V**
e	**100 V**
L	**2 mH**
R	**0 ohm**
f_s (switch- freq)	**3.33 kHz**
i_{avg} (constant)	**5 A**



Solution 1.5

1) Time for current to rise from 0 to 5 A

$$T_{igbt\_on} = \frac{L}{e} \cdot \Delta i = \frac{0.002}{(300-100)} \cdot 5 = 50\,\mu s$$

2) Time for current to fall from 5 back to 0 A

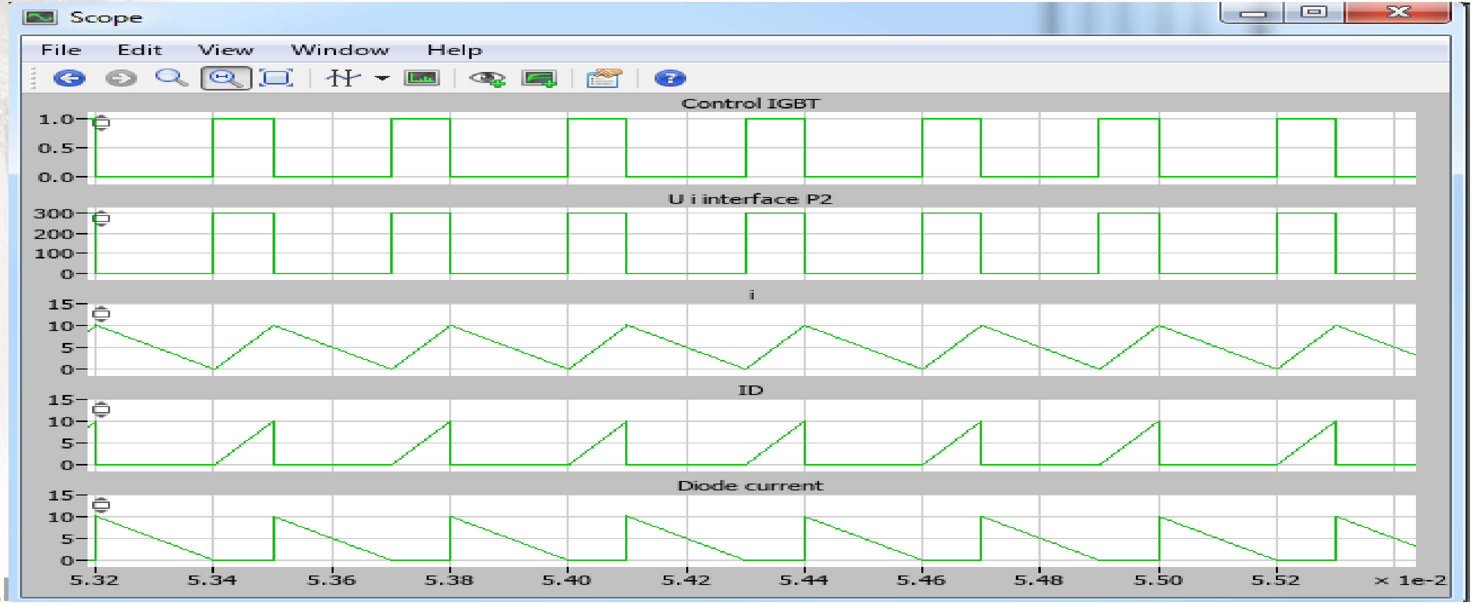
$$T_{igbt\_off} = \frac{L}{e} \cdot \Delta i = \frac{0.002}{100} \cdot 5 = 100\,\mu s$$

Solution 1.5

3) Phase voltage when transistor is off current =0

When the current =0 the diode is not conducting, and as the transistor is off the phase voltage is "floating" and there is no voltage drop over the inductor.
Thus, the phase voltage = 100 V

4. Phase current, Phase voltage and dclink current graph



Solution 1.5

5) Average current and average voltage at $p_{1,}$ p_2 and p_3

$$\begin{cases} U_{avg\_P_1} = U_d = 300V \\ U_{avg\_P_2} = \dfrac{(300\cdot 50\mu s + 0\cdot 100\mu s + 100\cdot(300-50-100)\mu s)}{300\mu s} = 100V \\ U_{avg\_P_3} = e = 100V \end{cases}$$

$$\begin{cases} I_{avg\_P_1} = \dfrac{5\cdot 50\mu s}{2}\cdot\dfrac{1}{300\mu s} = 0.4167A \\ I_{avg\_P_2} = \dfrac{5\cdot 150\mu s}{2}\cdot\dfrac{1}{300\mu s} = 1.25A \\ I_{avg\_P_3} = I_{P_2} \end{cases}$$

6) Power at $p_{1,}$ p_2 and p_3

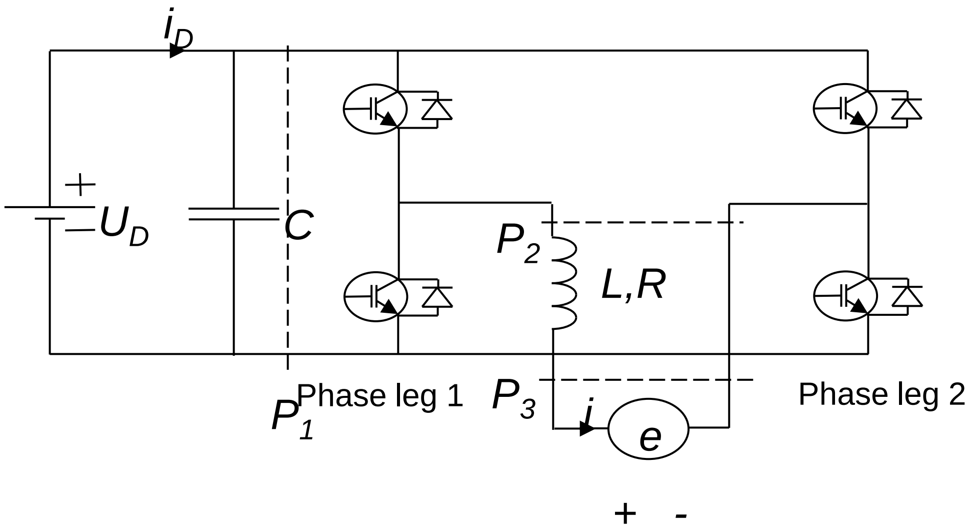
$$\begin{cases} P_{P_1} = U_d \cdot I_{avg\_P_1} = 300\cdot 0.4167 = 125W \\ P_{P_2} = U_{avg\_P_2}\cdot I_{avg\_P_2} = 100\cdot 1.25 = 125W \\ P_{P_3} = e\cdot I_{avg\_P_3} = 100\cdot 1.25 = 125W \end{cases}$$

Exercise 1.6 4QC Bridge converter

- Determine
  - *Phase voltages incl graphs*
  - *DC-link current incl graphs*
  - *Power at P_1, P_2, and P_3*

Data for the Bridge converter

U_D	300 V
e	100 V
L	2 mH
R	0 ohm
F_s(switch- freq)	3.33 kHz
i_{avg} (constant)	10 A



Solution 1.6

Calculation steps

1. Duty cycle

2. Avg phase voltage

3. Phase current. Ripple and min and max current

4. Phase current graph, phase voltage graph and dclink current graph

5. Average current and average voltage at $p_{1,} p_2$ and p_3

6. Power at $p_{1,} p_2$ and p_3

1) $duty_{cycle\_1} = 1 - Duty_{cycle\_2}$

$$e = U_d \cdot duty_{cycle\_1} - U_d \cdot duty_{cycle\_2} = U_d \cdot duty_{cycle\_1} - U_d \cdot \left(1 - duty_{cycle\_1}\right) =$$

$$= U_d \cdot \left(2 \cdot duty_{cycle\_1} - 1\right)$$

$$\begin{cases} duty_{cycle\_1} = \dfrac{1 + \dfrac{e}{U_d}}{2} = \dfrac{1 + \dfrac{100}{300}}{2} = 0.67 \\ duty_{cycle\_2} = 1 - duty_{cycle\_1} = 0.33 \end{cases}$$

Solution 1.6

2) Avg phase voltage

$$\begin{cases} U_{phase\_1\_avg} = U_d \cdot duty_{cycle\_1} = 300 \cdot 0.67 = 200V \\ U_{phase\_2\_avg} = U_d \cdot duty_{cycle\_2} = 300 \cdot 0.33 = 100V \end{cases}$$

Solution 1.6

3. Phase current. Ripple and min and max current

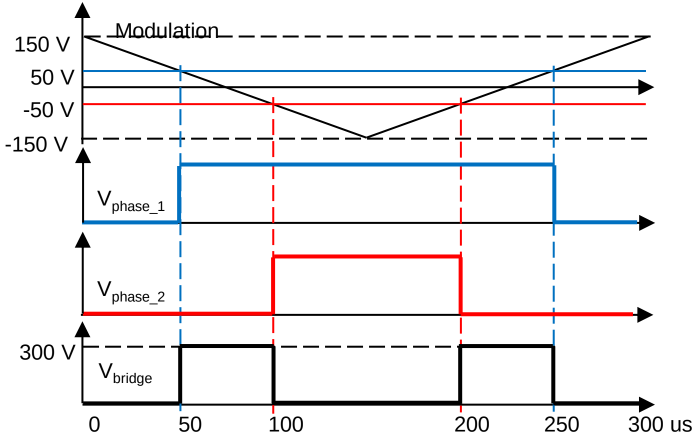
$$V_{phase\_1} = \begin{cases} 0 & 0-50\mu s \\ 300V & 50-250\mu s \\ 0 & 250-300\mu s \end{cases}$$

$$V_{phase\_2} = \begin{cases} 0 & 0-100\mu s \\ 300V & 100-200\mu s \\ 0 & 200-300\mu s \end{cases}$$

$$V_{bridge} = V_{phase\_1} - V_{phase\_2} = \begin{cases} 0 & 0-50\mu s \\ 300V & 50-100\mu s \\ 0 & 100-200\mu s \\ 300V & 200-250\mu s \\ 0 & 250-300\mu s \end{cases}$$

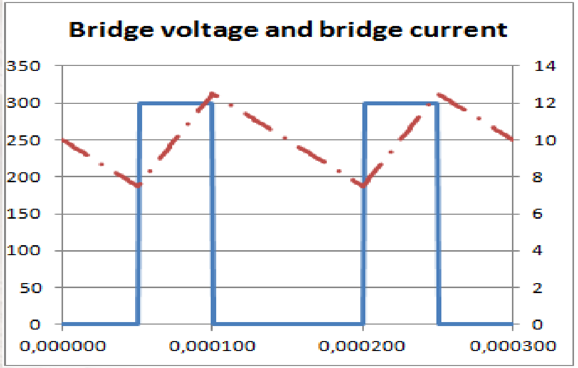
$$I_{ripple} = \Delta I = \frac{U_{bridge} - e}{L} \cdot \Delta t = \frac{300-100}{0.002} \cdot 50\mu s = 5\ A$$

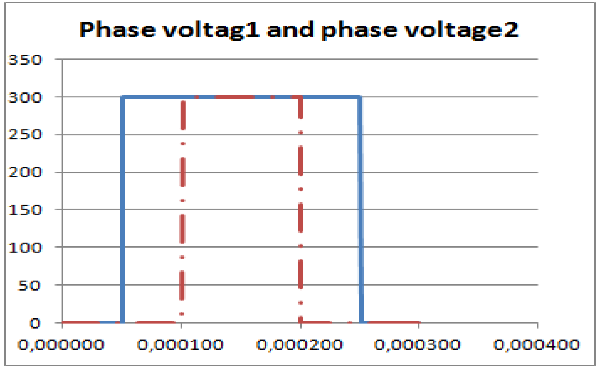
$$\begin{cases} I_{\min} = I_{avg} - 0.5 \cdot I_{ripple} = 7.5\,A \\ I_{\max} = I_{avg} + 0.5 \cdot I_{ripple} = 12.5\,A \end{cases}$$

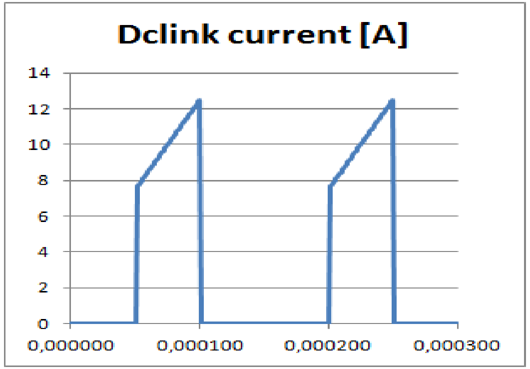


Solution 1.6

4) Phase current graph, phase voltage graph and dclink current graph







Solution 1.6

5. Average current and average voltage at $p_{1,} p_2$ and p_3

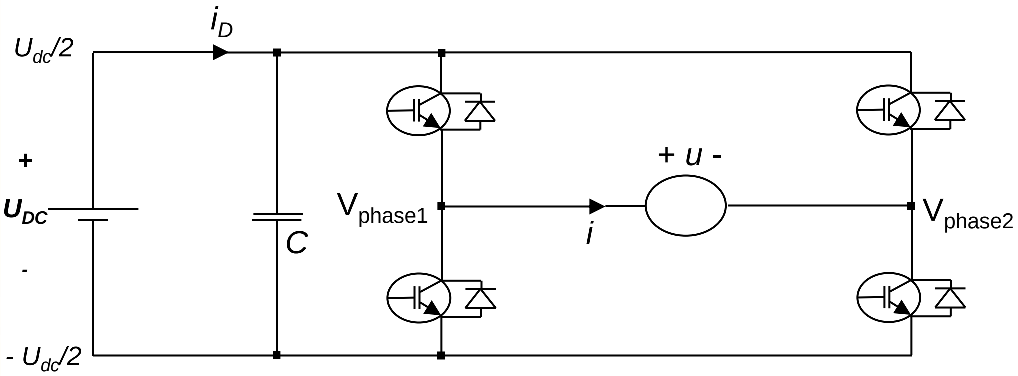
$$\begin{cases} I_{avg\_P_1} = duty_{cycle\_1} - duty_{cycle\_2} \cdot 10A = 3.333A \\ I_{avg\_P_2} = 10A \\ I_{avg\_P_3} = 10A \end{cases}$$

$$\begin{cases} U_{avg\_P_1} = 300\ V \\ U_{avg\_P_2} = 100\ V \\ U_{avg\_P_3} = 100V \end{cases}$$

6. Power at $p_{1,} p_2$ and p_3

$$\begin{cases} P_{p1} = U_{avg\_p_1} \cdot I_{avg\_p_1} = 300 \cdot 3.333 = 1.0\,kW \\ P_{p2} = U_{avg\_p_2} \cdot I_{avg\_p_2} = 100 \cdot 10 = 1.0\,kW \\ P_{p3} = U_{avg\_p_3} \cdot I_{avg\_p_3} = 100 \cdot 10 = 1.0\,kW \end{cases}$$

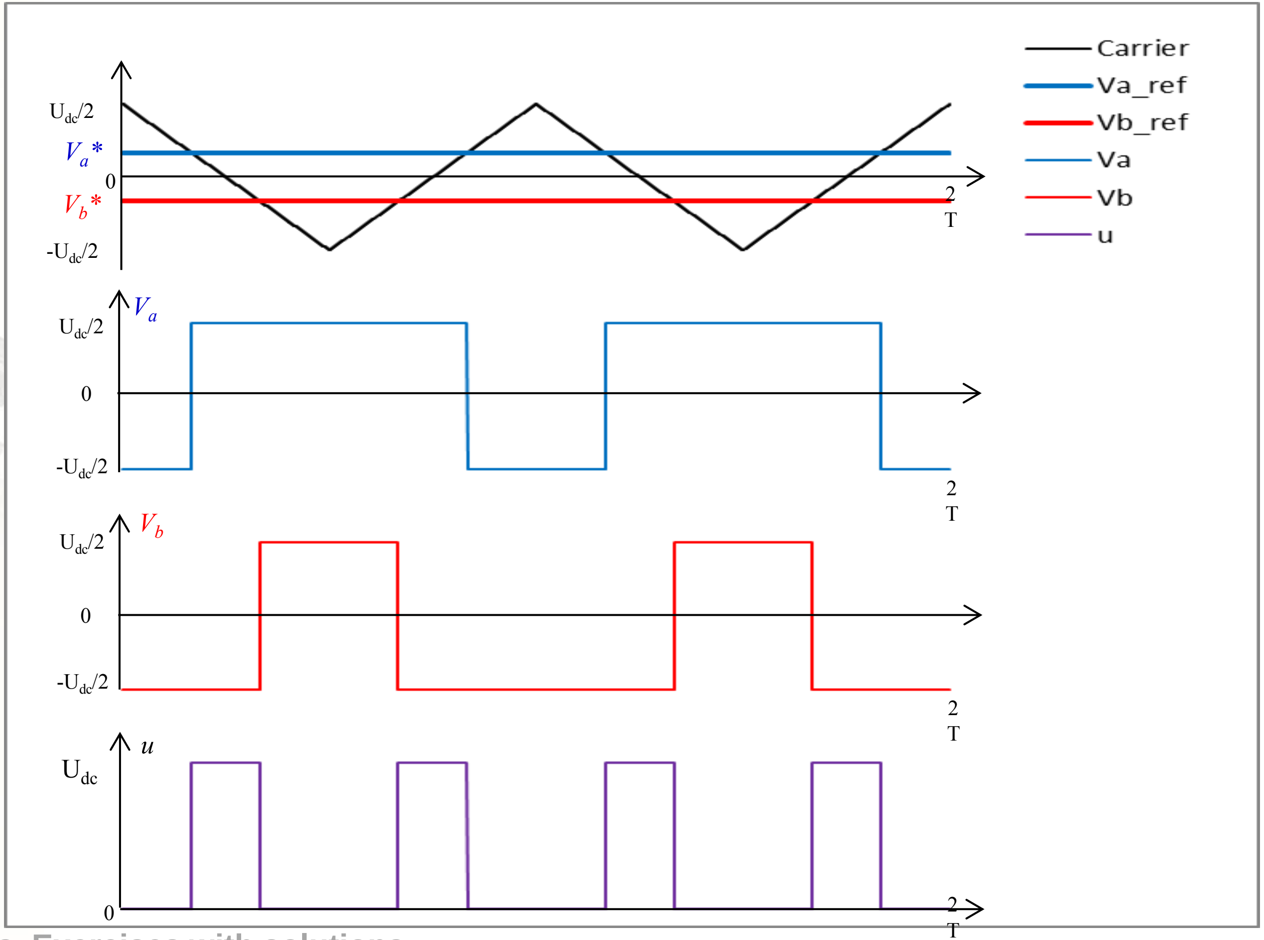
Exercise 1.7 4QC Bridge converter modulation



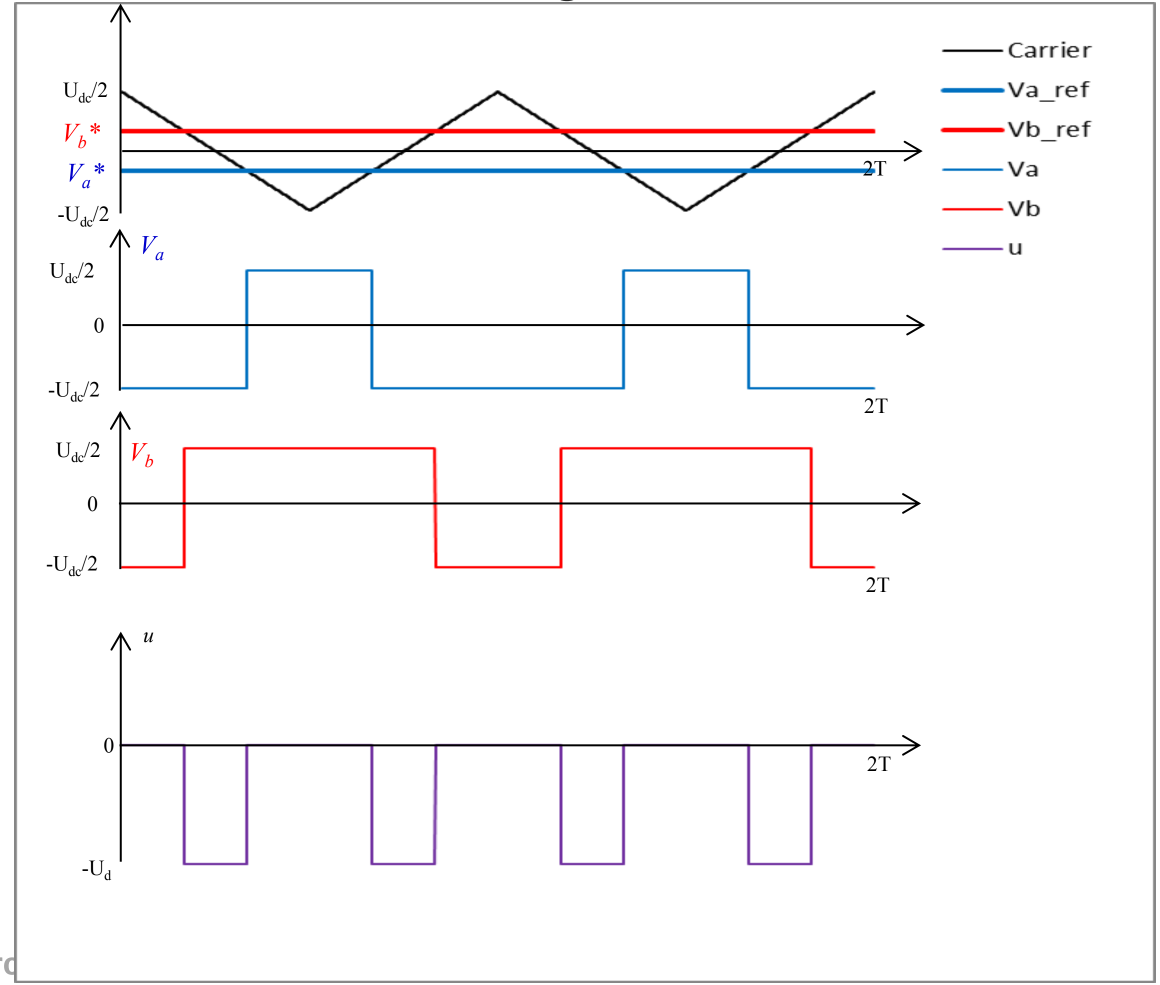
The resistance of the load is neglected, stationary state is assumed.

- *The ouput voltage reference is:*
  a) **u* =** $U_{dc}/3$
  b) **u* = -** $U_{dc}/3$
- *The phase potential references are:*
  - $v_a^* = u^*/2$
  - $v_b^* = -u^*/2$
- *Draw the phase potentials $v_a(t)$ and $v_b(t)$ together with the output voltage u(t) for cases a) and b) for two carrier wave periods!*

Solution 1.7a Modulation with positive reference



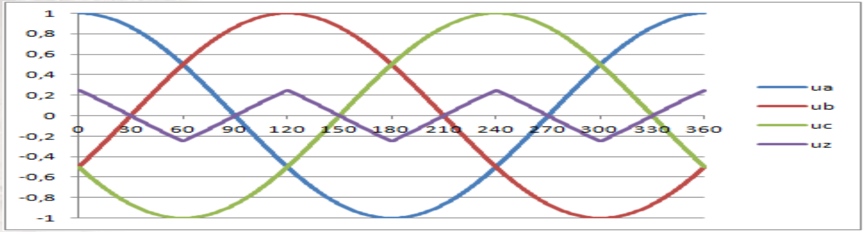
Solution 1.7b Modulation with negative reference

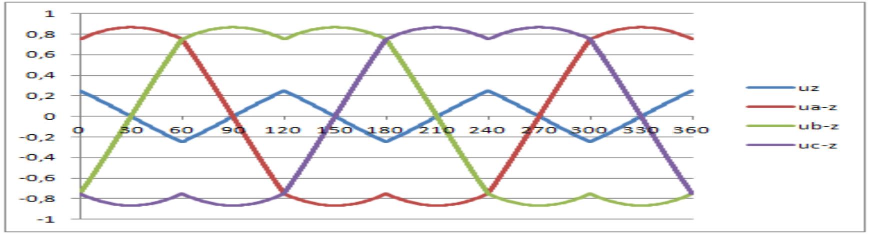


Exercise 1.8 Symmetrized 3phase voltage

- The sinusoidal reference curves (v_a^*, v_b^*, v_c^*) for a three phase constant voltage converter can be modified with a zero-sequence signal:
  - *v_z^* = [max(a, b, c)+min(a, b, c)]/2*

  according to the figure below.
- Determine the analytical expression for e.g. a-z in one of the 60˚ intervals!
- Determine the ratio between the maxima of the input and output signals!





Solution 1.8

The interval 0-60 deg is used (any such 60 degree can be used)
Find the maximum in this interval.

$$u_{az,0-60} = u_a - u_z = u_a - \left(\frac{u_a + u_c}{2}\right) = \frac{u_a}{2} - \frac{u_c}{2} = \frac{\cos(x)}{2} - \frac{\cos\left(x - \frac{4\pi}{3}\right)}{2}$$

$$\frac{du_{az,0-60}}{dx} = -\sin(x) + \sin\left(x - \frac{4\pi}{3}\right) = \sin(x)\cdot\cos\left(\frac{4\pi}{3}\right) - \cos(x)\cdot\sin\left(\frac{4\pi}{3}\right) - \sin(x) = -\frac{3}{2}\cdot\sin(x) + \frac{\sqrt{3}}{2}\cdot\cos(x)$$

$$\frac{du_{az,0-60}}{dx} = 0 \Rightarrow \frac{3}{2}\cdot\sin(x) = \frac{\sqrt{3}}{2}\cdot\cos(x) \Rightarrow \tan(x) = \frac{2\cdot\sqrt{3}}{2\cdot 3} = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{\pi}{6} = 30^o$$

$$Check\ if\ \max\ \frac{d^2u_{az,0-60}}{dx^2} = -\frac{3}{2}\cdot\cos(x) - \frac{\sqrt{3}}{2}\cdot\sin(x) = \left\{x = \frac{\pi}{6}\right\} = -\frac{3\sqrt{3}}{4} - \frac{\sqrt{3}}{4} < 0 \Rightarrow \max$$

$$u_{az,0-60}\left(\frac{\pi}{6}\right) = \frac{\cos\left(\frac{\pi}{6}\right)}{2} - \frac{\cos\left(\frac{\pi}{6} - \frac{4\pi}{3}\right)}{2} = \frac{\cos\left(\frac{\pi}{6}\right) - \cos\left(-\frac{7\pi}{6}\right)}{2} = \frac{\frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right)}{2} = \frac{\sqrt{3}}{2} \approx 0.866$$

$$The\ quote\ between\ the\ input\ and\ the\ output\ signals\ = \frac{1}{0.866} = 1.155$$

Exercise 1.9 Sinusoidal 3phase voltage

E.g.use Excel
The following data are choosen.

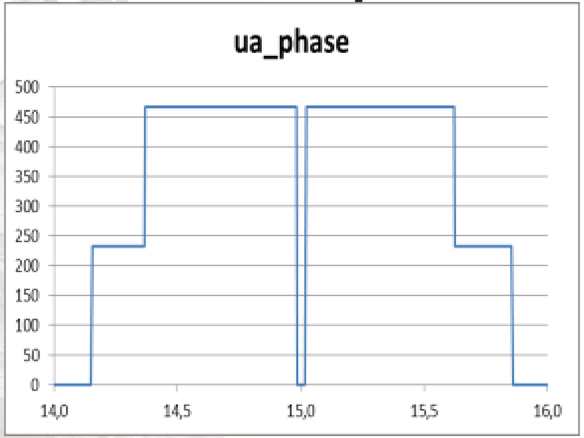
Amplitude of the fundamental voltage	$350\ V_{ac}$
The fundamental voltage frequency	$50\ Hz$
Angle in the fundamental voltage	15^o
The dclink voltage	$700 V_{dc} (\pm 350\ _{dc})$
The carrier wave frequency is set to	$18\,kHz (360\ times\ the\ fundamental\ 50\ Hz)$
One carrier wave period time equals	$\frac{1}{18000} \approx 0.056\,ms$
1^o *in the fundamental equals*	$\frac{1}{50} \cdot \frac{1}{360} \approx 0.056\,ms$
The zero voltage potential equals	$v_0 = \frac{(v_a + v_b + v_c)}{3}$

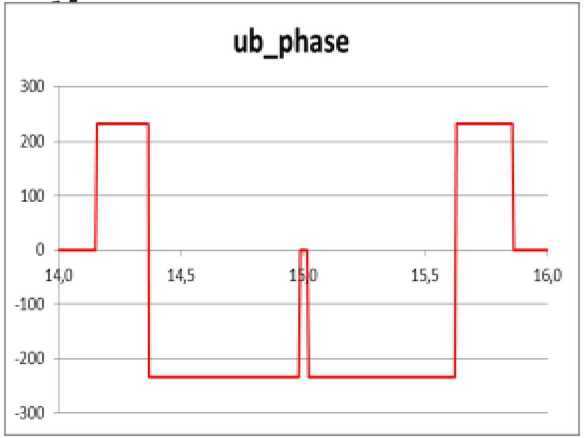
Solution 1.9

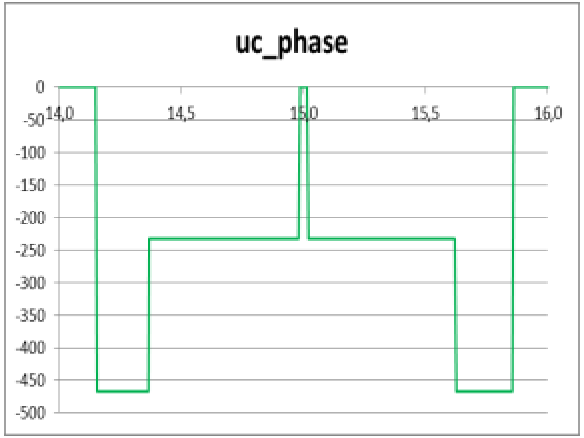
RESULT in the interval 14,5-15,5 deg with numerical integration	Phase a	Phase b	Phase c
4.15a) Average sinusoidal phase voltage	338	-91	-248
4.15b) Average symmetrised phase voltage	338	-91	-248
Compared to Sinusoidal phase voltage reference at 15 degrees	338	-91	-248

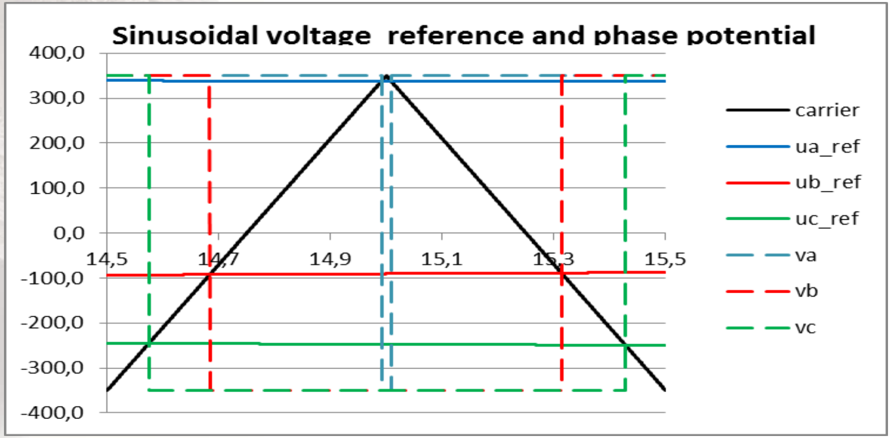
See diagrams at following 2 pages

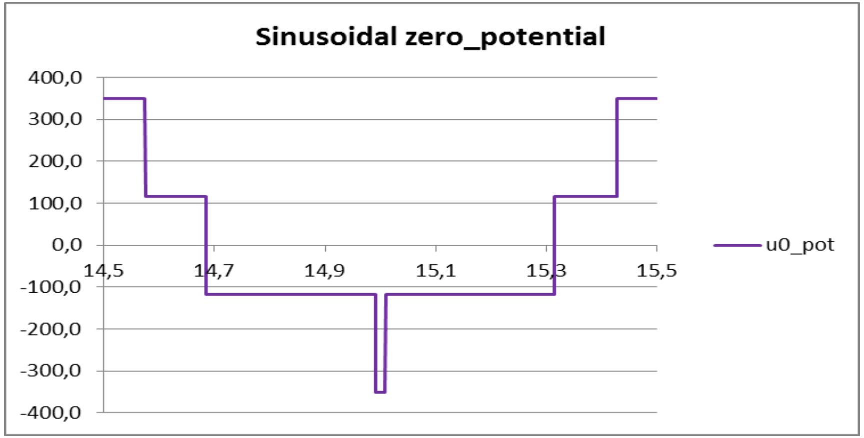
Solution 1.9a
Sinusoidal 3phase voltage reference



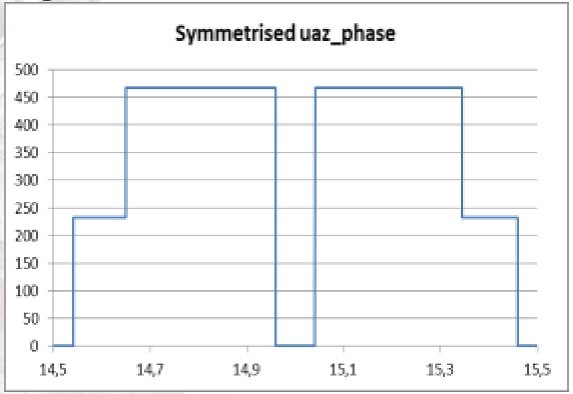


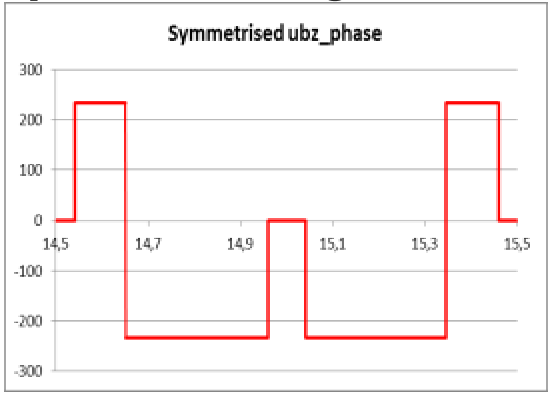


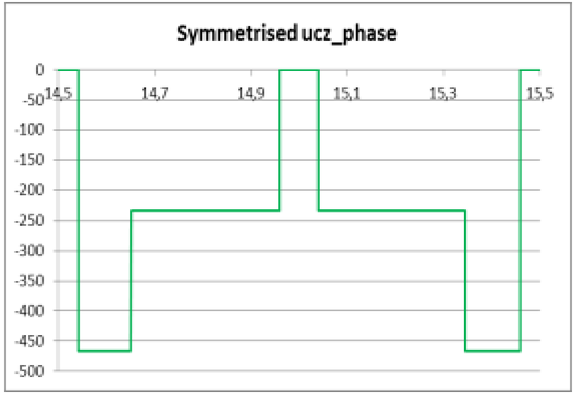


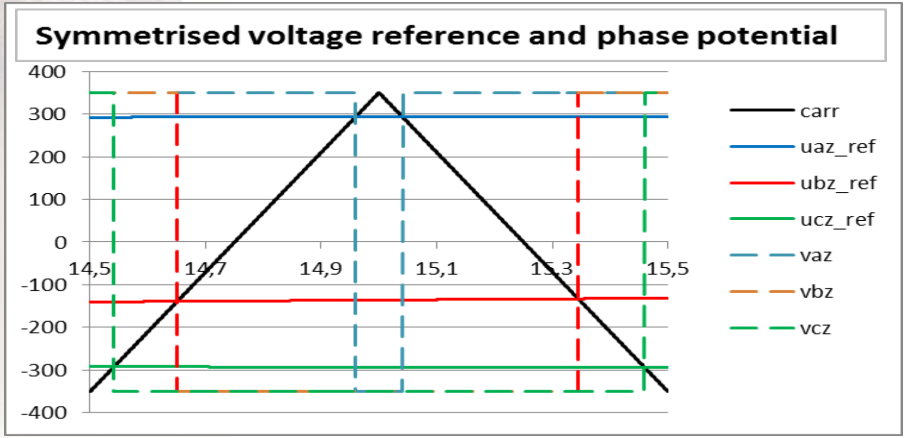


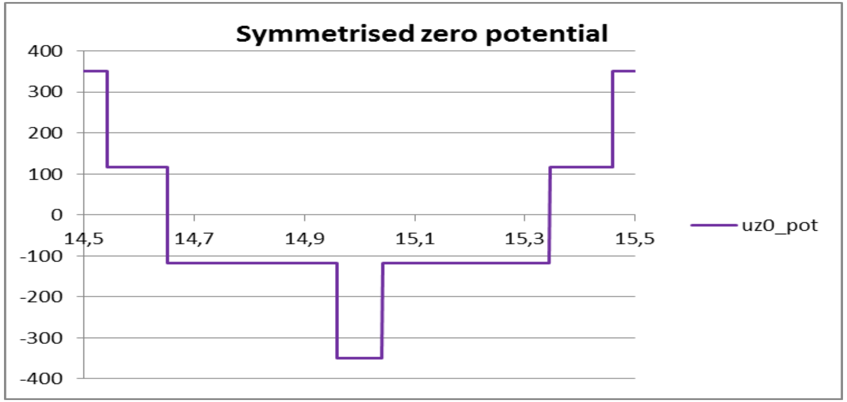
Solution 1.9b
Symmetrised reference 3phase voltage



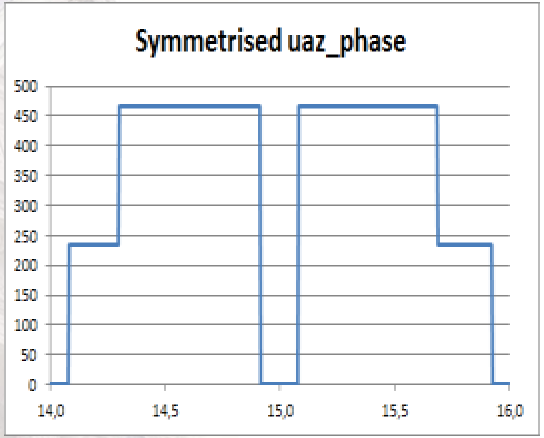


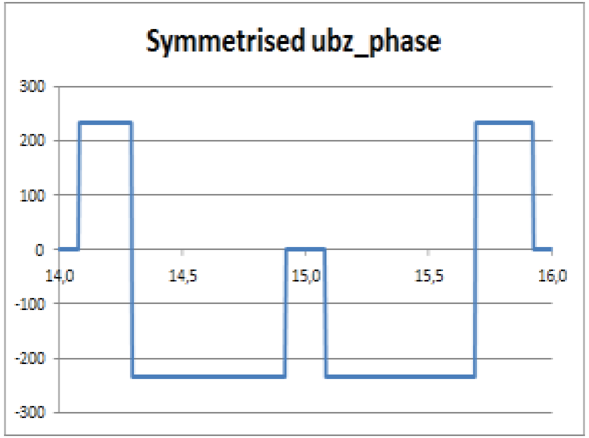


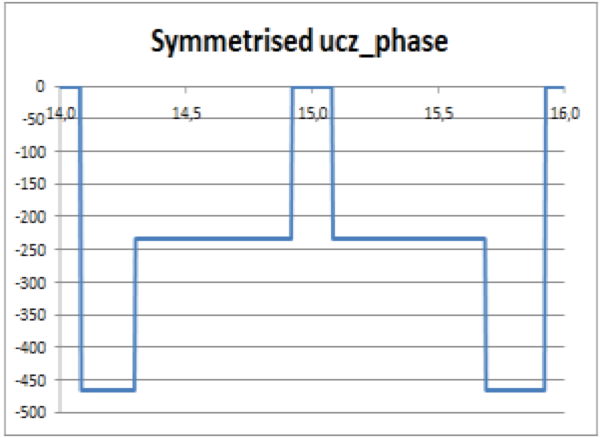




Solution 1.9b cont'd
Symmetrised reference 3phase voltage







Numerical integration in EXCEL
Average voltage 338 V
Voltage time area 0.0376 Vs

Analytical

$$350\cdot\cos(\omega t)=\{\omega t=15^o\}=338V$$

$$\int_{t_0}^{t_1}350\cdot\cos(\omega t)dt=\frac{1}{\omega}\cdot I_{t_0}^{t_1}\sin(\omega t)=$$

$$=\{\omega t_1=16^o,\omega t_0=14^o\}=0.0376Vs$$

Numerical integration in EXCEL
Average voltage -90.4 V
Voltage time area -0.01 Vs

Analytical

$$350\cdot\cos(\omega t-2\pi/3)=\{\omega t=15^o\}=-90.6V$$

$$\int_{t_0}^{t_1}350\cdot\cos(\omega t-2\pi/3)dt=\frac{1}{\omega}\cdot I_{t_0}^{t_1}\sin(\omega t-2\pi/3)=$$

$$=\{\omega t_1=16^o,\omega t_0=14^o\}=-0.0101Vs$$

Numerical integration in EXCEL
Average voltage -248 V
Voltage time area -0.0275 Vs

Analytical

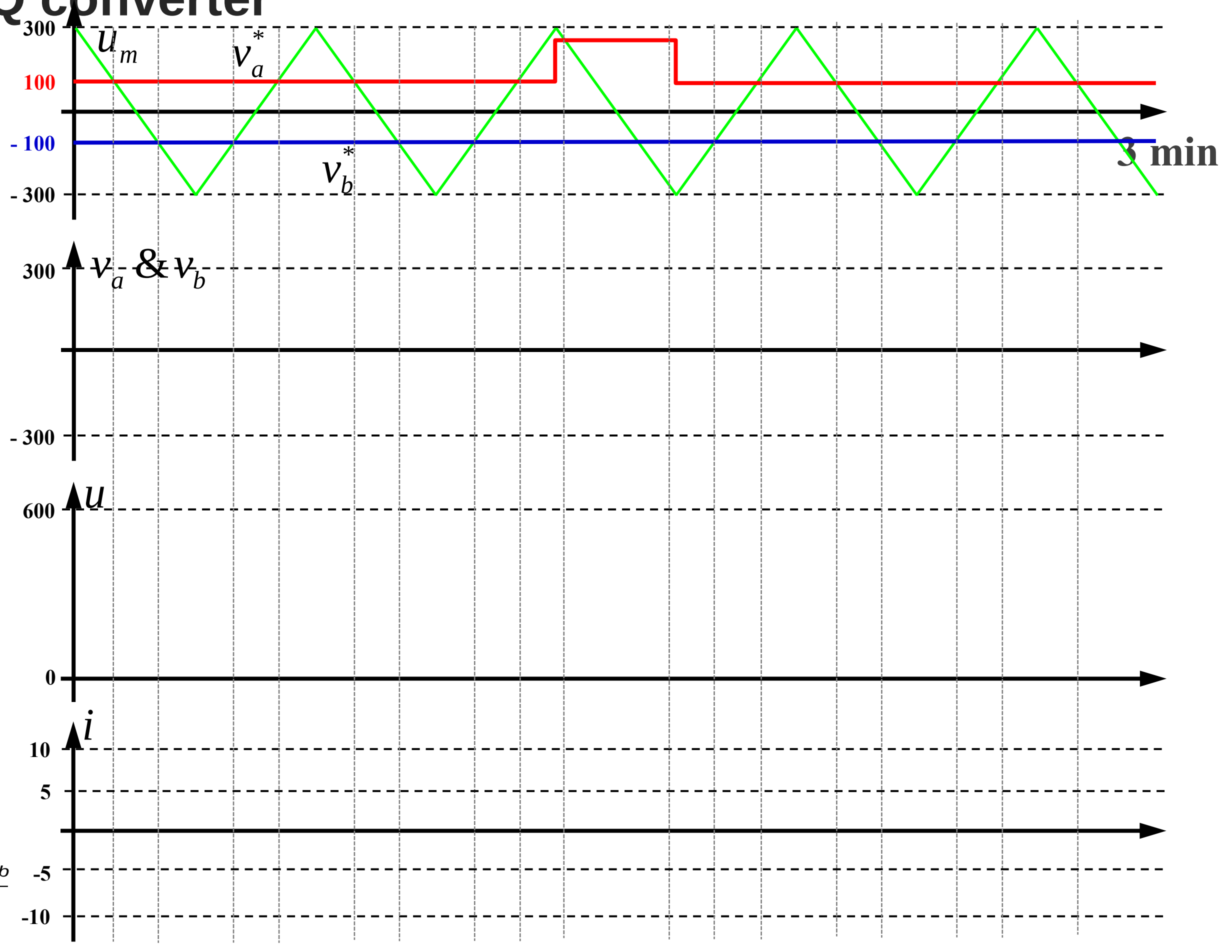
$$350\cdot\cos(\omega t-4\pi/3)=\{\omega t=15^o\}=-247.5V$$

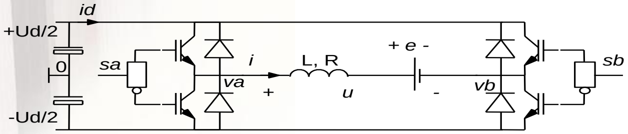
$$\int_{t_0}^{t_1}350\cdot\cos(\omega t-4\pi/3)dt=\frac{1}{\omega}\cdot I_{t_0}^{t_1}\sin(\omega t-4\pi/3)=$$

$$=\{\omega t_1=16^o,\omega t_0=14^o\}=-0.0275Vs$$

1.9 : Modulation of a 4Q converter

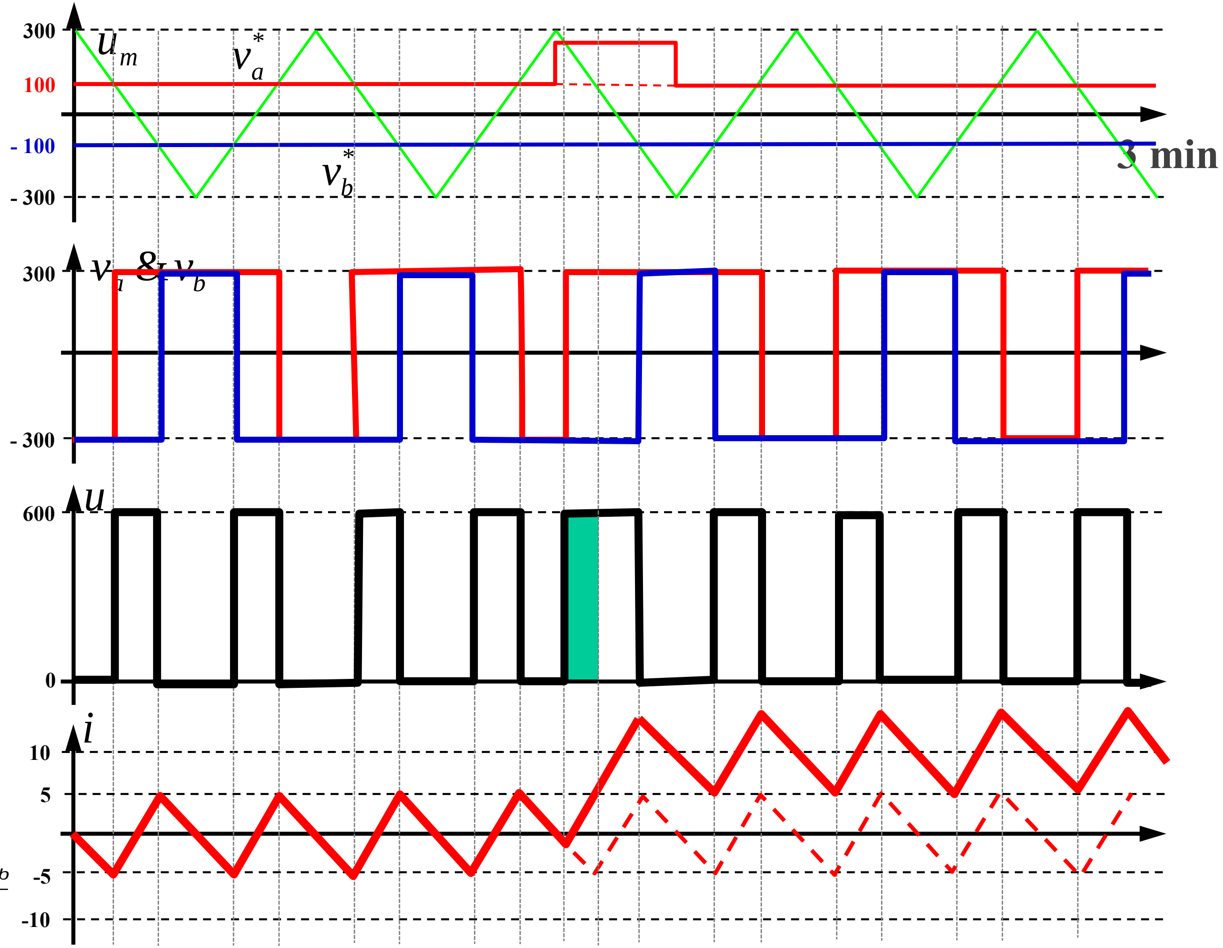
- **Given:**
  - *Udc = 600 V*
  - *e=200 V*
  - *i(t=0)=0*
  - *Voltage reference given*
- **Parameters:**
  - *L= 2 [mH]*
  - *Switchfrekvens: 6.67 [kHz]*
- **Draw:**
  - *Potentials v_a and v_b*
  - *Load voltage u*
- **Calculate**
  - *Positive current derivative*
  - *Negative current derivative*
- **Draw**
  - *Load current i*

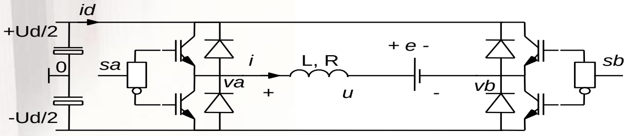




1.9 Solution

- Given:
  - *Udc = 600 V*
  - *e=200 V*
  - *i(t=0)=0*
  - *Voltage reference given*
- Parameters:
  - *L= 2 [mH]*
  - *Switchfrekvens: 6.67 [kHz]*
- Draw:
  - *Potentials v_a and v_b*
  - *Load voltage u*
- Calculate
  - *Positive current derivative*
  - *Negative current derivative*
- Draw
  - *Load current i*

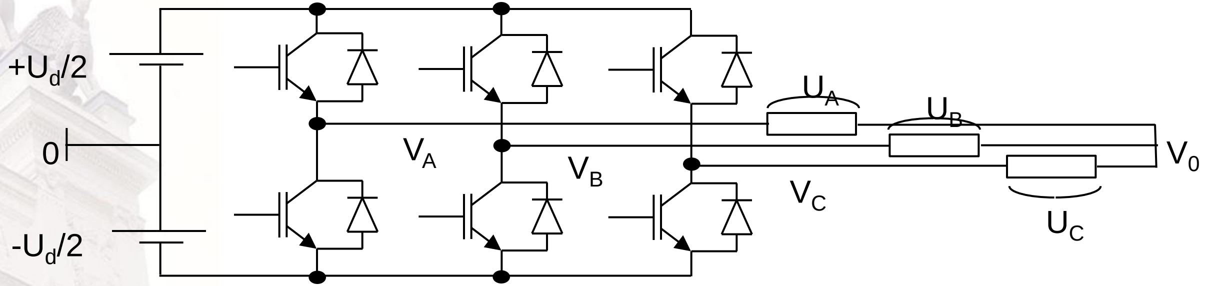




Exercise 1.10 Voltage vectors

Deduce the 8 voltage vectors that are created in a converter fed by a constant voltage!

Solution 1.10

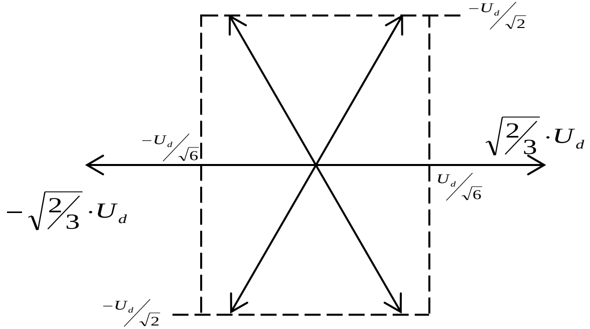


$$V_0 = \frac{V_A + V_B + V_C}{3}$$

$$\begin{cases} U_A = V_A - V_0 \\ U_B = V_B - V_0 \\ U_C = V_C - V_0 \end{cases}$$

$$\begin{cases} U_\alpha = \sqrt{3/2} \cdot U_A \\ U_\beta = \frac{1}{\sqrt{2}} \cdot (U_B - U_C) \end{cases}$$

modA	mod**B**	modC	V_A	V_B	V_C	V_0	U_A	U_B	U_C	ualfa	ubeta*j	U_{vector}
0	0	0	$-U_d/2$	$-U_d/2$	$-U_d/2$	$-U_d/2$	0	0	0	0	0	0
0	0	1	$-U_d/2$	$-U_d/2$	$U_d/2$	$-U_d/6$	$-U_d/3$	$-U_d/3$	$U_d 2/3$	$-U_d/\sqrt{6}$	$-U_d/\sqrt{2}$	$\sqrt{2/3} \cdot U_d \cdot e^{\frac{4\pi}{3}}$
0	1	1	$-U_d/2$	$U_d/2$	$U_d/2$	$U_d/6$	$-U_d 2/3$	$U_d/3$	$U_d/3$	$-U_d\sqrt{2/3}$	0	$-\sqrt{2/3} \cdot U_d$
0	1	0	$-U_d/2$	$U_d/2$	$-U_d/2$	$-U_d/6$	$-U_d/3$	$U_d 2/3$	$-U_d/3$	$-U_d/\sqrt{6}$	$U_d/\sqrt{2}$	$\sqrt{2/3} \cdot U_d \cdot e^{\frac{2\pi}{3}}$
1	1	0	$U_d/2$	$U_d/2$	$-U_d/2$	$U_d/6$	$U_d/3$	$U_d/3$	$-U_d 2/3$	$U_d/\sqrt{6}$	$U_d/\sqrt{2}$	$\sqrt{2/3} \cdot U_d \cdot e^{\frac{\pi}{3}}$
1	0	0	$U_d/2$	$-U_d/2$	$-U_d/2$	$-U_d/6$	$U_d 2/3$	$-U_d/3$	$-U_d/3$	$U_d\sqrt{2/3}$	0	$\sqrt{2/3} \cdot U_d$
1	0	1	$U_d/2$	$-U_d/2$	$U_d/2$	$U_d/6$	$U_d/3$	$-U_d 2/3$	$U_d/3$	$U_d/\sqrt{6}$	$-U_d/\sqrt{2}$	$\sqrt{2/3} \cdot U_d \cdot e^{\frac{5\pi}{3}}$
1	1	1	$U_d/2$	$U_d/2$	$U_d/2$	$U_d/2$	0	0	0	0	0	0



Exercise 1.11 i_D and i_Q in symmetric three phase

A three phase self commutated converter is connected to the three phase grid with net reactors. The fundamental current of from the converter to the net is a symmetric 3-phase system:

$i_a = \hat{\imath} \cdot \cos(\omega t)$

$i_b = \hat{\imath} \cdot \cos(\omega t - 2*pi/3)$

$i_c = \hat{\imath} \cdot \cos(\omega t - 4*pi/3)$

a) **Deduce the expressions for i_D and i_Q!**

b) **Determine the active and reactive power!**

c) **The DC voltage is U_d. Determine the highest possible grid voltage relative to U_d!**

Solution 1.11a

$a)$ *Symmetric* $3-$ *phase RST* $-$ *frame*

$$\begin{cases} i_{a(1)} = \hat{i}\cdot\cos(\omega\cdot t-\phi) \\ i_{b(1)} = \hat{i}\cdot\cos\left(\omega\cdot t-\frac{2\pi}{3}-\phi\right) \\ i_{c(1)} = \hat{i}\cdot\cos\left(\omega\cdot t-\frac{4\pi}{3}-\phi\right) \end{cases}$$

Transform from RST $-$ *frame to* $\alpha\beta -$ *frame*

$$i_{\alpha\beta} = \sqrt{\frac{2}{3}}\cdot\left(\hat{i}\cdot\cos(\omega\cdot t-\phi)\cdot e^{j0} + \hat{i}\cdot\cos\left(\omega\cdot t-\frac{2\pi}{3}-\phi\right)\cdot e^{j\frac{2\pi}{3}} + \hat{i}\cdot\cos\left(\omega\cdot t-\frac{4\pi}{3}-\phi\right)\cdot e^{j\frac{4\pi}{3}}\right) =$$

$$= \left\{\cos(x) = \frac{e^{jx}+e^{-jx}}{2}\right\} = \sqrt{\frac{2}{3}}\cdot\hat{i}\cdot\left(\frac{e^{j(\omega t-\phi)}+e^{-j(\omega t-j\phi)}}{2} + \frac{e^{j\left(\omega t-\frac{2\pi}{3}-\phi\right)}+e^{-j\left(\omega t+\frac{2\pi}{3}-\phi\right)}}{2}\cdot e^{j\frac{2\pi}{3}} + \frac{e^{j\left(\omega t-\frac{4\pi}{3}-\phi\right)}+e^{-j\left(\omega t+\frac{4\pi}{3}-\phi\right)}}{2}\cdot e^{j\frac{4\pi}{3}}\right) =$$

$$= \sqrt{\frac{2}{3}}\cdot\frac{\hat{i}}{2}\cdot\left(e^{j\omega t-j\phi}+e^{-j\omega t+j\phi}+\left(e^{j\omega t-j\frac{2\pi}{3}-j\phi}+e^{-j\omega t+j\frac{2\pi}{3}+j\phi}\right)\cdot e^{j\frac{2\pi}{3}}+\left(e^{j\omega t-j\frac{4\pi}{3}-j\phi}+e^{-j\omega t+j\frac{4\pi}{3}+j\phi}\right)\cdot e^{j\frac{4\pi}{3}}\right) =$$

$$= \sqrt{\frac{2}{3}}\cdot\frac{\hat{i}}{2}\cdot\left(e^{j\omega t-j\phi}+e^{-j\omega t+j\phi}+e^{j\omega t-j\frac{2\pi}{3}-j\phi+j\frac{2\pi}{3}}+e^{-j\omega t+j\frac{2\pi}{3}+j\phi+j\frac{2\pi}{3}}+e^{j\omega t-j\frac{4\pi}{3}-j\phi+j\frac{4\pi}{3}}+e^{-j\omega t+j\frac{4\pi}{3}+j\phi+j\frac{4\pi}{3}}\right) =$$

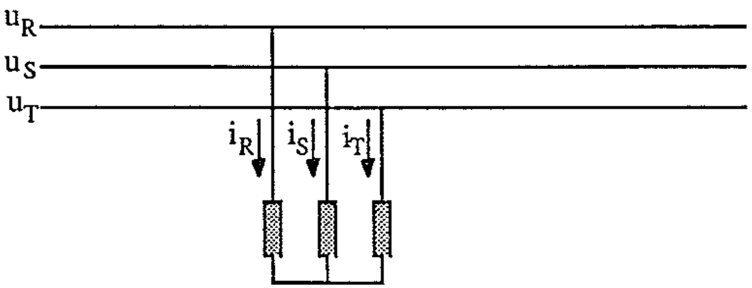
$$= \sqrt{\frac{2}{3}}\cdot\frac{\hat{i}}{2}\cdot\left(e^{j\omega t-j\phi}+e^{-j\omega t+j\phi}+e^{j\omega t-j\phi}+e^{-j\omega t+j\phi+j\frac{4\pi}{3}}+e^{j\omega t-j\phi}+e^{-j\omega t+j\phi+j\frac{8\pi}{3}}\right) = \sqrt{\frac{2}{3}}\cdot\frac{\hat{x}}{2}\cdot\left(3\cdot e^{j\omega t-j\phi}+e^{-j\omega t+j\phi}\cdot\left(1-\frac{1}{2}-\frac{\sqrt{3}}{2}-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\right) =$$

$$= \frac{3\sqrt{2}}{2\sqrt{3}}\cdot\hat{i}\cdot e^{j\omega t-j\phi} = \sqrt{\frac{3}{2}}\cdot\hat{i}\cdot e^{j(\omega t-\phi)} = \sqrt{\frac{3}{2}}\cdot\hat{i}\cdot\left(\cos(\omega t-\phi)+j\cdot\sin(\omega t-\phi)\right)$$

Flux coordinates $i_{dq} = i_{\alpha\beta}\cdot e^{-j\left(\omega t-\frac{\pi}{2}\right)} = \sqrt{\frac{3}{2}}\cdot\hat{i}\cdot e^{j\omega t-j\phi-j\omega t+j\frac{\pi}{2}} = \sqrt{\frac{3}{2}}\cdot\hat{i}\cdot e^{j\left(\frac{\pi}{2}-\phi\right)} = \sqrt{\frac{3}{2}}\cdot\hat{i}\cdot\left(\cos\left(\frac{\pi}{2}-\phi\right)+j\cdot\sin\left(\frac{\pi}{2}-\phi\right)\right)$

$$\begin{cases} i_d = \sqrt{\frac{3}{2}}\cdot\hat{i}\cdot\cos\left(\frac{\pi}{2}-\phi\right) \\ i_q = \sqrt{\frac{3}{2}}\cdot\hat{i}\cdot\sin\left(\frac{\pi}{2}-\phi\right) \end{cases}$$

Exercise 1.12 3 phase line



Symmetric three phase

A three phase grid with the voltages uR, uS, uT loaded by sinusoidal currents iR, iS, iT and the angle ϕ.

a) Derive the expression for the voltage vector

b) Derive the expression for the current vector

c) Determine the active power p(t)!

Do the same derivations as above with the vectors expressed in the flux reference frame!

Solution 1.12a

$$a)\ Equ\ \ 2.39,\ 2.40\ \ and\ exercise\ 4.26a\ \vec{u}_N = \sqrt{\frac{2}{3}} \cdot \left(u_R + u_S \cdot e^{j\frac{2\pi}{3}} + u_T \cdot e^{j\frac{4\pi}{3}} \right) = \sqrt{\frac{3}{2}} \cdot u_R + \frac{1}{\sqrt{2}} \cdot (u_S - u_T)$$

$$\left\{ \begin{aligned} u_R &= \hat{u} \cdot \cos(\omega \cdot t) \\ u_S &= \hat{u} \cdot \cos\left(\omega \cdot t - \frac{2\pi}{3} \right) \\ u_T &= \hat{u} \cdot \cos\left(\omega \cdot t - \frac{4\pi}{3} \right) \end{aligned} \right\} \Rightarrow \vec{u}_N = \sqrt{\frac{3}{2}} \cdot \hat{u} \cdot e^{j\omega t}$$

Solution 1.12b

$$b)\ In\ the\ same\ way\quad \vec{i}_N = \sqrt{\frac{2}{3}} \cdot \left(i_R + i_S \cdot e^{j\frac{2\pi}{3}} + i_T \cdot e^{j\frac{4\pi}{3}} \right) = \sqrt{\frac{3}{2}} \cdot i_R + \frac{1}{\sqrt{2}} \cdot (i_S - i_T)$$

$$\left\{ \begin{aligned} i_R &= \hat{i} \cdot \cos(\omega \cdot t - \varphi) \\ i_S &= \hat{i} \cdot \cos\left(\omega \cdot t - \frac{2\pi}{3} - \varphi \right) \\ i_T &= \hat{i} \cdot \cos\left(\omega \cdot t - \frac{4\pi}{3} - \varphi \right) \end{aligned} \right\} \Rightarrow \vec{i}_N = \sqrt{\frac{3}{2}} \cdot \hat{i} \cdot e^{j(\omega t - \varphi)}$$

$$The\ power\ in\ dq-frame,\ the\ dot-product\ P = e^{dq} \cdot i^{dq} = e_d \cdot i_d + e_q \cdot i_q$$

$$\left\{ \begin{aligned} e_a &= \hat{e} \cdot \cos(\omega \cdot t) \\ e_b &= \hat{e} \cdot \cos\left(\omega \cdot t - \frac{2\pi}{3} \right) \\ e_c &= \hat{e} \cdot \cos\left(\omega \cdot t - \frac{4\pi}{3} \right) \end{aligned} \right\} \Rightarrow \vec{e}^{\alpha\beta} = \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot e^{j\omega t}$$

$$Transform\ from\ \alpha\beta-frame\ to\ dq-flux\ frame$$

$$\vec{e}^{dq} = \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot e^{j\omega t} \cdot e^{-j\left(\omega t - \frac{\pi}{2}\right)} = \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot e^{j\omega t - j\omega t + j\frac{\pi}{2}} = \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot e^{j\cdot\frac{\pi}{2}} = j \cdot \sqrt{\frac{3}{2}} \cdot \hat{e} = \vec{e}^{\,q}, \left(\vec{e}^{\,d} = 0 \right)$$

$$\begin{cases} The\ active\ power & P = e_q \cdot i_q = \sqrt{\frac{3}{2}} \cdot \hat{e} \cdot \sqrt{\frac{3}{2}} \cdot \hat{i} \cdot \sin\left(\frac{\pi}{2} - \varphi \right) = \frac{3}{2} \cdot \hat{e} \cdot \hat{i} \cdot \cos(\varphi) = \sqrt{3} \cdot e_{Heff} \cdot i_{eff} \cdot \cos(\varphi) \\ The\ reactive\ power & Q = \sqrt{3} \cdot e_{Heff} \cdot i_{eff} \cdot \sin(\varphi) \end{cases}$$

Solution 1.12c

$$c)\begin{cases} Sinusoidal\ modulation\ U_{LNrms} = \dfrac{U_{dc}}{2\cdot\sqrt{2}} = \dfrac{U_{dc}}{\sqrt{8}} \approx 0.35\cdot U_{dc} \\ Sinusoidal\ modulation\ U_{LLrms} = \dfrac{\sqrt{3}}{\sqrt{2}}\cdot\dfrac{U_{dc}}{2} = \sqrt{\dfrac{3}{8}}\cdot U_{dc} \approx 0.61\cdot U_{dc} \\ Symmetriced\ modulation\ U_{LNrms} = \dfrac{U_{dc}}{\sqrt{3}\cdot\sqrt{2}} = \dfrac{U_{dc}}{\sqrt{6}} \approx 0.41\cdot U_{dc} \\ Symmetriced\ modulation\ U_{LLrms} = \dfrac{U_{dc}\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{2}} = \dfrac{U_{dc}}{\sqrt{2}} \approx 0.71\cdot U_{dc} \end{cases}$$

$$Power\ in\ \alpha\beta-frame\ P(t)=\left\{P=\mathrm{Re}\left(\vec{u}_N\cdot\vec{i}_N^{\,*}\right)\right\}=\mathrm{Re}\left(\sqrt{\frac{3}{2}}\cdot\hat{u}\cdot e^{j\omega t}\cdot\sqrt{\frac{3}{2}}\cdot\hat{i}\cdot e^{-j(\omega t-\varphi)}\right)=\frac{3}{2}\cdot\mathrm{Re}\left(\hat{u}\cdot\hat{i}\cdot e^{j\omega t-j\omega t+j\varphi}\right)=$$

$$=\frac{3}{2}\cdot\mathrm{Re}\left(\hat{u}\cdot\hat{i}\cdot e^{j\varphi}\right)=\frac{3}{2}\cdot\hat{u}\cdot\hat{i}\cdot\cos(\varphi)=\sqrt{3}\cdot U_{Heff}\cdot I_{eff}\cdot\cos(\varphi)$$

$$In\ flux\ \acute{o}rientation,\ flux\ is\ \frac{\pi}{2}\ after\ voltage$$

$$\begin{cases} u^{dq}=u^{\alpha\beta}\cdot e^{-j\omega t+j\frac{\pi}{2}}=\sqrt{\dfrac{3}{2}}\cdot\hat{u}\cdot e^{j\omega t}\cdot e^{-j\omega t+j\frac{\pi}{2}}=\sqrt{\dfrac{3}{2}}\cdot\hat{u}\cdot e^{j\omega t-j\omega t+j\frac{\pi}{2}}=\sqrt{\dfrac{3}{2}}\cdot\hat{u}\cdot e^{j\frac{\pi}{2}} \\ i^{dq}=i^{\alpha\beta}\cdot e^{-j\omega t+j\frac{\pi}{2}}=\sqrt{\dfrac{3}{2}}\cdot\hat{i}\cdot e^{j(\omega t-\varphi)}\cdot e^{-j\omega t+j\frac{\pi}{2}}=\sqrt{\dfrac{3}{2}}\cdot\hat{i}\cdot e^{j\omega t-j\varphi-j\omega t+j\frac{\pi}{2}}=\sqrt{\dfrac{3}{2}}\cdot\hat{i}\cdot e^{j\frac{\pi}{2}-j\varphi} \end{cases}$$

$$P(t)=\mathrm{Re}\left(u^{dq}\cdot i^{dq^*}\right)=\mathrm{Re}\left(\sqrt{\frac{3}{2}}\cdot\hat{u}\cdot e^{j\frac{\pi}{2}}\cdot\sqrt{\frac{3}{2}}\cdot\hat{i}\cdot e^{-\left(j\frac{\pi}{2}-j\varphi\right)}\right)=\frac{3}{2}\cdot\mathrm{Re}\left(\hat{u}\cdot\hat{i}\cdot e^{j\frac{\pi}{2}-j\frac{\pi}{2}+j\varphi}\right)=$$

$$=\frac{3}{2}\cdot\mathrm{Re}\left(\hat{u}\cdot\hat{i}\cdot e^{j\varphi}\right)=\frac{3}{2}\cdot\hat{u}\cdot\hat{i}\cdot\cos(\varphi)$$

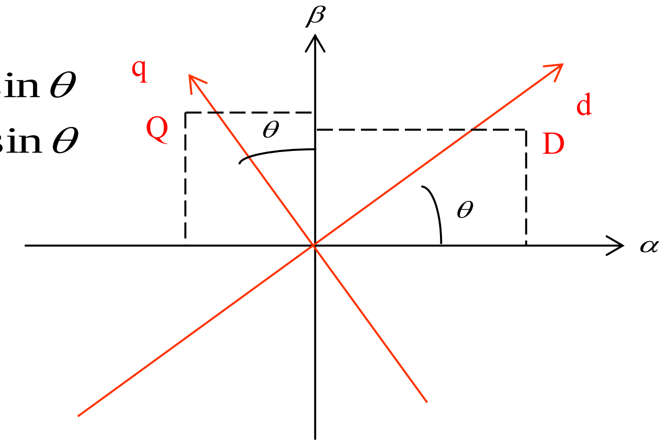
Exercise 1.13 Symmetric 3-phase transformation

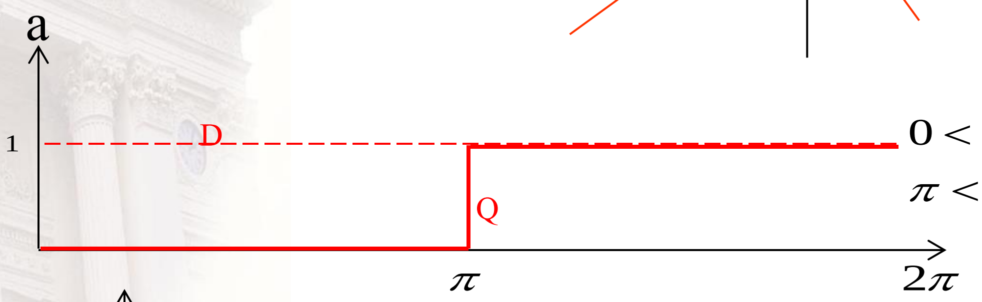
Symmetric three phase

Do the inverse coordinate transformation from the (d,q) reference frame to (a, b) reference frame and the two phase to three phase transformation as well. Express the equations in component form.
Apply the coordinate transform on the following signals.

Solution 1.13

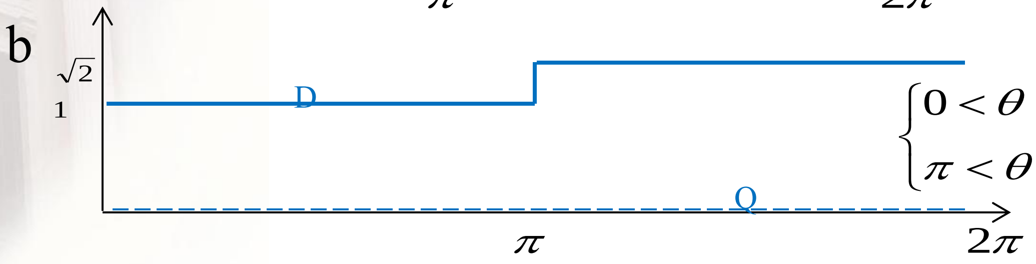
$$\begin{cases} \alpha = D\cdot\cos\theta - Q\cdot\sin\theta \\ \beta = Q\cdot\cos\theta + D\cdot\sin\theta \end{cases}$$





$0<\theta<\pi \qquad \alpha=\cos\theta, \beta=\sin\theta$

$\pi<\theta<2\pi \qquad \alpha=\cos\theta-\sin\theta, \beta=\cos\theta+\sin\theta$



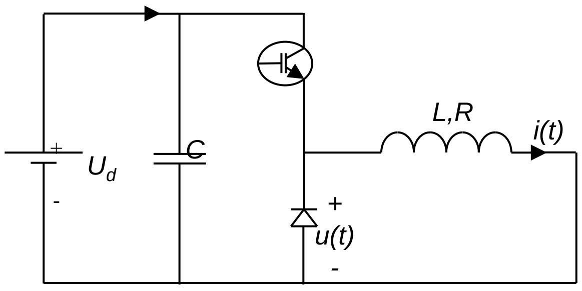
$$\begin{cases} 0<\theta<\pi & \alpha=\cos\theta, \beta=\sin\theta \\ \pi<\theta<2\pi & \alpha=\sqrt{2}\cos\theta, \beta=\sqrt{2}\sin\theta \end{cases}$$

2

Exercises on Current Control

Exercise 2.1 Current increase

a. The coil in the figure to the right has the inductance L and negligible resistance. It has no current when t<0. The current shall be increased to the value $i_1=0,1\ U_d/L$ in the shortest possible time.
Determine the voltage u(t) and the current i(t) for t>0!



b. The switch s is operated with the period time T=1ms. The time constant of the coil is L/R=10T. The average of the current is 0,1 U_d/R. Determine the voltage u(t) and the current i(t)!

Solution 2.1

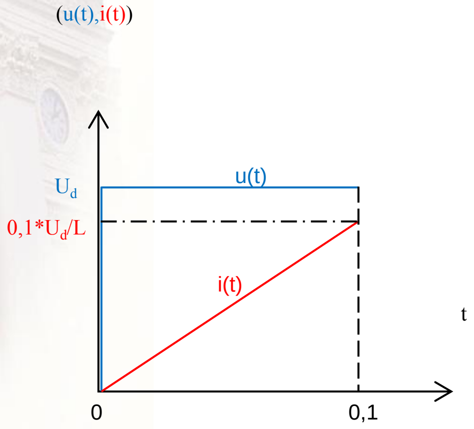
$a)\ R = 0$

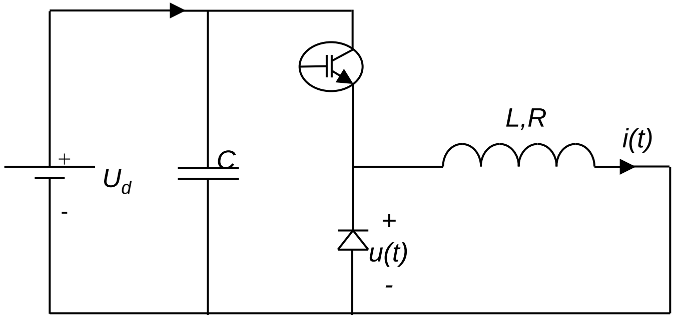
Calculate the shortest time for the current to increase from 0 to $0.1\dfrac{U_d}{L}\ A$

$$U = L \cdot \frac{di}{dt} \Rightarrow \text{the higher the voltage}$$

the faster the current increase. Use the highest voltage U_d

$$\Delta t = \frac{L \cdot \Delta i}{U_d} = \frac{L \cdot 0.1 \cdot U_d}{U_d \cdot L} = 0.1 \sec$$





Solution 2.1 Continued

$b)$ Average current $\quad i_{avg} = 0.1 \cdot \frac{U_d}{R}$

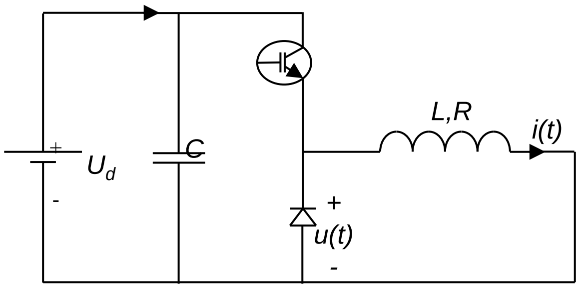
Average voltage $\quad u_{avg} = R \cdot i_{avg} = R \cdot 0.1 \cdot \frac{U_d}{R} = 0.1 \cdot U_d$

Duty cycle $\quad x = \frac{u_{avg}}{U_d} = \frac{0.1 \cdot U_d}{U_d} = 0.1$

Period time $\quad T = 1ms$

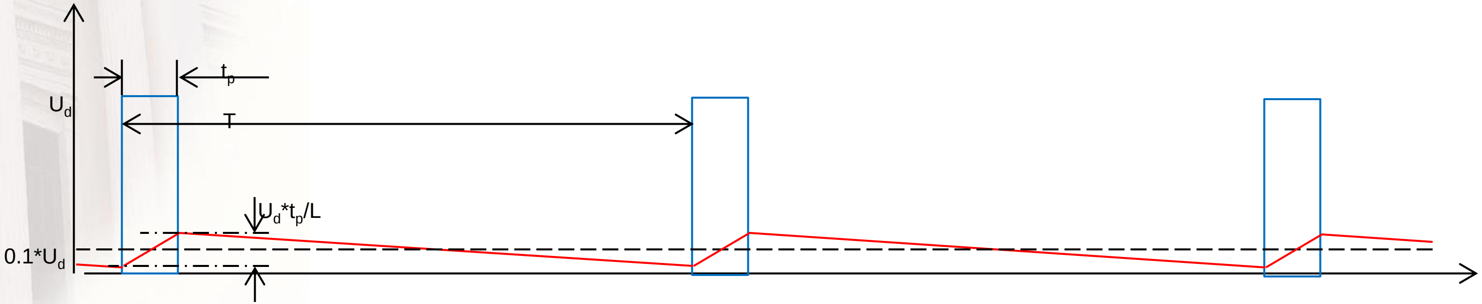
Time const. $\quad \tau = \frac{L}{R} = 10 \cdot T = 10\,ms$

Voltage pulse time $\quad t_p = x \cdot T = 0.1 ms \qquad (0 \le t_p \le x\,ms)$



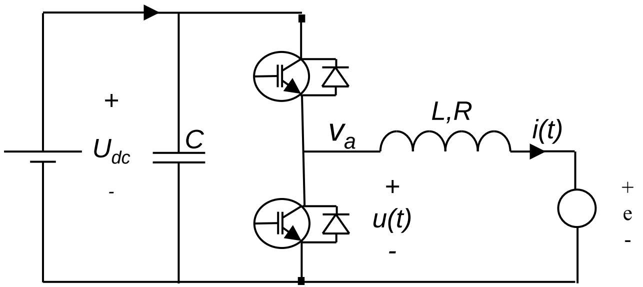
As $R \neq 0$ $\quad i(t) = \frac{U_d}{R} \cdot \left(1 - e^{-\frac{x}{\tau}}\right) = \{t << \tau\} \approx \frac{U_d}{R} \cdot \left(1 - 1 + \frac{x}{\tau}\right) = \frac{U_d \cdot x}{R \cdot \tau} = \frac{U_d \cdot t_p}{R \cdot \frac{L}{R}} = \frac{U_d \cdot t_p}{L}$

$$\begin{cases} i(t)_{start} = 0.1 \cdot \frac{U_d}{R} - \frac{U_d \cdot t_p}{2 \cdot L} \\ i(t)_{end} = 0.1 \cdot \frac{U_d}{R} + \frac{U_d \cdot t_p}{2 \cdot L} \end{cases}$$



2.2 2Q Current Control without load resistance

- A 2 quadrant DC converter with a constant voltage load has the following data:
  - *Udc = 600 V*
  - *L = 1 mH*
  - *R = 0*
  - *Ts = 0.1 ms*
  - *E = 200 V*
- Calculate and draw the output voltage patterns before, during and after a current step from 0 to 50 A and then back to 0 A again a few modulation periods after the positive step.



2.2 Solution

- **Calculation steps:**
  1. ***Calculate the voltage reference before the positive step, between the steps and after the negative step***
  2. ***Calculate how many sampling periods that are needed for the positive and negative steps***
  3. ***Calculate the current derivative and ripple***
  4. ***Draw the waveform***

2.2 Solution, continued

- **Step 1**

$$u^*(k)=\frac{L}{T_s}\cdot\left(i^*(k)-i(k)\right)+e=\begin{cases} e=200\text{ V } \textit{before} \text{ the positive step} \\ \frac{1\cdot 10^{-3}}{0.1\cdot 10^{-3}}\cdot(50-0)+200=700\text{ V } \textit{during} \text{ the positive step} \\ e=200\text{ V } \textit{between} \text{ the steps} \\ \frac{1\cdot 10^{-3}}{0.1\cdot 10^{-3}}\cdot(0-50)+200=-300\text{ V } \textit{during} \text{ the negative step} \\ e=200\text{ V } \textit{after} \text{ the negative step} \end{cases}$$

- **Step 2**
  - *The positive step requires 200+500V=700V (back-emf+current increase), but the DC link only provides 600 V, i.e two sampling periods are needed, one with 200+400V and one with 200+100 V.*
  - *The negative step requires 200-500V=-300V (back-emf+current decrease), but the DC link only provides 0 V, i.e three sampling periods are needed, two with 200-200V and one with 200-100 V.*

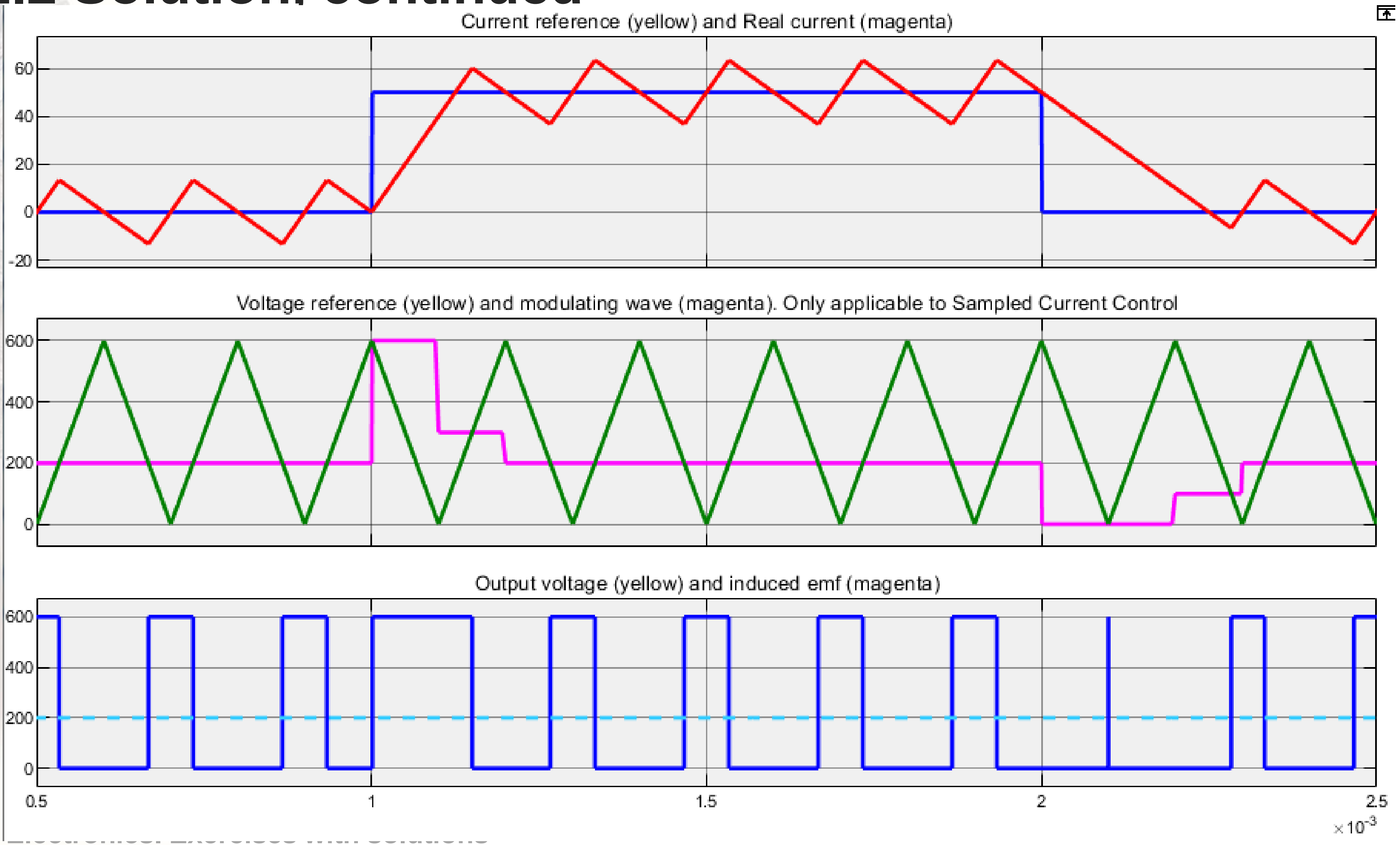
2.2 Solution, continued

- **Step 3**

$$\left.\begin{aligned} \frac{di}{dt} &= \frac{u_L}{L} \\ t_{pulse} &= \frac{e}{U_d}\cdot T_s \end{aligned}\right\} \rightarrow \Delta i = \frac{u_L}{L}\cdot t_{pulse} = \frac{(U_d - e)}{L}\cdot\frac{e}{U_d}\cdot T_s = \frac{(600-200)}{1\cdot 10^{-3}}\cdot\frac{200}{600}\cdot 0.1\cdot 10^{-3} = 13.3\ \text{A}$$

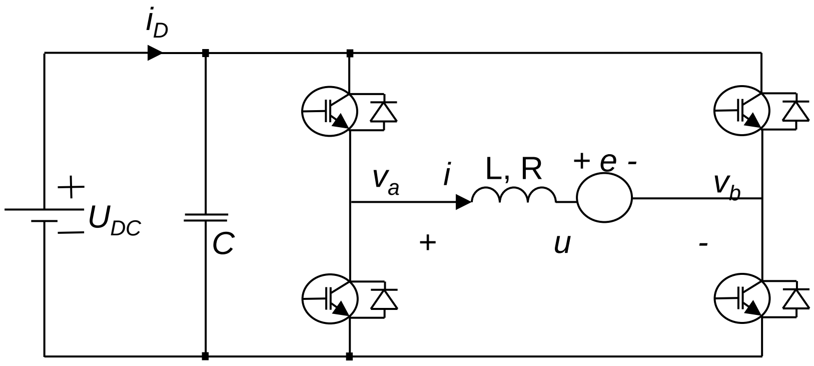
- **Step 4**
  - *Draw the carrier wave and the voltage reference wave as calculated. This gives the switching times*
  - *Note the time instants when the current will pass its reference values = when the carrier wave turns*
  - *See next page*

2.2 Solution, continued



2.3 4Q Current Control without load resistance

- A 4 quadrant DC converter with a constant voltage load has the following data:
  - *Udc = 600 V*
  - *L = 1 mH*
  - *R = 0*
  - *Ts = 0.1 ms*
  - *E = 200 V*
- Calculate and draw the output voltage patterns before, during and after a current step from 0 to 50 A and then back to 0 A again a few modulation periods after the positive step.



2.3 Solution

- **Calculation steps:**
  1. ***Calculate the voltage reference before the positive step, between the steps and after the negative step***
  2. ***Calculate how many sampling periods that are needed for the positive and negative steps***
  3. ***Calculate the current derivative and ripple***
  4. ***Draw the waveform***

2.3 Solution, continued

- **Step 1**

$$u^*(k)=\frac{L}{T_s}\left(i^*(k)-i(k)\right)+e=\begin{cases} e=200\text{V before the positive step} \\ \frac{1\cdot 10^{-3}}{0.1\cdot 10^{-3}}\cdot(50-0)+200=700\text{V during the positive step} \\ e=200\text{V between the steps} \\ \frac{1\cdot 10^{-3}}{0.1\cdot 10^{-3}}\cdot(0-50)+200=-300\text{V during the negative step} \\ e=200\text{V after the negative step} \end{cases}$$

- **Step 2**
  - *The positive step requires 200+500V=700V (back-emf+current increase) = +/- 350, but the DC link only provides +/-300 V, i.e two sampling periods are needed, one with 200+400V=+/-300 and one with 200+100 V=+/-150V.*
  - *The negative step requires 200-500V=-300V (back-emf+current decrease)=-/+150. The DC link provides down to -300 V, i.e 1 sampling periods is enough*

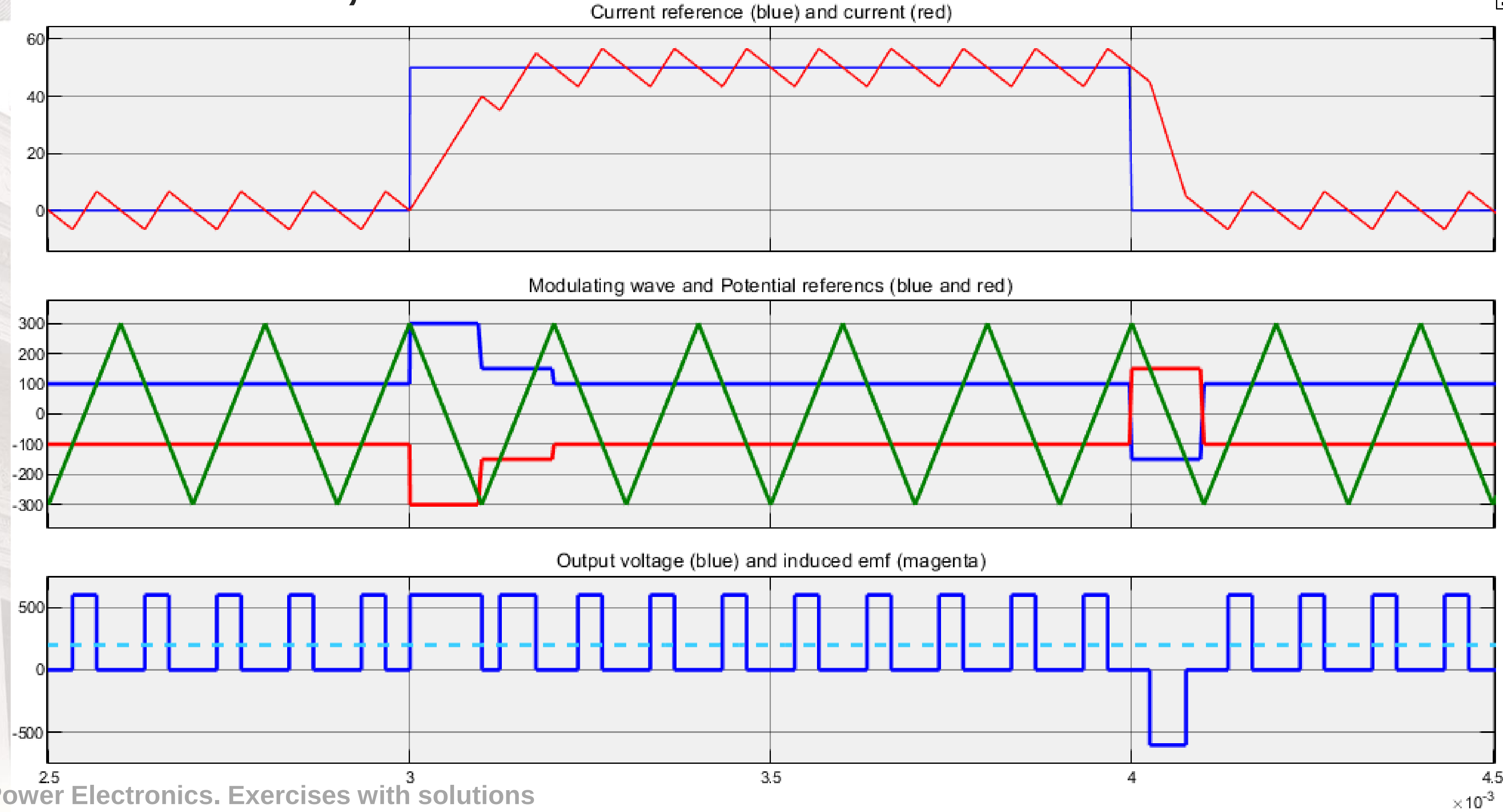
2.3 Solution, continued

- **Step 3**

$$\left.\begin{array}{l} \dfrac{di}{dt}=\dfrac{u_L}{L} \\ t_{pulse}=\dfrac{e}{U_d}\cdot T_s \end{array}\right\} \rightarrow \Delta i=\frac{u_L}{L}\cdot t_{pulse}=\frac{(U_d-e)}{L}\cdot\frac{e}{U_d}\cdot T_s=\frac{(600-200)}{1\cdot 10^{-3}}\cdot\frac{200}{600}\cdot 0.1\cdot 10^{-3}=13.3\text{ A}$$

- **Step 4**
  - *Draw the carrier wave and the voltage reference wave as calculated. This gives the switching times*
  - *Note the time instants when the current will pass its reference values = when the carrier wave turns*
  - *See next page*

2.3 Solution, continued



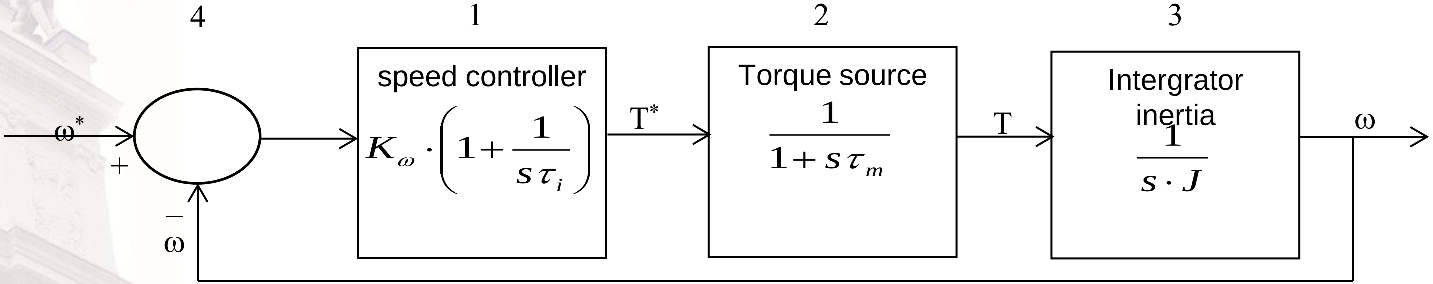
3

Exercises on Speed Control

Exercise 3.1 Cascade control

- **The speed of a motor shaft shall be controlled by so called cascade control. The torque source is modelled by a first order time constant.**
  - *Draw a block diagram of the system with speed control, torque source model and inertia*
  - *Include the load torque in the block diagram.*
  - *How large is the stationary error with a P-controller and constant load torque?*
  - *Show two different ways to eliminate the stationary fault.*

Solution 3.1a



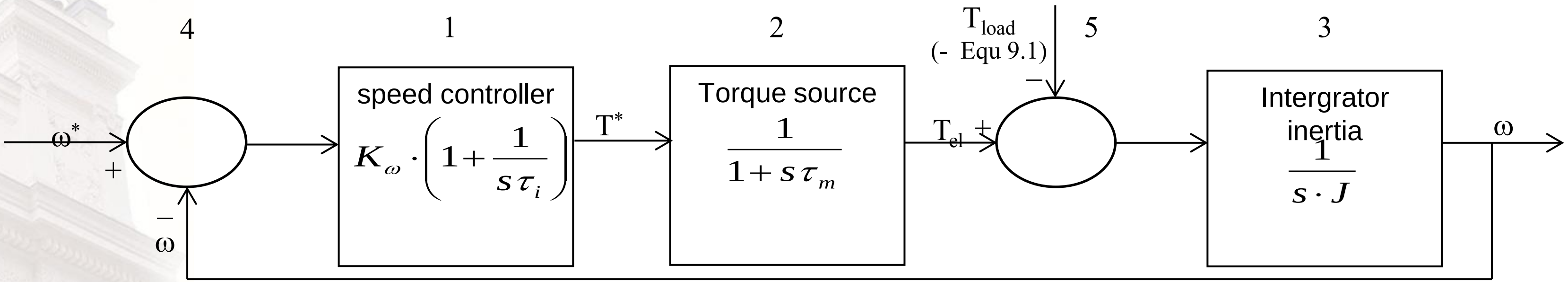
1 *The PI-controller with the gain K_ω*

2 *The torque controller is modelled as a first order filter*

3 *By dividing the torque with the inertia J the angular acceleration is achieved. By integration, dividing with the LaPlace 1/s, the angular speed is achieved.*

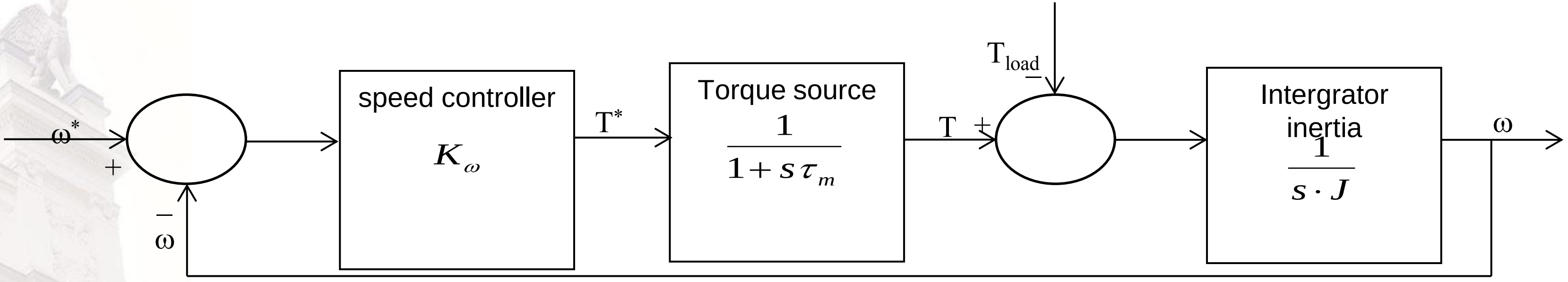
4 *By subtracting the angular speed from its reference the control error is achieved*

Solution 3.1b



See equation 9.1. The load torque is subtracted from the achieved electric torque at the output of the torque congtroller.

Solution 3.1c



$$\begin{cases} T = \left(\omega^* - \omega\right)\cdot K_\omega \cdot \frac{1}{\left(1+s\tau_m\right)} & (1) \\ \omega = \left(T - T_{load}\right)\cdot \frac{1}{s\cdot J} & (2) \end{cases}$$

Set (1) *in* (2)

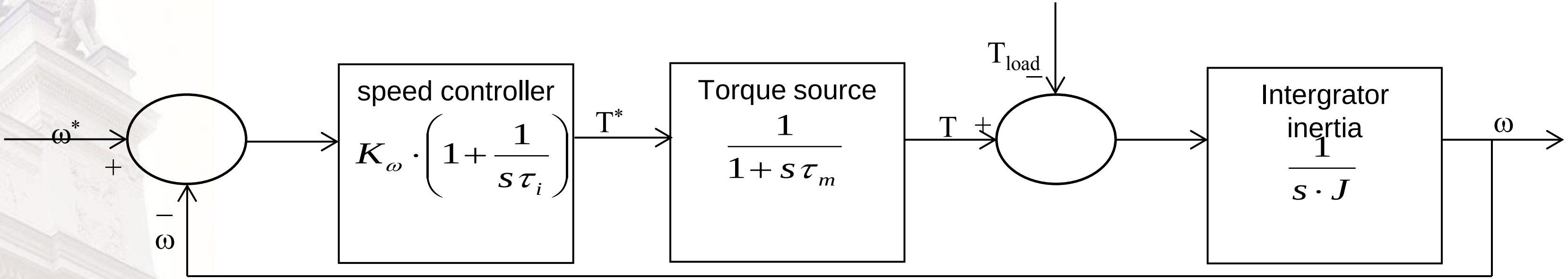
$$\omega = \left(\left(\omega^* - \omega\right)\cdot K_\omega \cdot \frac{1}{\left(1+s\tau_m\right)} - T_{load}\right)\cdot \frac{1}{s\cdot J}$$

$$s\cdot J\cdot \omega = \left(\left(\omega^* - \omega\right)\cdot K_\omega \cdot \frac{1}{\left(1+s\tau_m\right)} - T_{load}\right)$$

$$\lim\ t \to \infty \to s = 0 \qquad 0 = \left(\left(\omega^* - \omega\right)\cdot K_\omega \cdot \frac{1}{\left(1+0\right)} - T_{load}\right)$$

$$Stationary\ \ error\ \left(\omega^* - \omega\right) = \frac{T_{load}}{K_\omega}$$

Solution 3.1d -Alt.1 Replace the speed P-controller with a PI-controller



$$\begin{cases} T = \left(\omega^* - \omega\right)\cdot K_\omega \cdot \left(1+\frac{1}{s\tau_i}\right)\cdot \frac{1}{\left(1+s\tau_m\right)} \\ \omega = \left(T - T_{load}\right)\cdot \frac{1}{s\cdot J} \end{cases}$$

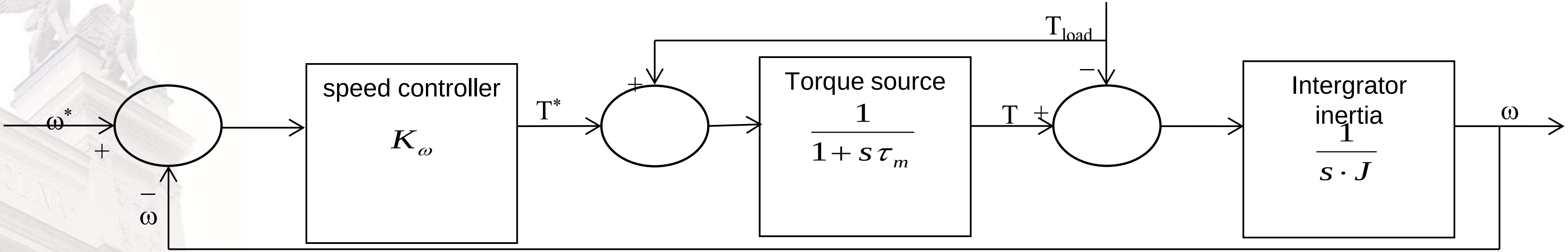
$$\begin{cases} s\tau_i \cdot T = \left(\omega^* - \omega\right)\cdot K_\omega \cdot \left(s\tau_i + 1\right)\cdot \frac{1}{\left(1+s\tau_m\right)} \\ s\cdot J\cdot \omega = \left(T - T_{load}\right) \end{cases}$$

$$\lim\ t \to \infty \to s = 0$$

$$\begin{cases} 0 = \left(\omega^* - \omega\right)\cdot K_\omega \cdot \left(0+1\right)\cdot \frac{1}{\left(1+0\right)} \\ 0 = \left(T - T_{load}\right) \end{cases}$$

$$\left(\omega^* - \omega\right) = \frac{0}{K_\omega} = 0$$

Solution 3.1d. Alt.2 Add the load before the torque source



$$\begin{cases} T = \left(\left(\omega^* - \omega\right)\cdot K_\omega + T_{load}\right)\cdot \dfrac{1}{\left(1+s\tau_m\right)} \\ \omega = \left(T - T_{load}\right)\cdot \dfrac{1}{s\cdot J} \end{cases}$$

$$\begin{cases} T = \left(\left(\omega^* - \omega\right)\cdot K_\omega + T_{load}\right)\cdot \dfrac{1}{\left(1+s\tau_m\right)} \\ s\cdot J\cdot \omega = \left(T - T_{load}\right) \end{cases}$$

$$\lim\ t \to \infty \to s = 0$$

$$\begin{cases} T = \left(\left(\omega^* - \omega\right)\cdot K_\omega + T_{load}\right)\cdot \dfrac{1}{\left(1+0\right)} \\ 0 = T - T_{load} \quad \Rightarrow \quad T = T_{load} \end{cases}$$

$$T_{load} = \left(\omega^* - \omega\right)\cdot K_\omega + T_{load}$$

$$\left(\omega^* - \omega\right) = \frac{0}{K_\omega} = 0$$

Exercise 3.2 DC motor control

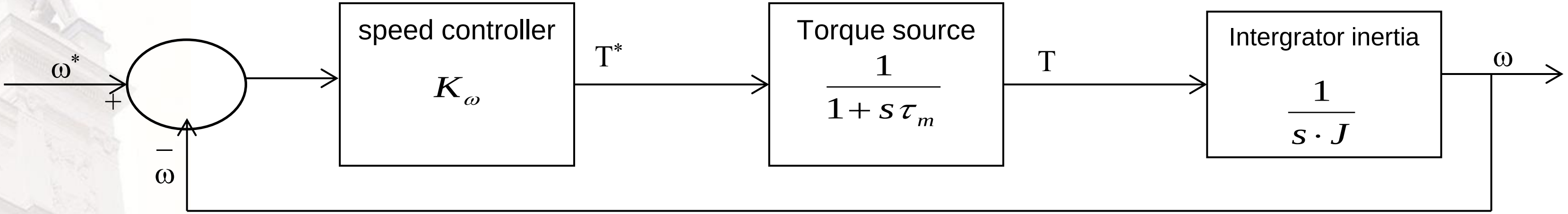
A DC motor with the inertia J=0,033 kgm^2 is driven by a converter with current control set for dead-beat current control at 3.33 ms sampling time. The speed of the DC motor is controlled by a P-regulator. The current loop is modelled with a first order time constant that equals the pulse interval of the converter.

a) ***Draw a block diagram of the system with speed control with the models of the current loop and the motor. Calculate K_p = the gain of the speed control for maximum speed without oscillatory poles.***

b) ***The motor is loaded with the torque T_l. How large is the speed stationary error?***

c) ***If the speed is measured with a tachometer and lowpass filtered, what does that mean for $K_{p?}$***

Solution 3.2a



$$\textit{Open circuit} \quad G = K_\omega \cdot \frac{1}{(1+s\tau_m)} \cdot \frac{1}{s\cdot J}$$

$$\textit{Closed loop} \quad = \frac{G}{1+G} = \frac{K_\omega \cdot \frac{1}{(1+s\tau_m)} \cdot \frac{1}{s\cdot J}}{1+K_\omega \cdot \frac{1}{(1+s\tau_m)} \cdot \frac{1}{s\cdot J}} = \frac{K_\omega}{s\cdot J\cdot(1+s\tau_m)+K_\omega} = \frac{K_\omega}{J\cdot\tau_m\cdot s^2 + s\cdot J + K_\omega}$$

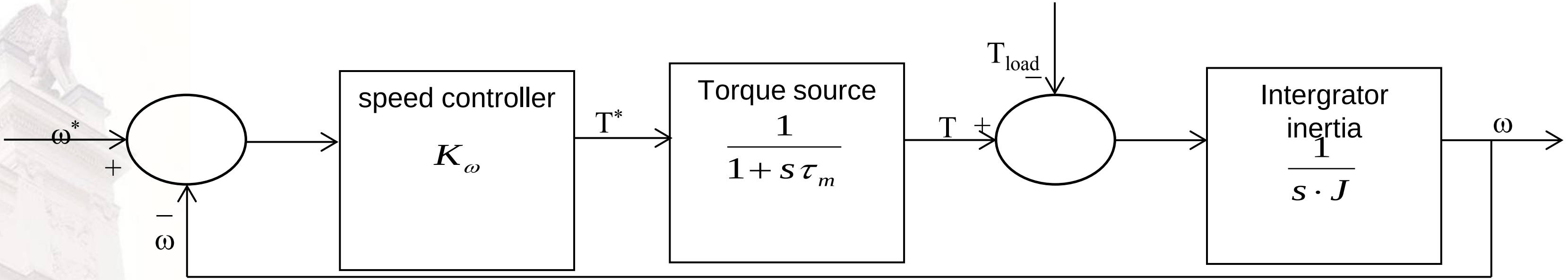
$$\textit{Characteristic equation } s^2 + \frac{s}{\tau_m} + \frac{K_\omega}{J\cdot\tau_m} = 0 \textit{ with the roots } \quad s = -\frac{1}{2\cdot\tau_m} \pm \sqrt{\frac{1}{4\cdot\tau_m^2} - \frac{K_\omega}{J\cdot\tau_m}}$$

$$\textit{Fastest operation is achived when the roots are the same } \frac{1}{4\cdot\tau_m^2} = \frac{K_\omega}{J\cdot\tau_m} \Rightarrow K_\omega = \frac{J}{4\cdot\tau_m}$$

$$6-\textit{pulse converter, assume } 50\,Hz \qquad \tau_m = 3.3\,ms$$

$$K_\omega = \frac{0.033}{4\cdot 3.3\cdot 10^{-3}} = 2.5$$

Solution 3.2b



$$\begin{cases} T = \left(\omega^* - \omega\right) \cdot K_\omega \cdot \frac{1}{\left(1 + s\tau_m\right)} & (1) \\ \omega = \left(T - T_{load}\right) \cdot \frac{1}{s \cdot J} & (2) \end{cases}$$

$Set\,(1)\,in\,(2)$

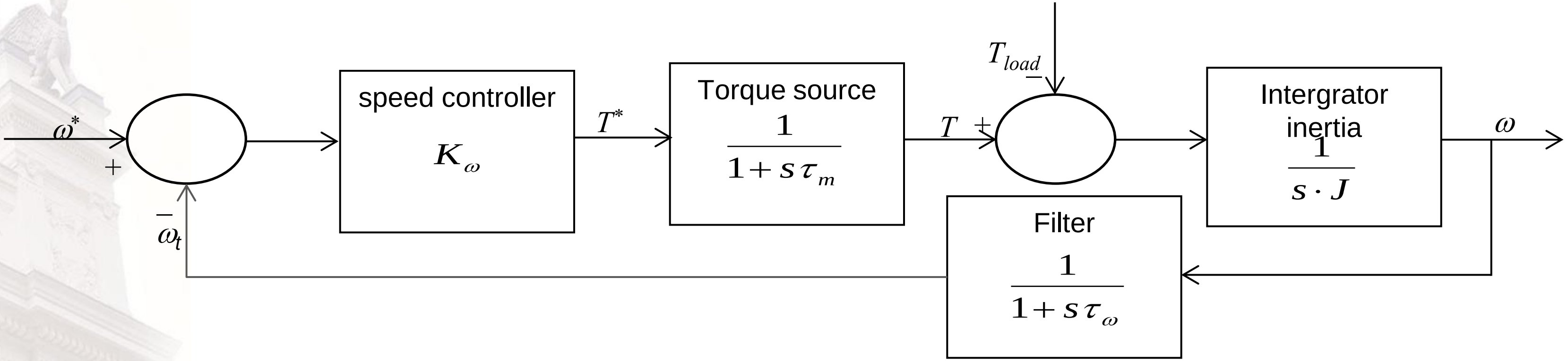
$$\omega = \left(\left(\omega^* - \omega\right) \cdot K_\omega \cdot \frac{1}{\left(1 + s\tau_m\right)} - T_{load}\right) \cdot \frac{1}{s \cdot J}$$

$$s \cdot J \cdot \omega = \left(\left(\omega^* - \omega\right) \cdot K_\omega \cdot \frac{1}{\left(1 + s\tau_m\right)} - T_{load}\right)$$

$$\lim\ t \to \infty \to s = 0 \qquad 0 = \left(\left(\omega^* - \omega\right) \cdot K_\omega \cdot \frac{1}{\left(1 + 0\right)} - T_{load}\right)$$

$$\left(\omega^* - \omega\right) = \frac{T_{load}}{K_\omega} = \frac{T_{load}}{2.5}$$

Solution 3.2c



It is now the measured speed (ω_t) that is controlled and the system becomes of 3rd order.

IF we assume that the filter time constant (τ_ω) is much longer that the torque control time constant (τ_m), then the system (including the filter) is now slower than the system without a filter, implying a need for a lower gain.

The solution to 3.2a can be applied, but with the torque control time constant replaced by the filter time constant, thus giving a lower speed controller gain.

Exercise 3.3 Pump control

- **A pump is driven at variable speed. It is therefore driven by an electric machine that is speed controlled. The speed controller is PI. The total inertia for both pump and electric machine is J=0,11. The power converter is a current controlled switched amplifier where the current control has an average response time of 100 µs, which is considered very fast if the integration time of the PI control is not of the same magnitude.**

  a) ***Draw the speed control system as a block diagram with the PI control and the models for torque source, load torque and inertia.***

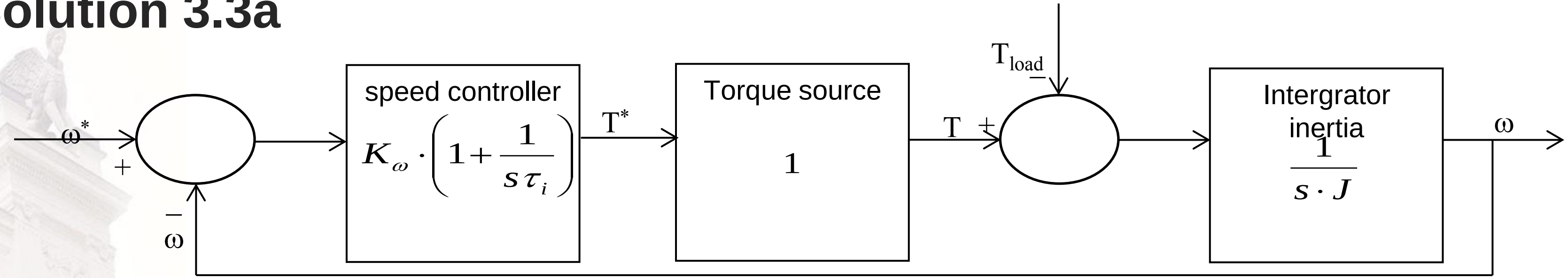
  b) ***Dimension the PI control so that the system has a double pole along the negative real axis.***

  c) ***If the integration part for some reason is excluded (Ti=∞), how large is the speed error then?***

  d) ***If the current loop can not be considered as very fast, how is the it modelled in the block diagram?***

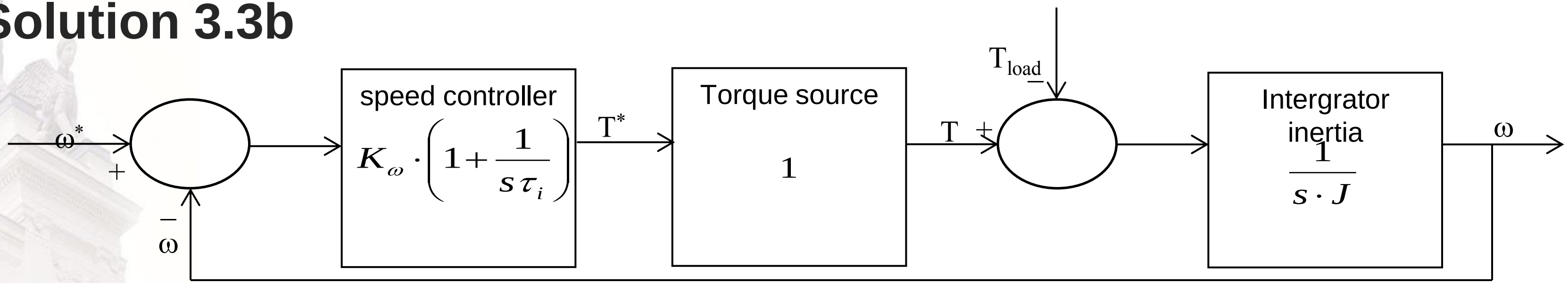
  e) ***There is a standard method for dimensioning the speed control in d). What is it called?***

Solution 3.3a



The torque controller is modelled as a 1st order low pass filter. As the integration time is much longer than the filter time constant, the filter time constant is set to zero

Solution 3.3b



$$Open\ circuit\ (The\ torque\ source\ is\ 100\mu s,\ very\ fast) \qquad G = K_\omega \cdot \left(1+\frac{1}{s\tau_i}\right)\cdot 1 \cdot \frac{1}{s \cdot J}$$

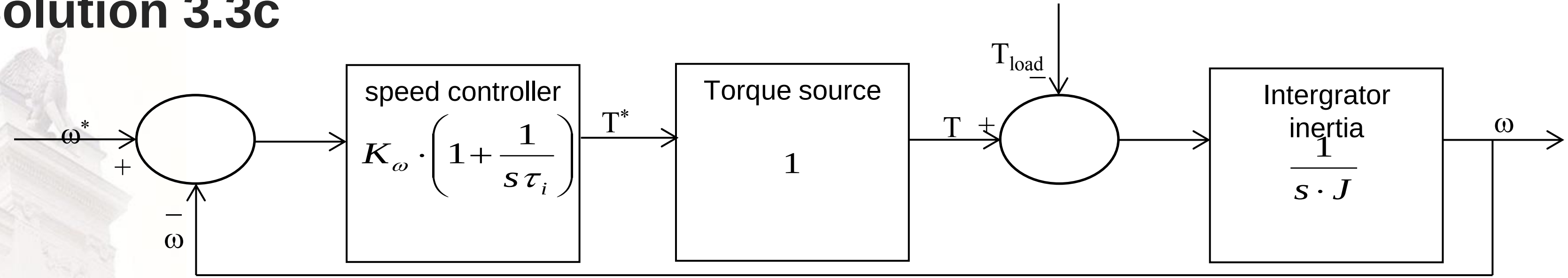
$$Closed\ loop \quad = \frac{G}{1+G} = \frac{K_\omega \cdot \left(1+\frac{1}{s\tau_i}\right)\cdot \frac{1}{s \cdot J}}{1+K_\omega \cdot \left(1+\frac{1}{s\tau_i}\right)\cdot \frac{1}{s \cdot J}} = \frac{K_\omega \cdot (s\tau_i+1)}{s \cdot J \cdot s\tau_i + K_\omega \cdot (s\tau_i+1)} = \frac{K_\omega \cdot (s\tau_i+1)}{s^2 \cdot J \cdot \tau_i + s \cdot K_\omega \cdot \tau_i + K_\omega}$$

$$Characteristic\ equation\ s^2 + s \cdot \frac{K_\omega}{J} + \frac{K_\omega}{J \cdot \tau_i} = 0\ with\ the\ roots\ s = -\frac{K_\omega}{2 \cdot J} \pm \sqrt{\frac{K_\omega^{\ 2}}{4 \cdot J^2} - \frac{K_\omega}{J \cdot \tau_i}}$$

$$Double\ roots \Rightarrow \frac{K_\omega^{\ 2}}{4 \cdot J^2} - \frac{K_\omega}{J \cdot \tau_i} = 0 \Rightarrow \frac{K_\omega^{\ 2}}{4 \cdot J^2} = \frac{K_\omega}{J \cdot \tau_i} \Rightarrow K_\omega = \frac{4 \cdot J}{\tau_i}$$

$$Assume\ \tau_i = 100\,ms\,(as\ \tau_i >> \tau_m, 100\mu s),\ J = 0.11 \qquad K_\omega = \frac{4 \cdot 0.11}{0.1} = 4.4$$

Solution 3.3c



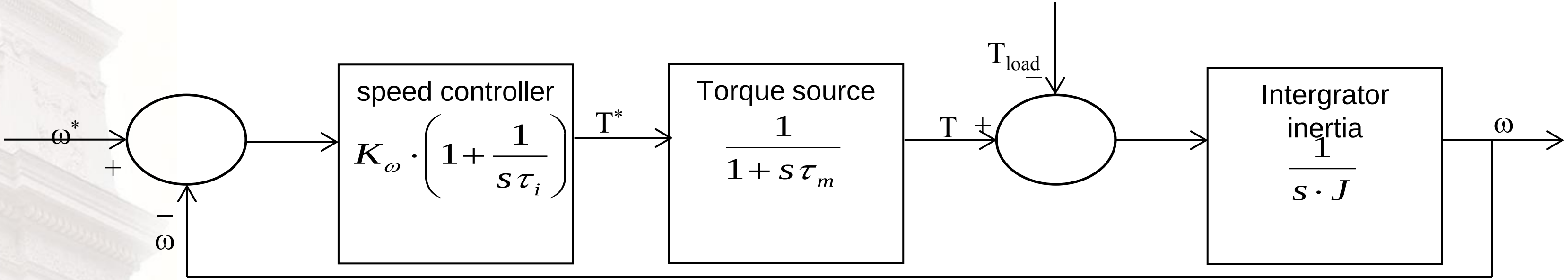
$$T = (\omega^* - \omega) \cdot K_\omega \cdot \left(1 + \frac{1}{s\tau_i}\right) = \{\tau_i \to \infty\} = (\omega^* - \omega) \cdot K_\omega$$

$$\begin{cases} T = (\omega^* - \omega) \cdot K_\omega \\ Stationary,\ T = T_{load} \end{cases}$$

$$T_{load} = (\omega^* - \omega) \cdot K_\omega$$

$$(\omega^* - \omega) = \frac{T_{load}}{K_\omega}$$

Solution 3.3d



Solution 3.3e

- With new closed loop system, there will a another third pole to place. This is more complicated, but a recommended method is the Symmetric optimum, see chapter 9.5

$$\omega_0 = \frac{1}{\sqrt{\tau_i \cdot \tau_m}} \qquad (eq\ 9.19)$$

$$\tau_i = a^2 \cdot \tau_m \qquad \tau_m < \tau_i, a > 1 \qquad (eq\ 9.20)$$

$$No\ complex\ poles,\ set\ a = 3 \qquad (chapter\ 9.5)$$

$$Set\ all\ three\ poles\ the\ same\ at\ \omega_0$$

$$\tau_i = a^2 \cdot \tau_m = 3^2 \cdot 100 \cdot 10^{-6} = 0.9\,ms$$

$$K_p = \frac{a \cdot J}{T_i} = \frac{3 \cdot 0.11}{0.9 \cdot 10^{-3}} = 367\ (!)$$

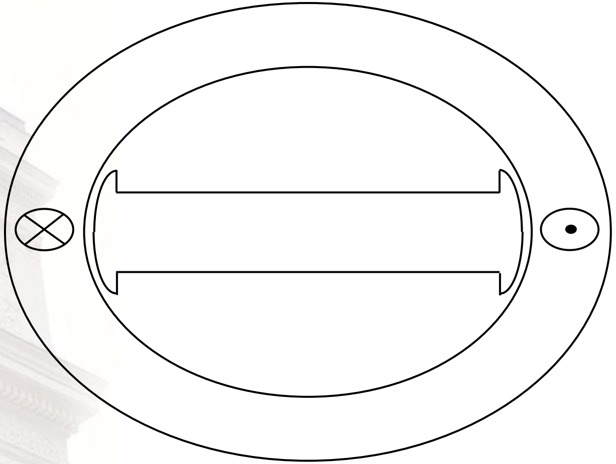
4

Exercises on MMF distribution

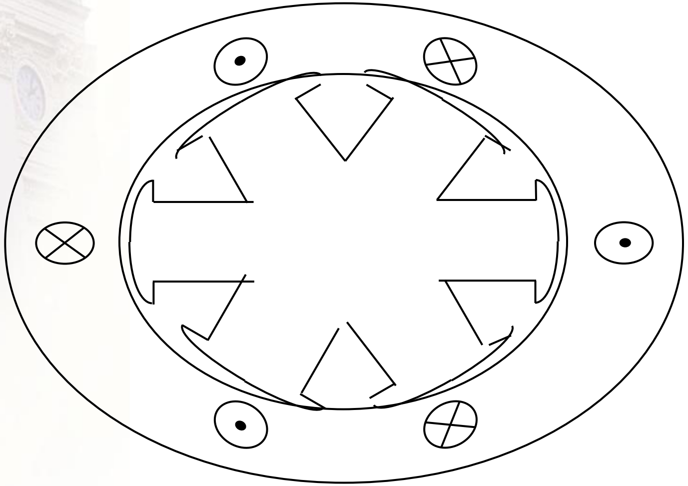
Exercise 4.1 2- and 6-pole motor

Draw a cross section of one two pole and one six pole synchronous machine with salient poles. Draw also a diameter harness (Swedish “diameterhärva”) which covers all poles.

Solution 4.1



2-pole synchrounous machine

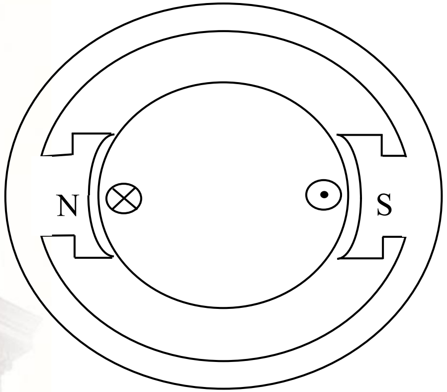


6-pole synchrounous machine

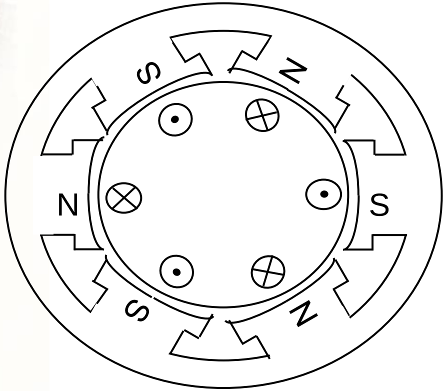
Exercise 4.2 DC machine

Draw a cross section of a DC machine with salient poles. Draw also a diameter harness which covers all poles.

Solution 3.2



2-pole DC machine



6-pole DC machine

Exercise 4.3 mmf

Explanation. ”In electrical engineering, an armature is the power producing component of an electric machine. The armature can be on either the rotor (the rotating part) or the stator (stationary part) of the electric machine”. [Wikipedia].

In the other part the field is produced.

A two pole armature winding in the stator of an alternating current machine is approximately sinusoidally distributed according to the figure below.

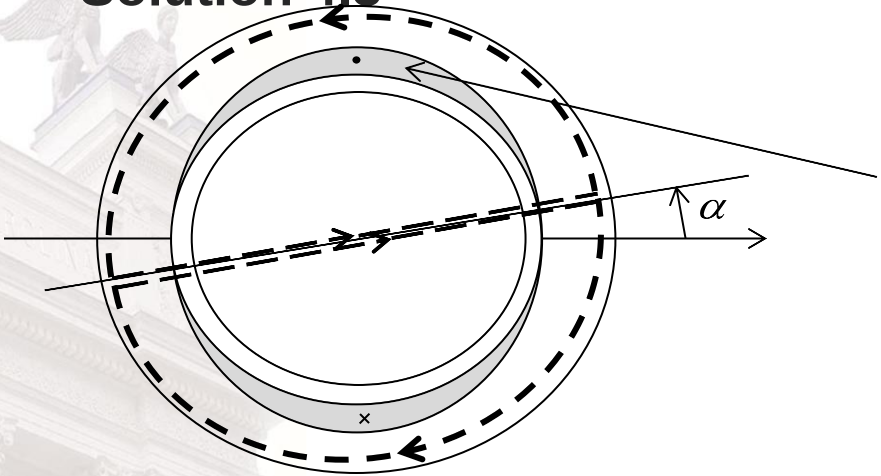
The current density (current per angle unit) is $J=J_{max}*\sin(\alpha)$ [A/radian]. The airgap is constant $\delta=\delta_0$.

Note that the outspread figure is done by spreading the windings from $\alpha=0$ and that the machine is seen from the back,that is why the current directions change.

How large is the magnetomotive force $F(\alpha)$?

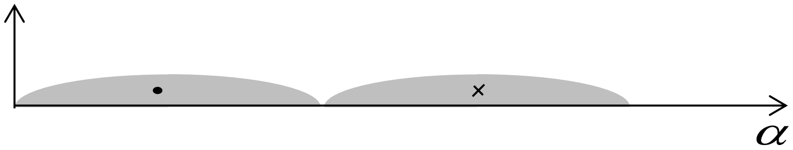
Where will the magnetomotive force be found?

Solution 4.3



A 2-pole machine

$$Current\ density\ J = J_{\max} \cdot \sin(\alpha)$$



The magnetic field in the lower closed loop is clock wise, while the magnetic field in the upper closed loop has opposite direction. However, along the center line, the direction from both loops has the same direction, and the contribution from both loops add.

Use ampèr's law in one loop. The magnetomotoric force in the air gap has contribution from both the lower and the upper loop.

$$F = 2 \cdot \int_{\alpha}^{180+\alpha} J d\alpha = J_{\max} \int_{\alpha}^{180+\alpha} \sin(\alpha) d\alpha = J_{\max} \cdot \left(\Big|_{\alpha}^{180+\alpha} \left(-\cos(\alpha)\right)\right) = 2 \cdot J_{\max} \cdot \left(\cos(\alpha) - \cos(180+\alpha)\right) =$$

$$= 2 \cdot J_{\max} \cdot \left(\cos(\alpha) - \underbrace{\cos(180)}_{=-1} \cdot \cos(\alpha) + \underbrace{\sin(180)}_{=0} \cdot \sin(\alpha) \right) = 2 \cdot J_{\max} \cdot \cos(\alpha)$$

Solution 4.3 cont'd

Where will the magnetomotive force be found in the magnetic circuit

mmf equals the total current inside in one loop.

$$N \cdot I = \oint \overline{H} \cdot d\overline{s} = H_{Fe} \cdot s_{Fe} + H_{air} \cdot \delta = \frac{B_{Fe}}{\mu\mu_0} \cdot s_{Fe} + \frac{B_{air}}{\mu_0} \cdot \delta =$$

$$= \frac{s_{Fe}}{\mu\mu_0 \cdot A_{Fe}} \cdot \psi + \frac{\delta}{\mu_0 \cdot A_\delta} \cdot \psi = R_{Fe} \cdot \psi + R_\delta \cdot \psi$$

$$R_{Fe} = \frac{s_{Fe}}{\mu\mu_0 \cdot A_{Fe}}, \; R_\delta = \frac{\delta}{\mu_0 \cdot A_\delta}$$

As the reluctance is proportionel to 1/μ, the reluctance in the iron can be neglected compared to the air gap reluctance.

I.e. the magnetomotive force will be concentrated in the two air gaps

$$F = J_{\max} \cdot \cos(\alpha)$$

The magnetomotive force in one air gap will be

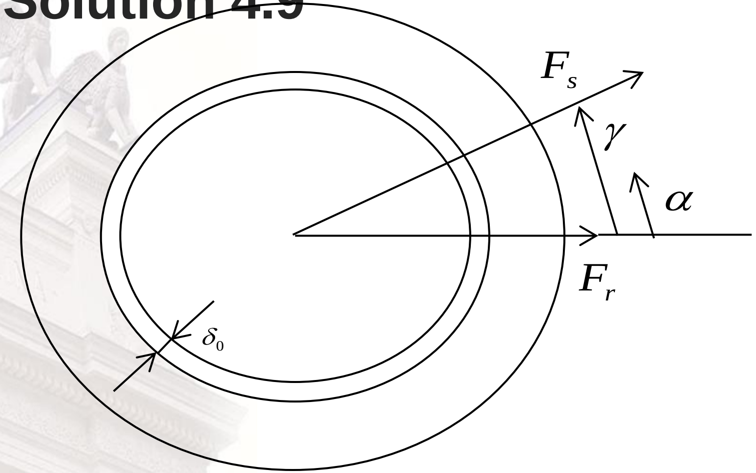
Exercise 4.9 2 wave mmf

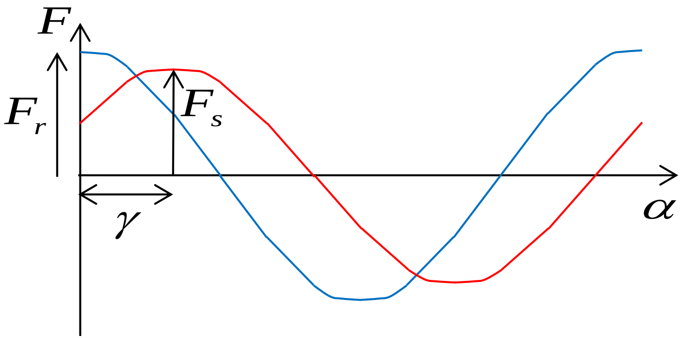
An electrical machine has two waves of magnetomotive force. One is caused by the current distribution in the rotor and the other by the current distribution in the stator, see figure below.

The machine has a constant airgap, $\delta=\delta_0$ and the iron in the stator and the rotor has infinite magnetic conductivity. The peak amplitude of the waves of the magnetomotive force are for the stator and for the rotor.

Calculate the energy in the airgap.

Solution 4.9





Both the stator and the rotor are cylindrical, thus the airgap reluctance R is the same in all directions

See equ(8.8) $\vec{F}_\delta = \vec{F}_r + \vec{F}_s = F_x + j \cdot F_y$

See equ(8.9) $F_x = \hat{F}_s \cdot \cos(\gamma) + \hat{F}_r$

See equ(8.9) $F_y = \hat{F}_s \cdot \sin(\gamma)$

See equ(8.10) $W_{magn} = \frac{1}{2} \cdot \frac{\hat{F}_x^2}{R} + \frac{1}{2} \cdot \frac{F_y^2}{R} = \frac{\hat{F}_s^2 \cdot \cos^2(\gamma) + 2 \cdot \hat{F}_s \cdot \hat{F}_r \cdot \cos(\gamma) + \hat{F}_r^2 + \hat{F}_s^2 \cdot \sin^2(\gamma)}{2R} =$

$$= \frac{\hat{F}_s^2 + 2 \cdot \hat{F}_s \cdot \hat{F}_r \cdot \cos(\gamma) + \hat{F}_r^2}{2R}$$

Exercise 4.10 Torque

Same as 3.9. Assume that no electric energy can be fed to or from the machine and that the system is lossless.

How large is the mechanical torque as a function of the angle γ?

Solution 4.10

See exercise 3.9

The airgap reluctance R is the same in all directions

No energy supplied to the system $W_{magn} + W_{magn} = constant$

thus $$\frac{dW_{magn}}{d\gamma} + \frac{dW_{mec}}{d\gamma} = 0 \Rightarrow \frac{dW_{mec}}{d\gamma} = -\frac{dW_{magn}}{d\gamma}$$

See equ(8.11) $$T = \frac{dW_{mec}}{d\gamma}$$

See equ(8.13) $$T = -\frac{dW_{magn}}{d\gamma} = -\frac{1}{2} \cdot \frac{d\left(\frac{F_s^2 + 2 \cdot F_s \cdot F_r \cdot \cos(\gamma) + F_r^2}{R}\right)}{d\gamma} =$$

$$= \frac{F_s \cdot F_r \cdot \sin(\gamma)}{R} = \frac{F_{sy} \cdot F_r}{R}$$

There is no reluctance torque

Exercise 4.11 Flux

A machine with salient poles in the rotor and cylindrical stator has its armature winding in the stator. The effective number of winding turns is $N_{a,\,eff}$ and the magnetized rotor contributes to the air gap flux with Φ_m The main inductances are L_{mx} and L_{my} in the x- and y directions.

a) How large is the flux contribution from the rotor that is linked to the armature winding?

b) How large is the resulting flux that is interconnected with the armature winding?

c) Draw a figure of how the armature current vector is positioned in the x-y plane to be perpendicular to the resulting air gap flux !

Solution 4.11a

The magnetized rotor contribution to the airgap flux Φ_m

The effective number of winding turns in the stator $N_{a,eff}$

The linked flux contribution from the rotor to the armature winding $\Psi_m = N_{a,eff} \cdot \Phi_m$

Solution 4.11b

The stator main inductance in x-direction L_{mx}

The stator main inductance in y-direction L_{my}

The armature winding current in x-direction I_{ax}

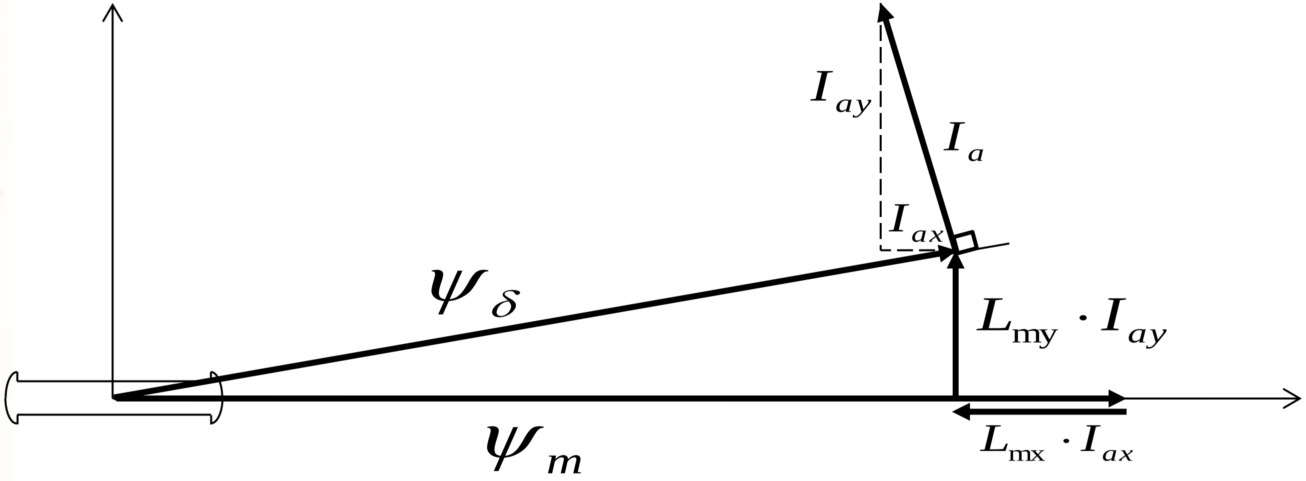
The armature winding current in y-direction I_{ay}

The magnetizing flux in x-direction Ψ_m

The resulting flux, interconnected with the armaturewinding

$$\Psi_a = \left(\Psi_m + L_{\mathrm{mx}} \cdot I_{ax}\right) + j \cdot L_{\mathrm{my}} \cdot I_{ay}$$

Solution 4.11c



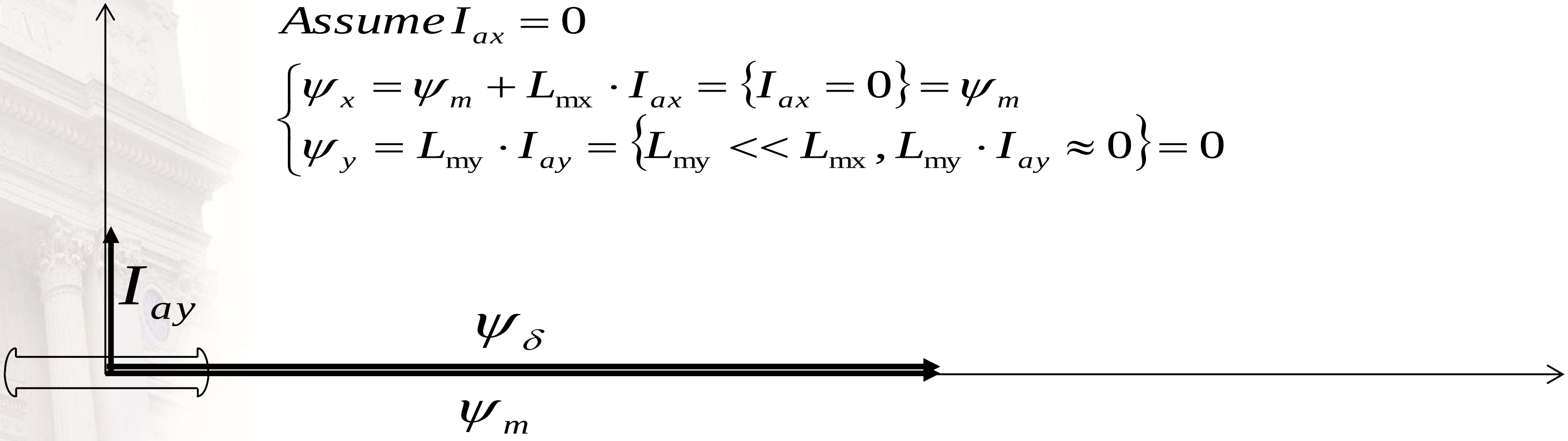
Exercise 4.12 Flux vector

Same as 4.11 but L_{my}<< L_{mx}.
Draw a stylized picture of a cross section of the machine and draw a figure of how the armature current vector is positioned in the x-y plane to be perpendicular to the resulting air gap flux !

Solution 4.12

$$Assume\, I_{ax} = 0$$

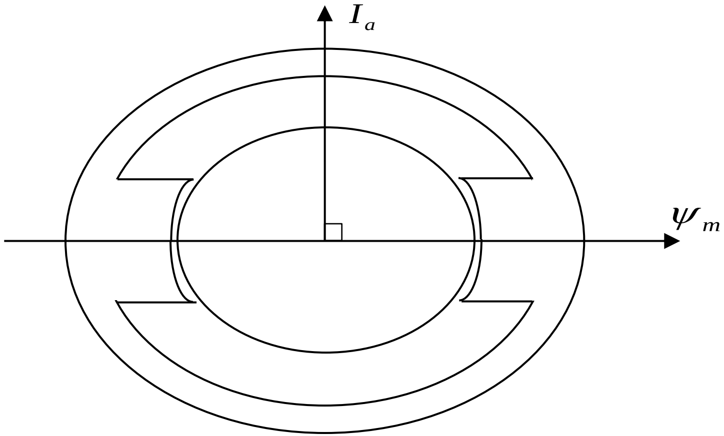
$$\begin{cases} \psi_x = \psi_m + L_{\text{mx}} \cdot I_{ax} = \{I_{ax} = 0\} = \psi_m \\ \psi_y = L_{\text{my}} \cdot I_{ay} = \{L_{\text{my}} << L_{\text{mx}}, L_{\text{my}} \cdot I_{ay} \approx 0\} = 0 \end{cases}$$



Exercise 4.13 Armature current vector

Same as 4.12 but now the armature winding is in the rotor, which is cylindrical, and the stator has salient poles. Draw a stylized picture of a cross section of the machine and draw a figure of how the armature current vector is positioned in the x-y plane to be perpendicular to the resulting air gap flux !

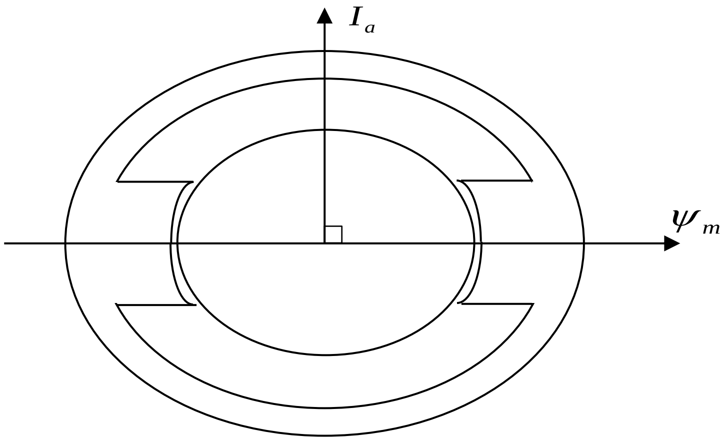
Solution 4.13



Exercise 4.14 Rotation problem

Suggest two ways of solving tha rotation problem, i. e. how the angle of the armature current vector to the air gap flux vector can be maintained during rotation for the cases in 4.12 and 4.13!

Solution 4.14



See chapter 8.8 and 10.1. According to chapter 8.8 the armature DC-winding must not be fixed to the stator.

Alt 1 This can be achieved be means of two or three phase AC-windings, see figure 8.9.

Exercise 4.15 Voltage equation

A three phase armature winding with the resistances R_a, the leakage inductances $L_{a\lambda}$ and the fluxes ψ_1, ψ_2, and ψ_3 that are linked to the respective armature windings.

a) Form the voltage equations first for each phase and then jointly in vector form!

b) Express all vectors in rotor coordinates instead of stator coordinates and separate the equation into real and imaginary parts.

Solution 4.15a

$$Equation\ \ 8.28$$

$$\begin{cases} U_a = R_a \cdot i_a + \dfrac{d\psi_1}{dt} = R_a \cdot i_a + \dfrac{d\left(\psi_{\delta 1} + L_{a\lambda} \cdot i_a\right)}{dt} \\ U_b = R_a \cdot i_b + \dfrac{d\psi_2}{dt} = R_a \cdot i_b + \dfrac{d\left(\psi_{\delta 2} + L_{a\lambda} \cdot i_b\right)}{dt} \\ U_c = R_a \cdot i_c + \dfrac{d\psi_3}{dt} = R_a \cdot i_c + \dfrac{d\left(\psi_{\delta 3} + L_{a\lambda} \cdot i_c\right)}{dt} \end{cases}$$

$$Equation\ \ 8.29$$

$$\vec{U}_s^{\alpha\beta} = R_a \cdot \vec{i}_s^{\alpha\beta} + \frac{d\vec{\psi}_s^{\alpha\beta}}{dt} = R_a \cdot \vec{i}_s^{\alpha\beta} + \frac{d\left(\vec{\psi}_\delta^{\alpha\beta} + L_{a\lambda} \cdot \vec{i}_s^{\alpha\beta}\right)}{dt}$$

Solution 4.15b

Perform a transformation from the $\alpha\beta-$ frame to $xy-$ frame.
Assume you are "sitting" on the $xy-$ frame, which is rotating in positive direction,
then you will see the $\alpha\beta-$ frame rotating in negative direction
Equation 8.30

$$u_s^{\alpha\beta} = R_a \cdot \vec{i}_s^{\alpha\beta} + \frac{d\left(\vec{\psi}_\delta^{\alpha\beta} + L_{a\lambda} \cdot \vec{i}_s^{\alpha\beta}\right)}{dt}$$

Transform by multiply by $e^{-j\omega t}$ *(negative direction)*

$$\begin{cases} \vec{u}_s^{xy} = \vec{u}_s^{\alpha\beta} \cdot e^{-j\omega t} \Rightarrow \vec{u}_s^{\alpha\beta} = \vec{u}_s^{xy} \cdot e^{j\omega t} \\ \vec{i}_s^{xy} = \vec{i}_s^{\alpha\beta} \cdot e^{-j\omega t} \Rightarrow \vec{i}_s^{\alpha\beta} = \vec{i}_s^{xy} \cdot e^{j\omega t} \\ \vec{\psi}_s^{xy} = \vec{\psi}_s^{\alpha\beta} \cdot e^{-j\omega t} \Rightarrow \vec{\psi}_s^{\alpha\beta} = \vec{\psi}_s^{xy} \cdot e^{j\omega t} \end{cases}$$

Insert

$$\vec{u}_s^{xy} \cdot e^{j\omega t} = R_a \cdot \vec{i}_s^{xy} \cdot e^{j\omega t} + \frac{d\left(\vec{\psi}_s^{xy} \cdot e^{j\omega t} + L_{a\lambda} \cdot \vec{i}_s^{xy} \cdot e^{j\omega t}\right)}{dt} = R_a \cdot \vec{i}_s^{xy} \cdot e^{j\omega t} + \frac{d\left(\left(\vec{\psi}_s^{xy} + L_{a\lambda} \cdot \vec{i}_s^{xy}\right) \cdot e^{j\omega t}\right)}{dt} \Rightarrow$$

$$\vec{u}_s^{xy} \cdot e^{j\omega t} = R_a \cdot \vec{i}_s^{xy} \cdot e^{j\omega t} + \frac{d\left(\vec{\psi}_s^{xy} + L_{a\lambda} \cdot \vec{i}_s^{xy}\right)}{dt} \cdot e^{j\omega t} + j\omega \cdot e^{j\omega t} \cdot \left(\vec{\psi}_s^{xy} + L_{a\lambda} \cdot \vec{i}_s^{xy}\right)$$

$$\vec{u}_s^{xy} = R_a \cdot \vec{i}_s^{xy} + \frac{d\left(\vec{\psi}_s^{xy} + L_{a\lambda} \cdot \vec{i}_s^{xy}\right)}{dt} + j\omega \cdot \left(\vec{\psi}_s^{xy} + L_{a\lambda} \cdot \vec{i}_s^{xy}\right)$$

Solution 4.15b cont'd

$$\vec{U}_s^{xy} = R_s \cdot \vec{i}_s^{xy} + \frac{d\left(\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{xy}\right)}{dt} + j \cdot \omega \cdot \left(\vec{\psi}_\delta^{xy} + L_{s\lambda} \cdot \vec{i}_s^{xy}\right)$$

Separate the equation in a real and in a imaginary part

Equation 8.31

$$\begin{cases} L_{sx} = \left(L_{mx} + L_{s\lambda}\right) \\ L_{sy} = \left(L_{my} + L_{s\lambda}\right) \end{cases}$$

$$U_{sx} = R_s \cdot i_{sx} + \frac{d\left(\psi_m + \left(L_{mx} + L_{s\lambda}\right) \cdot i_{sx}\right)}{dt} - \omega_r \cdot \left(L_{my} + L_{s\lambda}\right) \cdot i_{sy} =$$

$$= R_s \cdot i_{sx} + \frac{d\left(\psi_m + L_{sx} \cdot i_{sx}\right)}{dt} - \omega_r \cdot L_{sy} \cdot i_{sy}$$

$$U_{sy} = R_s \cdot i_{sy} + \frac{d\left(\left(L_{my} + L_{s\lambda}\right) \cdot i_{sy}\right)}{dt} + \omega_r \cdot \left(\psi_m + \left(L_{mx} + L_{s\lambda}\right) \cdot i_{sx}\right) =$$

$$= R_s \cdot i_{sy} + L_{sy} \cdot \frac{di_{sy}}{dt} + \omega_r \cdot \left(\psi_m + L_{sx} \cdot i_{sx}\right)$$

Solution 4.15b cont'd

$$Separate\ the\ equation\ below\ in\ real\ and\ imaginary\ parts,\ see\ equation\ (8.31)$$

$$\vec{u}_s^{xy} = R_a \cdot \vec{i}_s^{\,xy} + \frac{d\left(\vec{\psi}_s^{\,xy} + L_{a\lambda} \cdot \vec{i}_s^{\,xy}\right)}{dt} + j\omega \cdot \left(\vec{\psi}_s^{\,xy} + L_{a\lambda} \cdot \vec{i}_s^{\,xy}\right)$$

$$\begin{cases} u_{sx} = R_a \cdot i_{sx} + \frac{d}{dt}(\psi_m + L_{mx} \cdot i_{sx} + L_{a\lambda} \cdot i_{sx}) - \omega_r \cdot \left(L_{my} \cdot i_{sy} + L_{a\lambda} \cdot i_{sy}\right) = R_a \cdot i_{sx} + \frac{d}{dt}(\psi_m + L_{sx} \cdot i_{sx}) - \omega_r \cdot L_{sy} \cdot i_{sy} \\ u_{sy} = R_a \cdot i_{sy} + \frac{d}{dt}\left(L_{my} \cdot i_{sy} + L_{a\lambda} \cdot i_{sy}\right) + \omega_r \cdot (\psi_m + L_{mx} \cdot i_{sx} + L_{a\lambda} \cdot i_{sx}) = R_a \cdot i_{sy} + L_{sy} \cdot \frac{di_{sy}}{dt} + \omega_r \cdot (\psi_m + L_{sx} \cdot i_{sx}) \end{cases}$$

Exercise 4.16 DC machine voltage equation

An armature winding is designed as a commutator winding, positioned in the rotor.

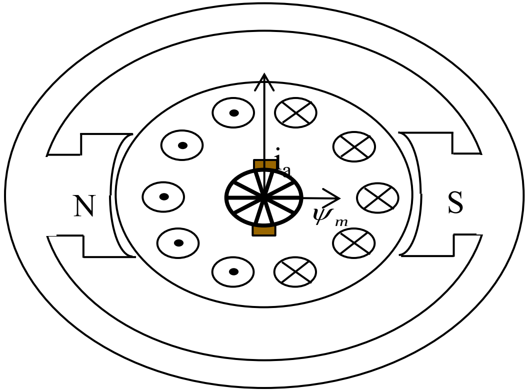
a. Draw a stylized picture of a cross section of the machine and show the resulting current distribution in the armature circuit that gives maximum torque if $L_{my}=0$.

b. Given the position of the commutator as in a), form an expression of the torque!

c. Give the voltage equation for the aramature circuit as it is known via the sliding contacts positioned as in b)

Solution 4.16a,b

a) See figure 10.2



$$b)\ Torque\ \left(Equation\ 10.1\right) \qquad T = \psi_m \cdot i_a$$

Solution 4.16c

$$c)\,Voltage\ \ (Equation\ \ 8.31)$$

$$See\ \ paragraph\ 10.2,\ the\ \ x-axis\ \ windings\ \ are\ never\ used,$$

$$the\ \ x-axis\ \ current\ \ is\ \ always\ \ zero, see\ equation\ 10.1$$

$$u_{ax}=R_s\cdot\underbrace{i_{ax}}_{=0}+\underbrace{\frac{d}{dt}\underbrace{\psi_m}_{constant}}_{=0}+\frac{d}{dt}\left(L_{mx}\cdot\underbrace{i_{ax}}_{=0}+L_{a\lambda}\cdot\underbrace{i_{ax}}_{=0}\right)-\omega_r\cdot\underbrace{L_{sy}}_{=0}\cdot i_{ay}=0$$

$$u_{ay}=u_a=R_a\cdot i_a+\frac{d}{dt}\left(\underbrace{L_{my}}_{=0}\cdot i_a+L_a\cdot i_a\right)+\omega_r\cdot\left(\psi_m+L_a\cdot\underbrace{i_{ax}}_{=0}\right)=$$

$$=R_a\cdot i_a+L_a\cdot\frac{di_a}{dt}+\omega_r\cdot\psi_m$$

Exercise 4.17 DC machine torque, power and flux

A DC machine has the following ratings:

U_{an}=300V

I_{an}=30A

R_a=1Ω

L_a=5mH

n_n=1500 rpm

Determine the rated torque T_n , the rated power P_n and the rated magnetization ψ_{mn}.

Solution 4.17

U_{an}= 300 V
I_{an}= 30 A
R_a= 1 ohm
L_a= 5 mH
n_a= 1500 rpm
At the nominal point, all values are constant

$$U_a = R_a \cdot i_a + L_a \cdot \frac{di_a}{dt} + e_a = \left\{ \frac{di_a}{dt} = 0 \right\} = R_a \cdot i_a + e_a \Rightarrow$$

SOLUTION

$$\begin{cases} Power \quad P_{motor} = e_a \cdot i_a = [U_a - R_a \cdot i_a] \cdot i_a = (300 - 30 \cdot 1) \cdot 30 = 8100\ W \\ Torque\, T_n = \frac{P_{motor}}{\omega_n} = \frac{8100}{\frac{1500}{60} \cdot 2\pi} = 51.6\ Nm \\ Flux \quad \psi_{\omega n} = \frac{e_a}{\omega_n} = \frac{U_a - R_a \cdot i_a}{\omega_n} = \frac{300 - 30 \cdot 1}{\frac{1500}{60} \cdot 2\pi} = 1.72\ Vs \end{cases}$$

Exercise 4.18 DC machine controller

Same data as in 4.17. The machine is fed from a switched converter with the sampling interval T_s =1ms, and the DC voltage U_{d0} =300V.

Derive a suitable controller for torque control at constant magnetization. The current is measured with sensors that give a maximum signal for $i_a = I_0$ =30A.

A DC machine has the following ratings:

R_a=1Ω

L_a=5mH

n_n=1500 rpm

Solution 4.18

$$U_a = R_a \cdot i_a + L_a \cdot \frac{di_a}{dt} + e_a$$

$$u_a^*(k) = R_a \cdot \frac{i_a(k+1) + i_a(k)}{2} + L_a \cdot \frac{i_a(k+1) - i_a(k)}{T_s} + e_a(k) = R_a \cdot \frac{i_a^*(k) - i_a(k)}{2} + R_a \cdot i_a(k) + \frac{L_a}{T_s} \cdot \left(i_a^*(k) - i_a(k)\right) + e_a(k)$$

$$u_a^*(k) = \frac{R_a}{2} \cdot \left(i_a^*(k) - i_a(k)\right) + R_a \cdot i_a(k) + \frac{L_a}{T_s \cdot} \cdot \left(i_a^*(k) - i_a(k)\right) + e_a(k) = \left(\frac{R_a}{2} + \frac{L_a}{T_s}\right) \cdot \left(i_a^*(k) - i_a(k)\right) + R_a \cdot \sum_{n=0}^{k-1} \left(i_a^*(n) - i_a(n)\right) + e_a(k)$$

$$u_a^*(k) = \left(\frac{R_a}{2} + \frac{L_a}{T_s}\right) \cdot \left(\left(i_a^*(k) - i_a(k)\right) + \frac{R_a}{\left(\frac{R_a}{2} + \frac{L_a}{T_s}\right)} \cdot \sum_{n=0}^{k-1} \left(i_a^*(n) - i_a(n)\right) \right) + e_a(k)$$

$$u_a^*(k) = \left(\frac{R_a}{2} + \frac{L_a}{T_s}\right) \cdot \left(\left(i_a^*(k) - i_a(k)\right) + \frac{T_s}{\left(\frac{T_s}{2} + \frac{L_a}{R_a}\right)} \cdot \sum_{n=0}^{k-1} \left(i_a^*(n) - i_a(n)\right) \right) + e_a(k)$$

$$u_a^*(k) = \left(\frac{1}{2} + \frac{0.005}{0.001}\right) \cdot \left(\left(i_a^*(k) - i_a(k)\right) + \frac{0.001}{\left(\frac{0.001}{2} + \frac{0.005}{1}\right)} \cdot \sum_{n=0}^{k-1} \left(i_a^*(n) - i_a(n)\right) \right) + u_a(k) - 1 \cdot i_a(k) = \{R_a = 1, L_a = 0.005, T_s = 0.001,\} =$$

$$u_a^*(k) = 5.5 \cdot \left(\left(i_a^*(k) - i_a(k)\right) + 0.182 \cdot \sum_{n=0}^{k-1} \left(i_a^*(n) - i_a(n)\right) \right) + \left(u_a(k) - i_a(k)\right)$$

Exercise 4.20 PMSM controller

A permanently magnetized synchronous machine has the following ratings:

$U_{line\text{-}to\text{-}line}$=220V

I_{sn}=13A

n_n=3000 rpm

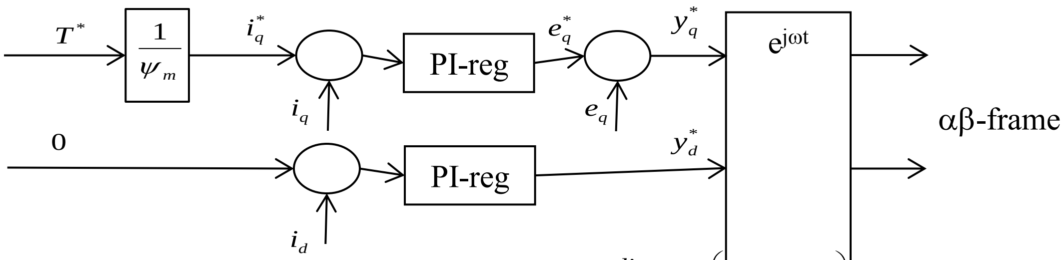
R_a=0,5Ω

L_d= L_q=7mH

The machine is driven by a switched amplifier with the DC voltage U_{d0} =350V. The frequency of the modulating triangular wave f_{tri} is 1000 Hz . The current sensor measures currents up to a maximum of I_0 =25A.

Suggest a structure for the control of the torque of the machine together with a set of relevant equations.

Solution 4.20



Equation (11.3)(*assume stationarity*) $u_{sq} = R_s \cdot i_{sq} + \underbrace{L_{sq} \cdot \frac{di_{sq}}{dt}}_{=0} + \omega \cdot \left(\psi + L_s \cdot \underbrace{i_{sd}}_{=0} \right) = R_s \cdot i_{sq} + \omega \cdot \psi$

Angular frequency $\omega = 2\pi \cdot \frac{3000}{60} = 314.2$

Assume $i_{sq} = i_{sn}, i_{sd} = 0$ $i_{sq} = \sqrt{\frac{3}{2}} \cdot 13 = 15.9\,A$

Voltage drop over res & ind $\Delta e_R = 0.5 \cdot 15.9 = 8V$

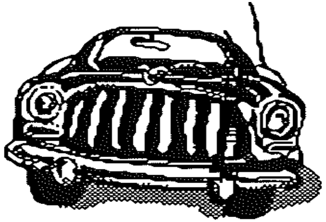
Max symmetrized voltage $u_{LL\_eff} = \frac{350}{\sqrt{2}} = 247.5\ V$

Back − emf voltage $e = u_{LL\_eff} - \Delta e_R = 247.5\ V - 8V = 239.5V = \omega \cdot \psi$

Exercise 4.25 Electric car

You are to design an electric car. You have a chassis with space for batteries and an electric motor. The battery weight is 265 kg, the storing capacity is
32 kWh and can be charged with 5 kW. The battery no load voltage e_0 ranges from 170V to 200V and its inner resistance is R_b =0,14Ω. The motor is a two-pole three phase alternating current motor with the rating 50 kW at the rated speed n_{nm} =3000 rpm. The car has two gears, which give the speed 120 km/h at the rated speed of the motor, corresponding to the net gear 1/2,83. The weight of the car is 1500 kg including the battery weight. The requirement is to manage a 30% uphill.

Data		
Motor, 2-pole, 3 phase AC		
Rated power	50 kW	
Rated motor speed		3000 rpm
Battery		
Voltage		170-200 V
Charge capacity		32 kWh
Max charging power	5 kW	
Internal resistance		0.14 ohm
Weight		265 kg
Vehicle		
Weight		1500 kg (incl battery)
Vehicle speed at rated motor speed	120 km/h	
Gear		1/2.83 at 120 km/h
Rated uphill	30%	



Exercise 4.25 cont'd

a) What is the rated torque of the motor?

b) What rated stator voltage would you choose when you order the motor?

c) Which is the minimum rated current for the transistors of the main circuit?

d) What gearing ratio holds for the low gear?

e) When driving in 120 km/h, the power consumption is 370 Wh/km. How far can you drive if the batteries are fully loaded when you start? For a certain drive cycle in city traffic, the average consumption is 190 Wh/km. How far can the car be driven in the city?

f) What is the cost/10 km with an energy price of 2 SEK/kWh?

Solution 4.25a,b

$$a)\ Angular\ speed\ at\ rated\ speed\ \omega=\frac{3000}{60}\cdot 2\cdot\pi=314$$

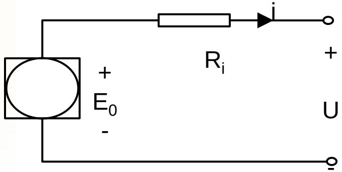
$$Torque\ at\ rated\ speed\ T=\{Power\ P=T\cdot\omega\}=\frac{P}{\omega}=\frac{50000}{314}=159\ Nm$$

$$b)\ P=50kW=u\cdot i=(e_0-R_i\cdot i)\cdot i=\left\{\begin{matrix}e_0=170-200V\\ use\ 170\ V\end{matrix}\right\}=(170-0.14\cdot i)\cdot i=170\cdot i-0.14\cdot i^2$$

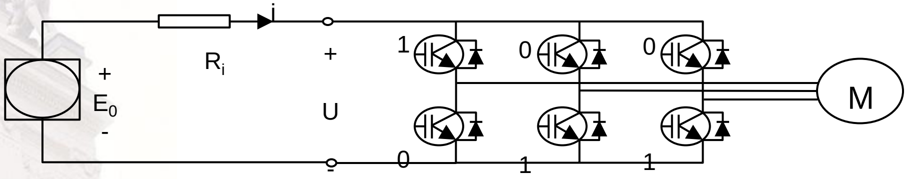
$$i^2-\frac{170\cdot i}{0.14}+\frac{50000}{0.14}=0\Rightarrow i=\frac{85}{0.14}\pm\sqrt{\left(\frac{85}{0.14}\right)^2-\frac{50000}{0.14}}=500\,A$$

$$u=170-0.14\cdot 500=100V_{dc}$$

$$With\ symmetrized\ 3-phase\ ac\ voltage\ \hat{u}_{LL}=100V_{dc}\Rightarrow u_{LL}=\frac{u_{dc}}{\sqrt{2}}\approx 71V$$



Solution 4.25c



$$c)\ Assume\ power\ factor = 0.9$$

$$P_{ac} = \sqrt{3} \cdot u_{LL} \cdot I_{phase} \cdot 0.9 \Rightarrow 50000 = \sqrt{3} \cdot 71 \cdot I_{phase} \cdot 0.9$$

$$I_{phase\_eff} = \frac{50000}{\sqrt{3} \cdot 71 \cdot 0.9} = 452\,A$$

$$\hat{I}_{phase} = 639\,A$$

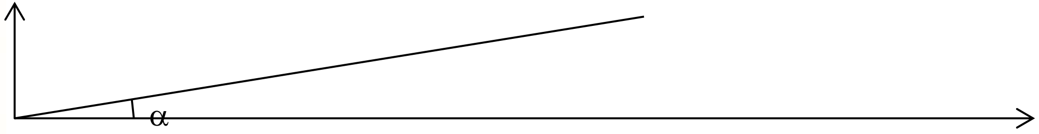
See figure above, "1" means the transistor is conducting, "0" the transistor is not conducting

E.g. the top left transistor is the only transistor in upper position which is conducting,

thus the full dc − current is flowing through this transistor,

Rated transistor current is 639 A

Solution 4.25d



$$d)\ The\ uphill\ slope\ is\ 30\%.\ \arctan(\alpha) = 0.3 \Rightarrow \alpha = 17^o$$

$$The\ requested\ force\ F = 1500 \cdot 9.81 \cdot \sin(17^o) = 4228\,N$$

$$Assume\ wheel\ radius\ r = 0.3\,m$$

$$Torque\ T = F \cdot r = 4228 \cdot 0.3 = 1268\,Nm$$

$$The\ motor\ torque\ at\ rated\ power\ T_{motor} = 159\,Nm$$

$$Assume\ the\ low\ gear,\ the\ gear\ ratio = \frac{1268}{159} = 8.0$$

Solution 4.25 e,f

$$e)\ Maximum\ battery\ charge = \quad 32kWh$$

$$Battery\ consumption\ at\ 120\,km/h = \quad 370\,Wh/km$$

$$How\ far\ with\ fully\ loaded\ battery\ at\ 120\,km/h = \frac{32}{0.37} \approx \quad 86\,km$$

$$Battery\ consumption\ in\ average\ city\ traffic = \quad 190\,Wh/km$$

$$How\ far\ in\ average\ city\ traffic = \frac{32}{0.19} \approx \quad 168\,km$$

$$f)\ Cost/10km\ at\ 120\,km/h = 10 \cdot 0.37 \cdot 2\,SEK/kWh = \quad 7.40\,SEK$$

$$Cost/10\,km\ in\ average\ city\ traffic = 10 \cdot 0.19 \cdot 2\,SEK/kWh = 3.80\,SEK$$

5

Exercises on PMSM

Exercise 5.1 Flux and noload voltage

A permanent magnetized synchronous machine is magnetized with at the most 0,7 Vs linkage flux in one phase. It is not connected.

a. How large is the flux vector as a function of the rotor position?
b. How large is the induced voltage vector as a function of rotor position and speed?
c. At which speed is the voltage too large for a frequency converter with a dc voltage of 600V?

Solution 5.1a

$$Given \quad \hat{\psi}_{phase} = 0.7Vs$$

$$No\ load,\ open\ stator$$

$$a)\ Sought \qquad \vec{\psi} = f(\Theta_r)$$

$$From\ equation\ (3.4)\ it\ is\ learned\ that\ the\ magnitude\ of\ the\ vector$$

$$equals\ the\ "phase-to-phase"\ RMS-value\ of\ the\ same\ quantity,$$

$$that\ means \qquad |\vec{\psi}_m| = \frac{\sqrt{3}}{\sqrt{2}} \cdot \hat{\psi}_{phase} = 0.86Vs$$

$$The\ flux\ vector\ is\ oriented\ along\ the\ PMSM\ rotor\ magnet\ position$$

Solution 5.1b,c

$b)$ *From equation* $(3.5)\, E\cdot e^{j\omega t} = \vec{e} = \omega\cdot\vec{\psi}_m \cdot e^{j\frac{\pi}{2}}$ *the induced voltage is* ω *times the flux and advanced* $\frac{\pi}{2}$ *in phase*

$c)$ *According to figure 2.24, the voltage vectors are* $\sqrt{\frac{2}{3}}\cdot U_{dc}$ *in magnitude.*

The longest vector that always can be sustained is the radius of the circle inscribed in the hexagon

$$|U|_{\max} = \sqrt{\frac{2}{3}}\cdot U_{dc}\cdot\cos\left(\frac{\pi}{6}\right) = \sqrt{\frac{2}{3}}\cdot U_{dc}\cdot\frac{\sqrt{3}}{2} = \frac{U_{dc}}{\sqrt{2}} = 424V$$

$$E_{\max} = \omega_{r,\max}\cdot\sqrt{\frac{3}{2}}\cdot\hat{\psi}_{phase} \Rightarrow \omega_{r,\max} = 424\cdot\frac{\sqrt{2}}{\sqrt{3}\cdot 0.7} = 495\, rad/s$$

Exercise 5.2 Inductance and torque generation

A permanent magnetized synchronous machine has a cylindrical rotor with $L_{mx} = L_{my} = L_m = 2$ mH. The magnetization is the same as in 4.1 0,7 Vs linkage flux in one phase. The machine is controlled so that the stator current along the x axle is zero ($i_{sx}=0$).

a) How large torque can the machine develop if the phase current s limited to 15 A RMS?

b) Draw the flux linkage from the permanent magnets and from the stator current in (x, y) coordinates together with induced voltage and voltage for the frequency 25 Hz and the stator resistance 0,2Ω!

c) How large stator current is required to reduce the flux to zero?

Solution 5.2a

Data	
Linked flux	$\psi_m = 0.7Vs$
Frequency	$25\,Hz$
Stator resistance	$0.2\,ohm$
Max current	$\begin{cases} i_{sx} = 0\,A \ (in\ exercise\ a)\ and\ b) \\ i_{phase\_RMS\_\max} = 15\,A \end{cases}$
Inductance	$L_{mx} = L_{my} = L_m = 2mH$
Assume	$L_{s\lambda} = 0$

a) MaxTorque, see equation (11.7)

$$\left|i_{sy\_RMS\_\max}\right| = \{abc \Rightarrow \alpha\beta\} = \sqrt{\frac{3}{2}} \cdot i_{phase\_RMS\_\max}$$

$$\left|\hat{i}_{sy\_\max}\right| = \sqrt{2} \cdot \left|i_{sy\_RMS\_\max}\right| = \sqrt{2} \cdot \sqrt{\frac{3}{2}} \cdot i_{phase\_RMS\_\max} = \sqrt{3} \cdot 15 = 26A$$

$$T_{\max} = \sqrt{\frac{3}{2}} \cdot i_{sy} \cdot \psi_m + \left(L_{mx} - L_{my}\right) \cdot i_{sx} \cdot i_{sy} = \{i_{sx} = 0\} = \sqrt{\frac{3}{2}} \cdot i_{sy} \cdot \psi_m = \sqrt{\frac{3}{2}} \cdot 26 \cdot 0.7 = 22.3\,Nm$$

Solution 5.2b,c

$b)\ See\ equation\ (11.3)$

$$\hat{u}_s = R_s \cdot \hat{i}_s + \frac{d}{dt}\left(\psi_m + L_s \cdot \hat{i}_s\right) + j \cdot \omega_r \cdot \left(\psi_m + L_s \cdot \hat{i}_s\right) = \left\{\hat{i}_s\ and\ \psi_m\ are\ const\right\} = R_s \cdot \hat{i}_s + j \cdot \omega_r \cdot \left(\psi_m + L_s \cdot \hat{i}_s\right) =$$

$$= 0.2 \cdot j26 + j \cdot 2\pi \cdot 25 \cdot \left(\sqrt{\frac{3}{2}} \cdot 0.7 + 0.002 \cdot j26\right) = -8.17 + j \cdot 134.4 = 135\ V$$

$c)\ Find\ the\ i_{sx}\ for\ which\ the\ flux\ is\ reduced\ to\ zero\ \vec{\psi}_s = \left(\vec{\psi}_m + L_{sx} \cdot i_{sx}\right) + j \cdot L_{sy} \cdot i_{sy} = 0 + j0$

$$i_{sy} = 0$$

$$i_{sx} = -\frac{\sqrt{\frac{3}{2}} \cdot \psi_m}{L_{sx}} = -428\,A$$

Exercise 5.3 PMSM Control

The machine in example 5.2 is vector controlled. The voltage is updated every 100 µs, i. e. the sampling interval is T_s=100 µs. The machine shall make a torque step from 0 to maximum torque when the rotor is at standstill. The DC voltage is 600V.

a. Determine the voltage that is required to increase the current i_{sy} from zero to a current that corresponds to maximum torque in one sample interval!

b. Is the DC voltage sufficient?

Solution 5.3a

Data

Sampling time	$T_s = 100\,\mu s$
Torque, see execisen 5.2a	$22.3\,Nm$
Dclink voltage	$U_{dc} = 600V$
Start from standstill	$\omega = 0$

a) It is a 3 − phase load, see the theory in chapter 3.7, particularly equ (3.17) and (3.18)
Since it will be a step in i_{sy} (called i_q in the equations) following expressions are valid

$$u_x^*(t) = \left(\frac{L}{T_s} + \frac{R}{2}\right) \cdot \left((0-0) + \frac{T_s}{\left(\frac{L}{R} + \frac{T_s}{2}\right)} \cdot \sum_0^{k-1} (0-0) \right) - \underbrace{\omega}_{=0} \cdot L_s \cdot i_q = 0$$

$$u_y^*(t) = \left(\frac{L}{T_s} + \frac{R}{2}\right) \cdot \left((26-0) + \frac{T_s}{\left(\frac{L}{R} + \frac{T_s}{2}\right)} \cdot \sum_0^{k-1} (0-0) \right) + \underbrace{\omega}_{=0} \cdot \left(\psi_m + L_s \cdot \underbrace{i_d}_{=0}\right) = \left(\frac{L}{T_s} + \frac{R}{2}\right) \cdot 26 = \left(\frac{2 \cdot 10^{-3}}{1 \cdot 10^{-4}} + \frac{0.2}{2}\right) \cdot 26 = 522V$$

Solution 5.3b

b) The maximum line-to-line voltage from a dclink voltage is $= \frac{U_{dc}}{\sqrt{2}} = 424V$

If we are lucky and the step in $u_y^*(k)$ happens to point in the direction of one of the six voltage vectors defining the hexagon we will have the voltage $\sqrt{\frac{2}{3}} \cdot U_{dc} = 490V$, still too low than the requested $522V$.

The step will take more than one sampling interval

Exercise 5.4 Torque

A two-pole permanently magnetized synchronous machine with the parameters Lmx=Lmy=Lm=15mH is used in an airplane and is therefore driven with stator frequencies up to 400 Hz. The stator resistance is negligable. The phase current is limited to 10A rms. The motor is fed by a converter with the DC voltage 600V.

a. Determine the magnetization from the permanent magnets considering the case when all voltage is needed and the machine is developing full torque (all the current along the q-axle) and 200 Hz stator frequency!
b. Determine the torque!

c. Determine the torque at 400 Hz stator frequency provided a part of the current is needed for demagnetization!

Solution 5.4a,b

Data

$PMSM$	$2-pole$
$L_{sx} = L_{sy}$	$15\,mH$
$f_{\max}$	$400\,Hz$
R_s	$0\,\Omega$
I_{phase}	$10\,A$
$Dclink\ voltage\,U_{dc} =$	$600V$

$a)$ Sought ψ_m at max voltage and full torque, and no need for field weakening at this low speed

$L_{sx} = L_{sy}$, thus no reluctance torque

$f = 200\,Hz$

Start with equation (11.2)

$$\vec{u}_s = R_s \cdot \vec{i}_s + \frac{d}{dt}\left(\psi_m + L_s \cdot \vec{i}_s\right) + j \cdot \omega_r \cdot \left(\psi_m + L_s \cdot \vec{i}_s\right) = \left\{ R_s = 0.\ Assume\ stationarity \Rightarrow \frac{d}{dt} = 0 \right\} =$$

$$= j \cdot \omega_r \cdot \psi_m + j \cdot \omega_r \cdot L_s \cdot \vec{i}_s \Rightarrow |\vec{u}| = \left| j \cdot \omega_r \cdot \psi_m + j \cdot \omega_r \cdot L_s \cdot \vec{i}_s \right|$$

$$\left|\frac{U_{dc}}{\sqrt{2}}\right| = \sqrt{\left(2\pi \cdot 200 \cdot \psi_m\right)^2 + \left(2\pi \cdot 200 \cdot L_s \cdot \vec{i}_s\right)^2} \Rightarrow \psi_m = \frac{\sqrt{\frac{600^2}{2} - \left(2\pi \cdot 200 \cdot 0.015 \cdot 10 \cdot \sqrt{2}\right)^2}}{2\pi \cdot 200} = 0.26Vs$$

$$b)\ T = \psi_m \cdot i_{sy} = 0.26 \cdot 10 \cdot \sqrt{3} = 4.5\,Nm$$

Solution 5.4c

$c)\ In\ this\ case\ i_{sx} <> 0\ since\ field\ weakening\ is\ used$

$L_{sx} = L_{sy},\ thus\ no\ reluctance\ torque$

$f = 400\,Hz$

$Assume\ stationarity \Rightarrow \frac{d}{dt} = 0$

$$\begin{cases} |\vec{u}| = \left|R_s \cdot \vec{i}_s + j\omega_r \cdot \psi_m + j\omega_r \cdot L_s \cdot \vec{i}_s\right| = \{R_s = 0\} = \sqrt{(\omega_r \cdot L_s \cdot i_{sy})^2 + (\omega_r \cdot \psi_m + \omega_r \cdot L_s \cdot i_{sx})^2} = 424V \\ |i_s| = \sqrt{i_{sx}^2 + i_{sy}^2} = 10\sqrt{3} \end{cases}$$

$$|u|^2 = \omega_r^2 \cdot L_s^2 \cdot i_{sy}^2 + \omega_r^2 \cdot \psi_m^2 + \omega_r^2 \cdot L_s^2 \cdot i_{sx}^2 + 2 \cdot \omega_r \cdot \psi_m \cdot \omega_r \cdot L_s \cdot i_{sx} =$$

$$= \underbrace{\omega_r}_{25133}{}^2 \cdot \left(\underbrace{L_s}_{0.015}{}^2 \cdot \underbrace{\left(i_{sx}^2 + i_{sy}^2\right)}_{10\sqrt{3}^2} + \underbrace{\psi_m}_{0.26}{}^2 + 2 \cdot \underbrace{\psi_m}_{0.26} \cdot \underbrace{L_s}_{0.015} \cdot i_{sx} \right) = 424^2$$

$$\begin{cases} i_{sx} = \dfrac{\left(\dfrac{424}{2513.3}\right)^2 - 0.015^2 \cdot 10^2 \cdot \sqrt{3}^2 - 0.26^2}{2 \cdot 0.26 \cdot 0.015} = 13.7\,A \\ i_{sy} = \sqrt{\left(10 \cdot \sqrt{3}\right)^2 - 13.7^2} = 10.6\,A \end{cases}$$

$T = 10.6 \cdot 0.26 = 2.76\,Nm$

$I.e.\ the\ torque\ has\ dropped\ to\ about\ half\ of\ the\ torque\ at\ 200\,Hz,\ which\ is\ not\ a\ surprise$

$(a\ bit\ more\ than\ half\ since\ we\ use\ 10.6\,A\ instead\ of\ 10\sqrt{3} = 17.3\,A)$

Exercise 5.5 Drive system

You are designing an electric bicycle with a synchronous machine as a motor, coupled to the chain by a planetary gear. The power of the motor is 200W and it has 10 poles. The speed of the motor is 1000rpm at full power. The motor is fed from a three phase converter with batteries of 20V. The stator resistance and inductance can be neglected.

a. Determine the magnetization expressed as a flux vector at rated operational with full voltage from the frequency converter!
b. Determine the phase current at full torque!
c. Determine the moment of inertia if the bicycle and its driver weigh 100kg, the gear ratio is 1:10 and the rated speed is 25 km/h!Om cykel med förare väger 100 kg, hur stort tröghetsmoment upplever drivmotorn om utväxlingen är 1:10 och märkhastigheten är 25 km/h?
d. How long is the time for accelaration?

Solution 5.5a

$$\begin{array}{lll} Data & & \\ PMSM & p = 10 - pole & \\ Power & & 200\,W \\ Dclink\ voltage\ U_{dc} = & & 20\,V \\ speed\ at\ full\ power & & 1000\,rpm \\ L_{sx} = L_{sy} & & 0\,mH,\ no\ reluct.\ torque \\ R_s & & 0\,\Omega \end{array}$$

$$a)\ Magnetization\ flux\ vector\ |\psi_s|\ at\ rated,\ nom\ speed.\ Assume\ stationarity,\ \Rightarrow \frac{d}{dt} = 0$$

$$|\vec{u}^{xy}| = |j\omega_r \cdot \psi_s|,\ see\ equ\ (11.2)$$

$$|\vec{\psi}_s| = \frac{\frac{U_{dc}}{\sqrt{2}}}{\omega_{r,el}} = \frac{U_{dc}}{\sqrt{2} \cdot \omega_{r,mech} \cdot \frac{p}{2}} = \frac{20}{\sqrt{2} \cdot 2\pi \cdot \frac{1000}{60} \cdot \frac{10}{2}} = 0.027\,Vs \approx \psi_{pm}\ ,as\ L_s = 0$$

Solution 5.5b,c,d

$$b)\,Torque \qquad T_{mech} = \frac{p}{2}\cdot T_{el} = \frac{p}{2}\cdot \psi_{pm}\cdot i_{sy} = \frac{P}{\omega_{r,mech}} = \frac{200}{2\pi\cdot\frac{1000}{60}} = 1.91\,Nm$$

$$i_{sy} = \frac{T_{mech}}{\frac{p}{2}\cdot\psi_{pm}} = \frac{1.91}{\frac{10}{2}\cdot 0.027} = 14.15\,A$$

$$I_s = \frac{14.15}{\sqrt{3}} = 8.17\,A$$

$$c)\ Inertia \qquad Energy\ \frac{1}{2}\cdot J\cdot\omega_{r,mech}{}^2 = \frac{1}{2}\cdot m\cdot v^2 \Rightarrow J_{ekv} = m\cdot\left(\frac{v}{\omega_{r,mech}}\right)^2 = 100\cdot\left(\frac{\frac{25}{3.6}}{104.72}\right)^2 = 0.44\,kgm^2$$

$$d)\,Acceleration\,time \qquad \omega_{r,mech} = \int\frac{T_{mech}}{J_{ekv}}dt = \frac{T_{mech}}{J_{ekv}}\cdot t_{acc} \Rightarrow t_{acc} = \frac{\omega_{r,mech}\cdot J_{ekv}}{T_{mech}} = \frac{104.72\cdot 0.44}{1.91} = 24.1\,s$$

Exam 2012-05-21

Exercise Exam 2012-05-21 1a - The four quadrant DC-DC converter

a) Draw a four quadrant DC/DC converter with a three phase diode rectifier connected to the power grid. Between the rectifier and the DC link capacitor is a BIG inductor connected. This inductor, the dc-link capacitor and protection against too high inrush currents should be included in the drawing. The transistors are of IGBT-type. (2 p.)

b) The three-phase grid, to which the three phase diode rectifier is connected, has the line-to-line voltage 400 V_{rms} at 50 Hz. The bridge output voltage of the four quadrant DC/DC converter is 430 V.
Calculate the average voltage at the rectifier dc output.
Calculate the duty cycle of the four quadrant dc/dc converter. (2 p.)

c) Due to the big inductor between the rectifier and the DC link capacitor the rectifier output DC-current can be regarded as constant, 172 A. The 4Q bridge load can be regarded as a constant voltage in series with a 5.1 mH inductance.

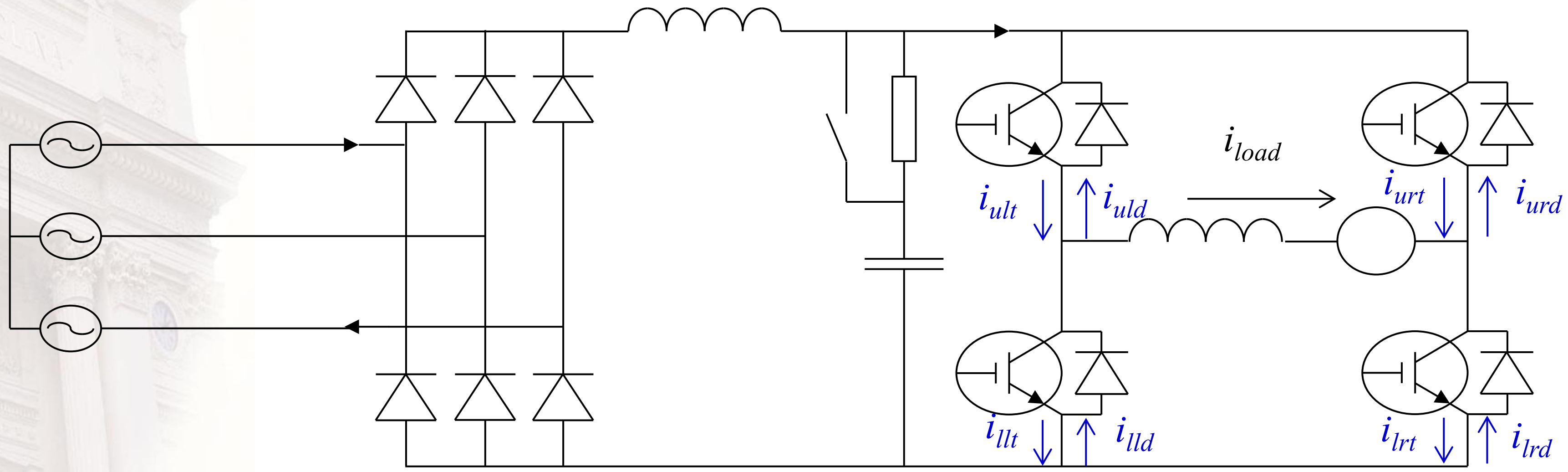
- The rectifier diode threshold voltage is 1.0 V and its differential resistance is 2.2 mohm.
- The rectifier diode turn-on and turn-off losses can be neglected
- The IGBT transistor threshold voltage is 1.4 V and its differential resistance is 12 mohm.
- The turn-on loss of the IGBT transistor is 65 mJ and its turn-off loss is 82 mJ.
- The IGBT diode threshold voltage is 1.1 V and its differential resistance is 9.5 mohm.
- The IGBT diode turn-off losses is 25 mJ, while the turn-on loss can be neglected
- Both the IGBT transistor and the IGBT diode turn-on and turn-off losses are nominal values at 900 V DC link voltage and 180 A turn-on and turn-off current.
- The switching frequency is 2 kHz.

Make a diagram of the 4Q load current
Calculate the rectifier diode losses.
Calculate the IGBT transistor losses of each IGBT in the four quadrant converter.
Calculate the IGBT diode losses of each IGBT in the four quadrant converter. (4 p.)

d) Which is the junction temperature of the IGBT transistor and of the IGBT diode, and which is the junction temperature of the rectifier diodes?

- The thermal resistance of the heatsink equals 0.025 K/W?
- The thermal resistance of the IGBT transistor equals 0.043 K/W?
- The thermal resistance of the IGBT diode equals 0.078 K/W?
- The thermal resistance of the rectifier diode equals 0.12 K/W?
- The ambient temperature is 42 ^{o}C.
- The rectifier diodes and the four quadrant converter IGBTs share the heatsink. (2 p.)

Solution Exam 2012-05-21 1a



Solution Exam 2012-05-21 1b

Average DC voltage

(Since the rectifier is loaded with a BIG inductor and in stationary state, the DC link voltage must be equal to the average of the rectified grid voltage)

$$U_{dc\_ave} = \frac{3}{\pi} \cdot 400 \cdot \sqrt{2}\ V = 540\ V$$

4Q average bridge voltage

(This is given in the question)

$$U_{dc4QC} = 430V$$

4Q output voltage duty cycle

(The 4Q output voltage is modulated to 430 V from 540 V DC))

$$D = \frac{430}{540} = 0.8$$

Solution Exam 2012-05-21 1c_1

Rectifier diode current	*172 A*
Rectifier diode threshold voltage	*1.0 V*
Rectifier diode diff resistance	*2.2 mohm*
Rectifer diode on state voltage	*1+172*0.0022=1.38 V*
Rectifier diode power loss	*1.38*172*0.33=78 W (conducting 33% of time)*
Rectifer diode thermal resistance	*0,12 K/W*

Continous rectifier output current	*172 A*
The continous 4Q load current	*172/0.8=215 A (to maintain the power)*

Solution Exam 2012-05-21 1c_2

4Q load current $I_{pulse,avg}=215A$
4Q load inductance $5.1\ mH$

Only the upper left and lower right transistors have losses and the lower left and upper right diodes have losses. The other semiconductors do not conduct since the 4Q output current is strictly positive.

The load current ripple can be calculated as:

$$\Delta i = \frac{u-e}{L}\Delta t = \frac{540-430}{0.0051}\,0.8 * \frac{1}{2*2000} = 4.3\ A$$

The "duty cycle" of the upper left, and lower right, transistor current is:

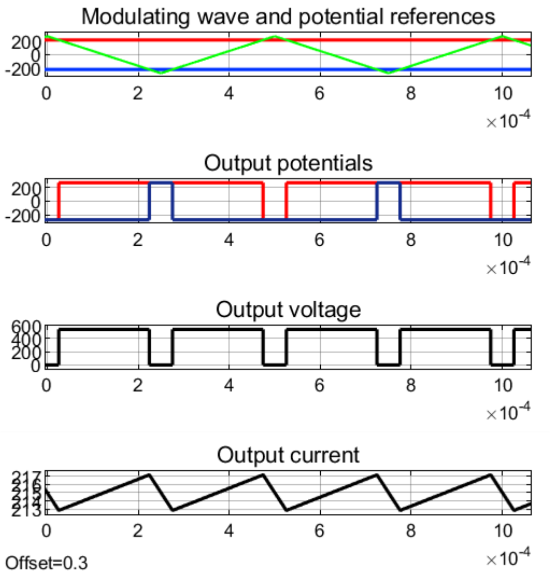
$$D_{tr} = 1 - \frac{D}{2} = 0.9$$

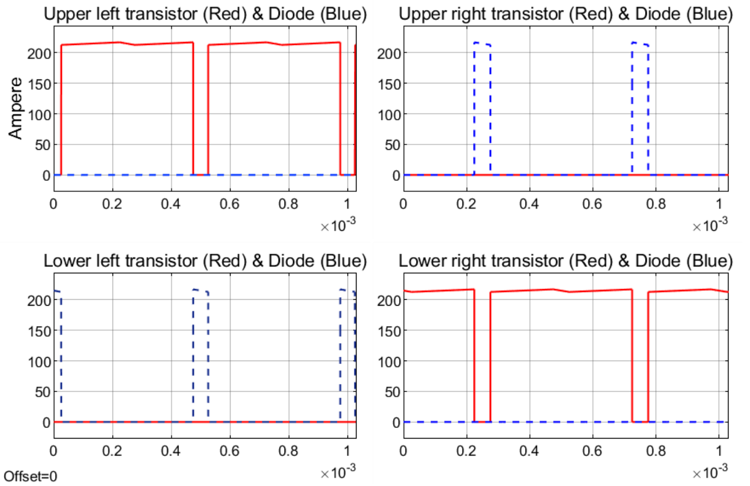
The average transistor current is

$$i_{T.ave} = D_{tr} * I_{pulse,avg} = 194\ A$$

The rms value of the transistor currents is:

$$i_{Tr,rms} = \sqrt{D_{tr} * (i_1^2 + \Delta i * i_1 + \frac{\Delta i^2}{3})} = 204\ A$$





Solution Exam 2012-05-21 1c_3

The "duty cycle" of the upper left, and lower right, diode current is:

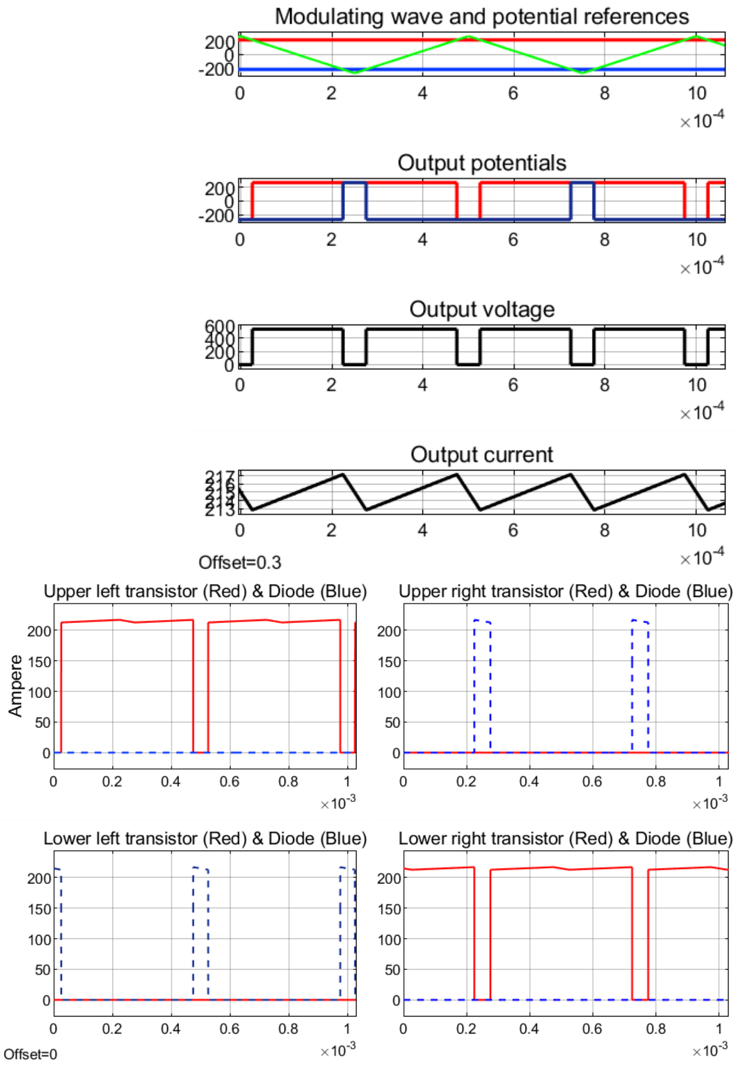
$$D_d \frac{D}{2} = 0.1$$

The average diode current is

$$i_{D,ave} = D_{tr} * I_{pulse,avg} = 21.5\ A$$

The rms value of the transistor currents is:

$$i_{D,rms} = \sqrt{D_d * (i_1^2 + \Delta i * i_1 + \frac{\Delta i^2}{3})} = 68\ A$$



Solution Exam 2012-05-21 1c_4

4QC transistor rms-current	*204A*
4QC transistor avg-current	*194A*
4QC transistor threshold voltage	*1.4 V*
4QC transistor diff resistance	*12 mohm*
4QC transistor turn-on loss	*65 mJ*
4QC transistor turn-off loss	*82 mJ*
4QC transistor thermal resistance	*0,043 K/W*

$$P_{onstate} = 1.4 \cdot 194 + 204^2 \cdot 0.012 = 771W$$

$$P_{switch} = 2000 \cdot \left(0.065 \cdot \frac{210.7}{180} + 0.082 \cdot \frac{219.3}{180} \right) \cdot \frac{540}{900} = 211W$$

$$P_{total} = 771 + 211 = 982W$$

Solution Exam 2012-05-21 1c_5

4QC diode threshold voltage 1.1 V
4QC diode diff resistance 9.5 mohm
4QC diode turn-off losses 25 mJ
4QC diode thermal resistance .078 W/K

$$\begin{cases} I_{\max} = 219.3\,A \\ I_{\min} = 210.7\,A \end{cases}$$

$$\begin{cases} I_{rms} = \sqrt{0.1\cdot\left(\dfrac{219.3^2 + 219.3\cdot 210.7 + 210.7^2}{3}\right)} = 68.0\,A \\ I_{avg} = 0.1\cdot\left(\dfrac{219.3+210.7}{2}\right) = 21.5\,A \end{cases}$$

$$\begin{cases} P_{onstate} = 1.1\cdot 21.5 + 68^2\cdot 0.0095 = 67.6W \\ P_{switch} = 2000\cdot 0.025\cdot \dfrac{210.7}{180}\cdot\dfrac{540}{900} = 35.1W \\ P_{total} = 67.6 + 35.1 = 103W \end{cases}$$

Solution Exam 2012-05-21 1c_6

Upper left IGBT transistor loss	*982 W*
Upper right IGBT transistor loss	*0 W*
Lower right IGBT transistor loss	*982 W*
Lower left IGBT transistor loss	*0 W*
Upper right IGBT diode loss	*103 W*
Upper left IGBT diode loss	*0 W*
Lower left IGBT diode loss	*103 W*
Lower right IGBT diode loss	*0 W*

Solution Exam 2012-05-21 1d

Rectifier diode (6)	IGBT diode (2)	IGBT transistor (2)
Loss each 78 W	Loss each 103 W	Loss each 982W
Rth diode 0.12 K/W	Rth diode 0.078K/W	Rth trans 0.043 K/W
Temp diff 9.4 ^{o}C	Temp diff 8.0 ^{o}C	Temp diff 42.2 ^{o}C

Heatsink

Contribution fron 6 rectifier diodes and from two IGBT.

Heatsink thermal resistance	*0.025 K/W*
Ambient temperature	*42 ^{o}C*
Total loss to heatsink	*6*78+2*103+2*982=2638 W*
Heatsink sink temperature	*42+2656*0.025=108 ^{o}C*

Junction temperature

Rectifier diode	*108+9.4=117 ^{o}C*
IGBT diode	*108+8=116 ^{o}C*
IGBT transistor	*108+42.2=150 ^{o}C*

Exam 20120521 2 - Snubbers, DC/DC converter, semiconductor

a. Draw an IGBT equipped step down chopper (buck converter)
with an RCD snubber. The dclink voltage on the supply side is 250V and the load voltage is 175 V.
Give a detailed description of how the RCD charge-discharge snubber should operate. Explain why the snubbers are needed (2 p.)

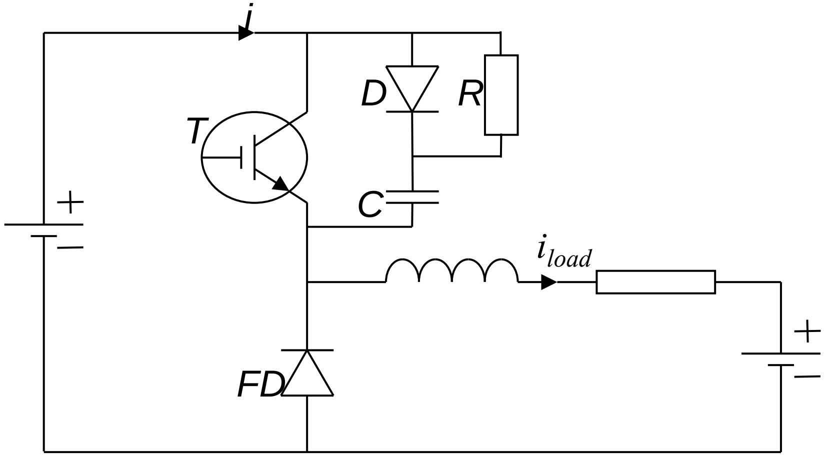
b. Calculate the snubber capacitor for the commutation time
0.012 ms.
The load current is 12 A, assumed constant during the commutation.
Calculate the snubber resistor so the discharge time (3 time constants)
of the snubber capacitor is less than the IGBT on state time.
The switch frequency is 1.5 kHz (4 p.)

Exam 20120521 2 - Snubbers, DC/DC converter, semiconductor

c. Draw the main circuit of a forward DC/DC converter.
The circuit should include DM-filter (differential mode) ,
CM (common mode)
filter, rectifier, dc link capacitors, switch transistor
and a simple output filter.
The circuit should also include snubbers. (2 p.)

d. Draw the diffusion structure of a MOSFET. In the figure the different doping areas
must be found. Draw where in the structure the unwanted
stray transistor effect can be found. What is done to avoid
this effect.
Also draw where in the structure
the anti-parallel diode effect can be found. (2 p.)

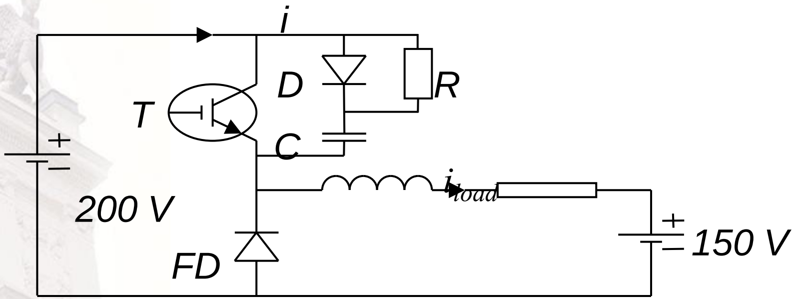
Solution Exam 2012-05-21 2a



At turn off of transistor *T*, the current *i* commtutates over to the capacitor *C* via diode *D*. The capacitor *C* charges until the potential of the transistor emitter reduces till the diode *FD* becomes forward biased and thereafter the load current i_{load} flows through diode *FD* and the current $i=0$.

A turn on of the transistor *T,* the capacitor *C* is discharged via the the transistor *T* and resistor *R*. *T*he diode *FD* becomes reverse biased and the current *i* commutates to the transistor *T*.

Solution Exam 2012-05-21 2b



Load current	*12 A*
Supply voltage	*250 V*
Load voltage	*175 V*
Commutation time	*0.012 ms*
Switching frequency	*1.5 kHz*

At turn off of transistor *T*, the capacitor *C* charges until the potential of the transistor emitter reduces till the diode *FD* becomes forward biased and thereafter the load current commutates to the freewheeling diode.

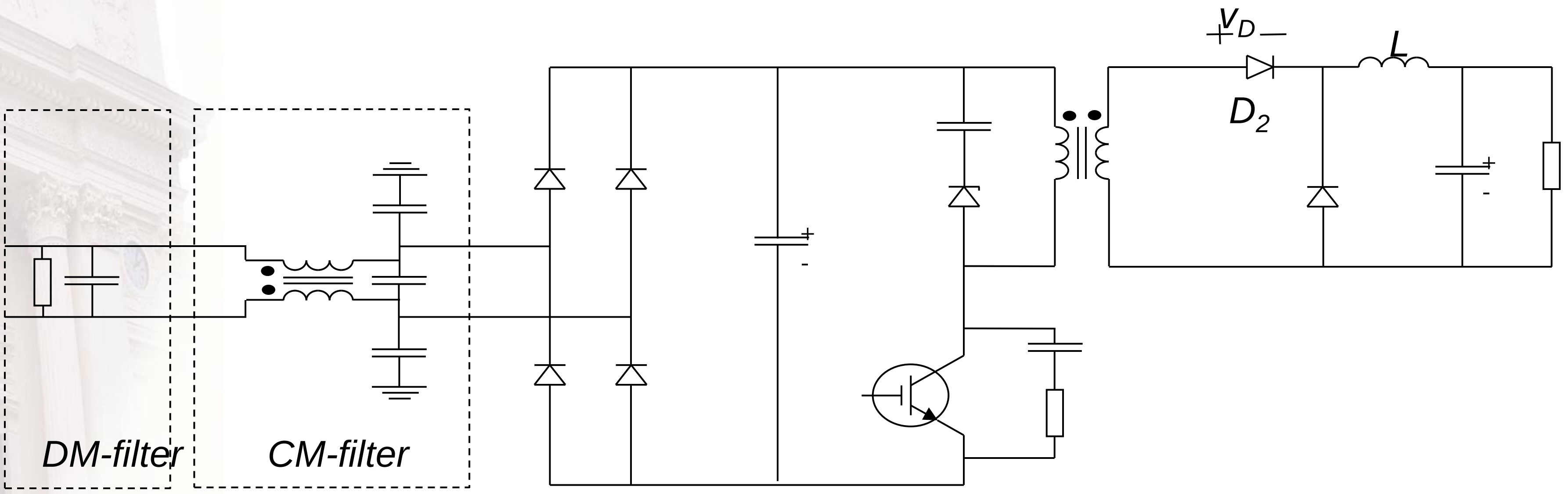
$$i = C \cdot \frac{du}{dt} \quad \Rightarrow \quad C = \frac{i \cdot dt}{du} = \frac{12 \cdot 12 \cdot 10^{-6}}{250} = 0.58\,\mu F$$

A turn on of the transistor *T*the current *i* commutates to the transistor *T*, and the capacitor *C* is discharged via the the transistor *T* and resistor *R*. As the load voltage is 175V the duty cycle is 70%. The switching frequency is 1.5 kHz and the on state time is 0.47 ms, and thus the time constant =0.16 ms

$$\tau = C \cdot R \quad \Rightarrow \quad R = \frac{\tau}{C} = \frac{156 \cdot 10^{-6}}{0.58 \cdot 10^{-6}} = 269\,\Omega$$

Solution Exam 2012-05-21 2c

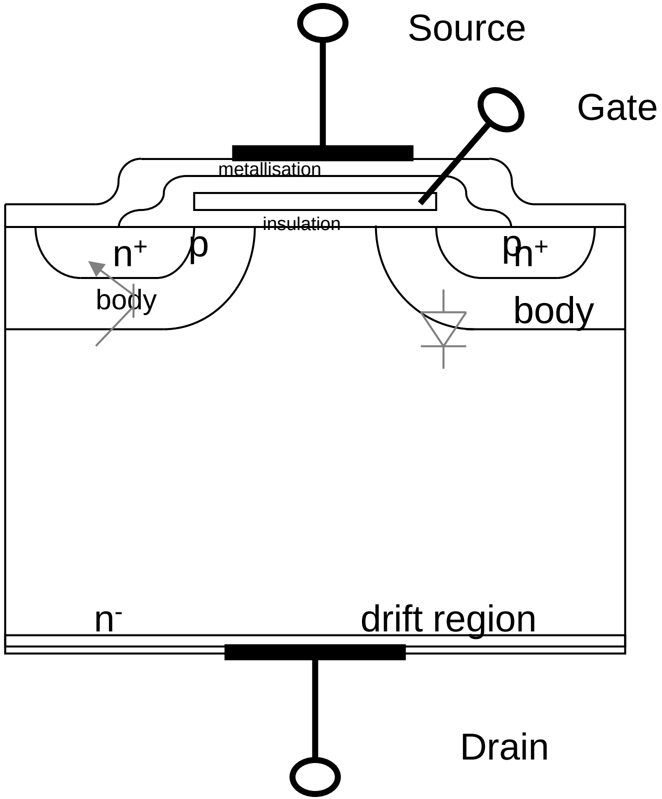
Forward converter with snubbers
and common mode (CM) and differential mode (DM) filter



Solution Exam 2012-05-21 2d

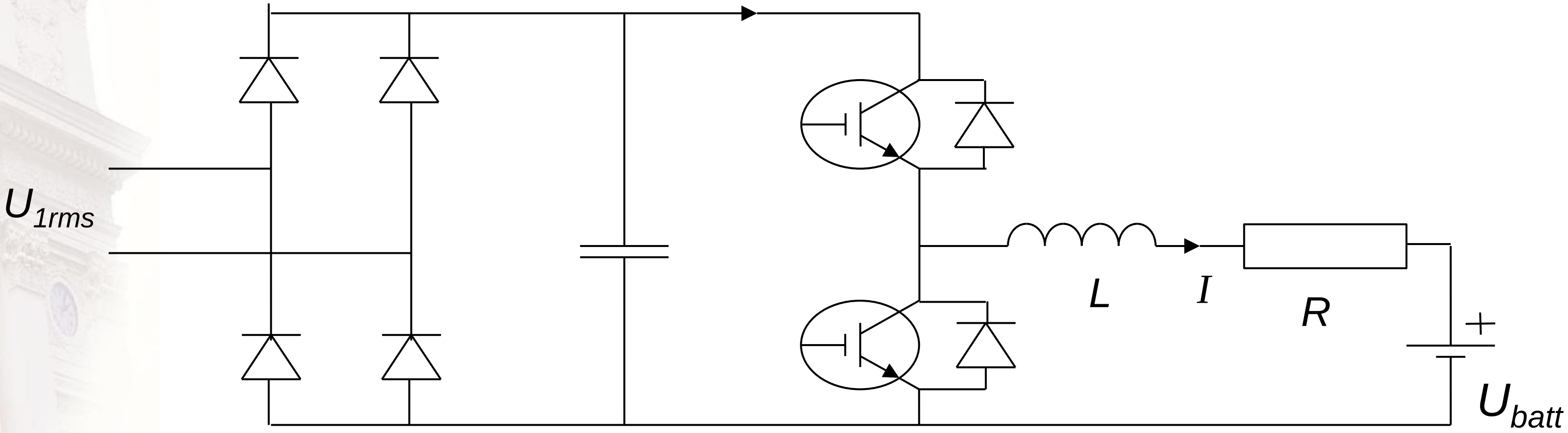
Diffusion structure of a MOSFET

The npn-transistor structure is formed of the n^+, the p (body) and the n^- (drift region), which cannot be turned off.
The gate metallisation short circuits the n+ and the p (body) to avoid turning on this unwanted transistor



Exercise Exam 20120521 3_1 - The buck converter as battery charger

A battery charger is supplied from a symmetrical single phase system.
A dc voltage is created by a two pulse diode bridge and a 2-quadrant dc-converter is used for the charge current control.



Data: U_{1rms}= the phase-voltage rms value = 220 V, 50 Hz.
The switching frequency is f = 4 kHz.
L = 4 mH and R=0 Ohms.
U_{batt} = 100 V and is approximated to be independent of the charge current.

Exam 20120521 3_2 - The buck converter as battery charger

a) What dc link voltage *Ud* will you get I) when the charging current is zero and II) when the charging current is non-zero with a perfectly smooth rectified current ? (2p)

b) Start with the electrical equation for the load and derive a suitable current control algorithm, giving all approximations you use.
(4p)

c) Draw a current step from 0 A till 10 A in the load current. The modulating wave (*um*), the voltage reference (*u**), the output voltage (*u*) and current (*ibatt*) must be shown. Indicate the sampling frequency you use in relation to the switching frequency. (4p)

Solution Exam 2012-05-21 3a

Average dc voltage with average dc current

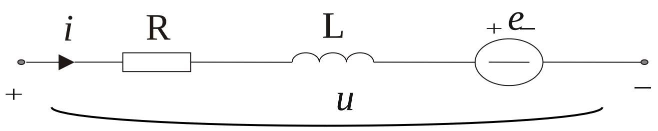
$$U_{dc\_ave} = \frac{2}{\pi} \cdot 220 \cdot \sqrt{2}\ V = 198\ V$$

Max dc voltage with zero dc current

$$U_{dc\_\max} = 220 \cdot \sqrt{2} V = 311 V$$

Solution Exam 20120521 3b_1

Current controller with fast computer



$$u = R \cdot i + L \cdot \frac{di}{dt} + e$$

$$\frac{\int\limits_{k \cdot T_s}^{(k+1)T_s} u \cdot dt}{T_s} = \frac{R \cdot \int\limits_{k \cdot T_s}^{(k+1)T_s} i \cdot dt + L \cdot \int\limits_{k \cdot T_s}^{(k+1)T_s} \frac{di}{dt} \cdot dt + \int\limits_{k \cdot T_s}^{(k+1)T_s} e \cdot dt}{T_s}$$

$$\overline{u}(k, k+1) = R \cdot \overline{i}(k, k+1) + L \cdot \frac{i(k+1) - i(k)}{T_s} + \overline{e}(k, k+1)$$

Solution Exam 20120521 3b_2

$$\bar{u}(k,k+1) = u^*(k) \qquad (a)$$

$$i(k+1) = i^*(k) \qquad (b)$$

$$\bar{i}(k,k+1) = \frac{i^*(k) + i(k)}{2} \qquad (c)$$

$$\bar{e}(k,k+1) = e(k) \qquad (d)$$

$$i(k) = \sum_{n=0}^{n=k-1} \left(i^*(n) - i(n)\right) \qquad (e)$$

$$u^*(k) = \{R=0\} = L \cdot \frac{i^*(k) - i(k)}{T_s} + e(k) = \frac{L}{T_s} \cdot \underbrace{\left(i^*(k) - i(k)\right)}_{Proportional} + \underbrace{e(k)}_{\substack{Feed \\ forward}}$$

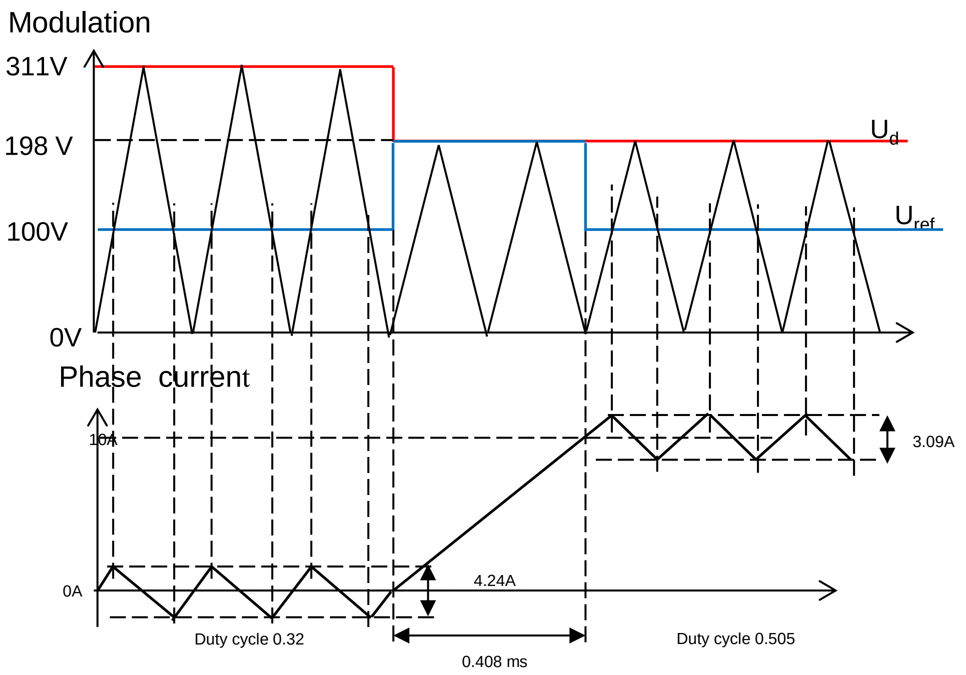
$$u^*(k) = \{R=0\} = L \cdot \frac{i^*(k) - i(k)}{T_s} + e(k) = \frac{L}{T_s} \cdot \underbrace{\Delta i}_{Proportional} + \underbrace{e(k)}_{\substack{Feed \\ forward}}$$

Solution Exam 20120521 3c

<u>*Constant 0 A*</u>
*Rectifier dc-voltage 220*1.414=311 V*
Voltage ref with const 0 A =100 V
Duty cycle 100/311=0.32
*On pulse 0.25 *0.32=0.080 ms*
*Current ripple =(311-100)/0.004*0.00008=4.24A*

<u>*Load current 0 to 10 A*</u>
*Rectifier dc-voltage 2/3.14*220*1.414=198 V*
Inductive voltage drop at current step =98 V Time to reach 10 A t=10.004/98=0.408 ms*
More than one sample time, set duty cycle =1

<u>*Constant 10 A*</u>
*Rectifier dc-voltage 2/3.14*220*1.414=198 V*
Duty cycle with 10 A = 100/198=0.505
*On pulse 0.25 *0.505=0.126 ms*
Voltage ref at const 10 A=100 V
Inductive voltage drop at current step =98 V
Current ripple=(198-100)/.004.000126=3.09 A*



Exam 20120521 4 - 4Q Converter & 3 Phase

In a 4Q DC/DC converter using PWM bipolar voltage switching, the bridge load consist of a constant voltage E (e.g. the back emf of a dc-motor) and an inductor L_a, the inductor resistance can be neglected.
The switching frequency is f_s, and the DC link voltage is V_d.

a. Calculate the maximum peak-to-peak load current ripple, expressed in V_d, L_a and f_s,. (5 p.)
b. Draw the circuit of a current control block for a generic three phase RLE load.
The drawing shall include three phase inverter, reference and load current measurement.
It must be clear in which blocks the different frame transformations occur. (5 p.)

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Solution Exam 2012-05-21 4a

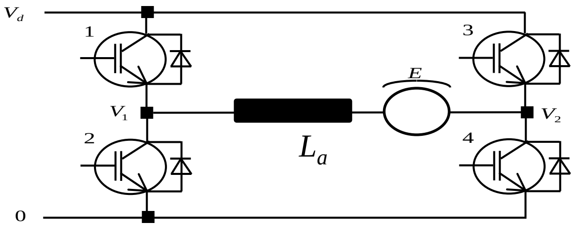


Fig 1

Control ratio $\quad x$

On – pulse duration $\quad \Delta t = x \cdot T_{s\_per} = \dfrac{x}{f_s}$

Phase voltages $\quad V_{1\_avg} = x \cdot V_d$

$$V_{2\_avg} = (1-x) \cdot V_d = V_d - x \cdot V_d$$

Voltage over motor $\quad e = V_{1\_avg} - V_{2\_avg} = x \cdot V_d - V_d + x \cdot V_d = 2 \cdot V_d \cdot x - V_d$

At current rise, switch 1
and 4 are turned – on $\quad V_1 = V_d$

$$V_2 = 0$$

Voltage over inductor $\quad V_L = V_1 - e - V_2 = V_d - e = V_d - 2 \cdot V_d \cdot x + V_d = 2 \cdot V_d \cdot (1-x)$

Current ripple via equation $\quad V_L = L\dfrac{di}{dt} \Rightarrow \Delta i = \dfrac{V_L \cdot \Delta t}{L} \;\Rightarrow\; \Delta i = \dfrac{2 \cdot V_d \cdot (1-x)}{L_a} \cdot \dfrac{x}{f_s} = \dfrac{2 \cdot V_d \left(x - x^2\right)}{f_s \cdot L_a}$

it's derivative $\quad \dfrac{\partial(\Delta i)}{\partial x} = \dfrac{2 \cdot V_d}{f_s \cdot L_a} \cdot (1-2x) \Rightarrow \dfrac{\partial(\Delta i)}{\partial x} = 0 \text{ when } x = 0.5$

it's second derivative $\quad \dfrac{\partial^2(\Delta i)}{\partial x^2} = -\dfrac{4 \cdot V_d}{f_s \cdot L_a} < 0 \Rightarrow \max \text{ at } x = 0.5$

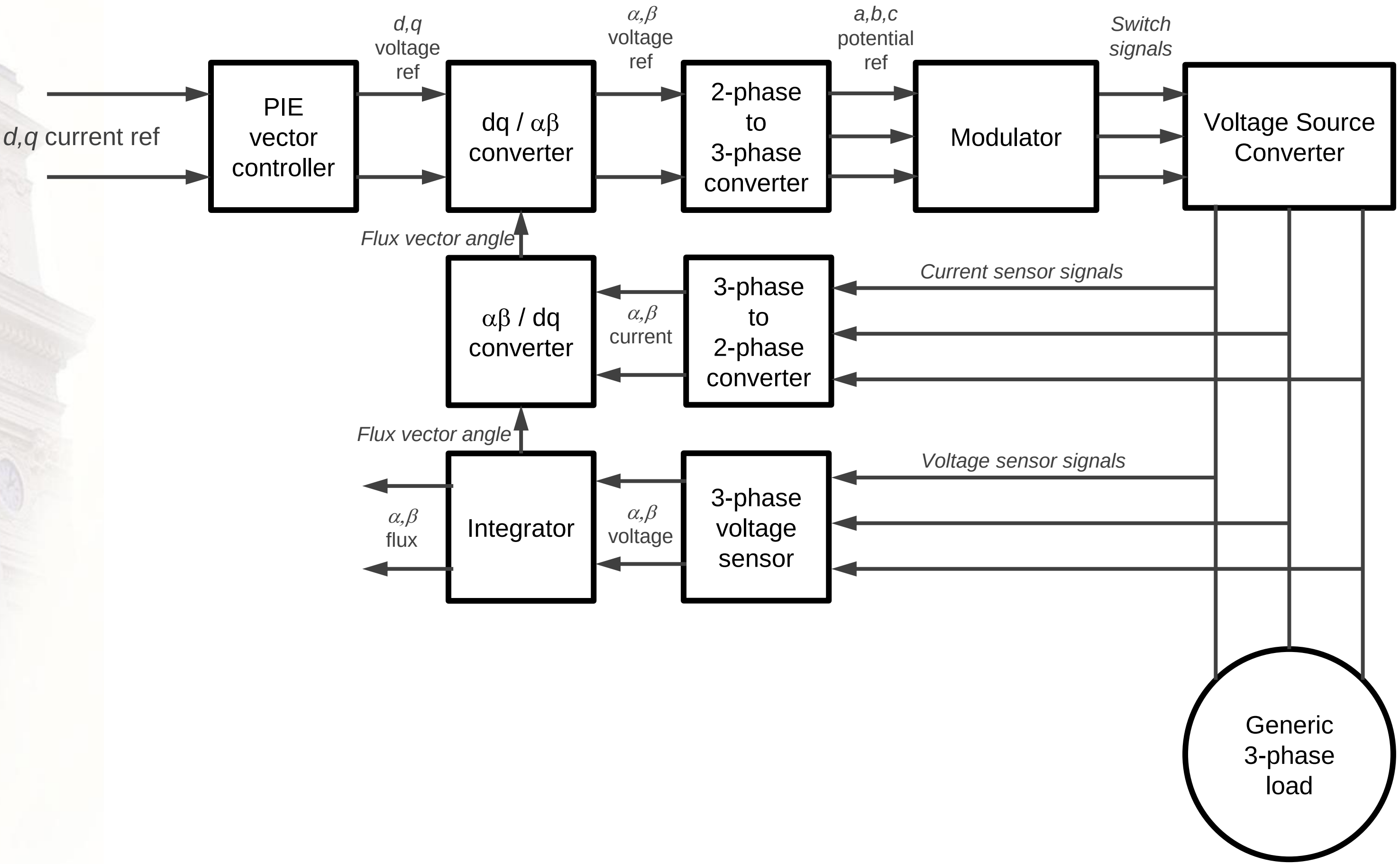
Phase voltages at max $\quad V_{1\_avg} = 0.5 \cdot V_d = 0.5 \cdot V_d$

$$V_{2\_avg} = (1-0.5) \cdot V_d = 0.5 \cdot V_d$$

$$e = V_{1\_avg} - V_{2\_avg} = 0.5 \cdot (V_d - V_d) = 0 = \frac{0}{V_d}$$

Max current ripple $\quad \Delta i_{\max} = \dfrac{2 \cdot V_d \cdot (1-0.5)}{L_a} \cdot \dfrac{0.5}{f_s} = \dfrac{V_d}{2 f_s L_a}$

Solution Exam 20120521 4b



Exam 20120521 5

A permanently magnetized synchronous machine with $L_{sy}>L_{sx}$ is used as a traction motor in an electric vehicle.

a. Draw the torque expression in rotor coordinates, and describe your interpretation of the terms in the expression, and how they relate to the rotor geometry and magnetization. (4p)

b. Explain, in a qualitative sense, what is the best locus for the stator current vector to minimize the amount of current needed for torque production. (3p)

c. Explain the restrictions to the stator current loci that are imposed when the need for stator voltage is higher than the maximum available voltage. (3p)

Solution Exam 20120521 5a_1

$$T = \vec{\psi}_s \, x \, \vec{i}_s = \psi_m x i_{sy} + \left(L_{mx} - L_{my}\right) \cdot i_{sx} \cdot i_{sy} \qquad (11.5)$$

ψ_m *is the permanent magetization along the positive $x-$axes*

i_{sx} *is the current along the permanent magetization*

i_{sy} *is the current perpendicular to the permanent magetization,*

$\pi/2$ *in positive direction*

L_{mx} *is the inductance in the $x-$direction*

L_{my} *is the inductance in the $x-$direction*

The more iron and the smaller the airgap in the

$x-$or $y-$direction, the higher is the inductance is that dircction

The permanent magetization material has no impact on the inductance

Solution Exam 20120521 5a_2

See the torque equation, the first part of the torque is achieved when the permanent flux ψ_m *is multiplied with the current* i_{sy}.
The second part of the torque is the so called reluctance torque. E.g. At high speed the drive system is in field weakening, and the permanent magnetisation must be reduced, which is done with a negative i_{sx}.
If $L_{mx} < L_{my}$ *the difference* $L_{mx} - L_{my}$ *is negative. When this difference is multiplied with the negative* i_{sx} *and the positive* i_{sy} *the result is a positive torque, called the reluctance torque.*

Solution Exam 20120521 5b

$$T = \vec{\psi}_s x \vec{i}_s = \psi_m x i_{sy} + \left(L_{mx} - L_{my}\right) \cdot i_{sx} \cdot i_{sy}$$

The first torque and the reluctance torque are added to the total torque, which can be achieved with different combinations of i_{sx} *and* i_{sy}.
The combination which gives the lowest

$$absolute\ sum\ of\ i_{sx}\ and\ i_{sy} = \sqrt{{i_{sx}}^2 + {i_{sy}}^2}$$

is the optimal combination of i_{sx} *and* i_{sy} *for a certain torque.*

Solution Exam 20120521 5c

We want to increase the voltage, more than the available voltlage.

This can be achieved by weaken the field further, by increasing the negative current i_{sx}.
However, this results in an increased total current, beyond the max current loci.

So, we have to reduce the i_{sy}, *to fullfill the the maximum current loci.*

See chapter 11.5

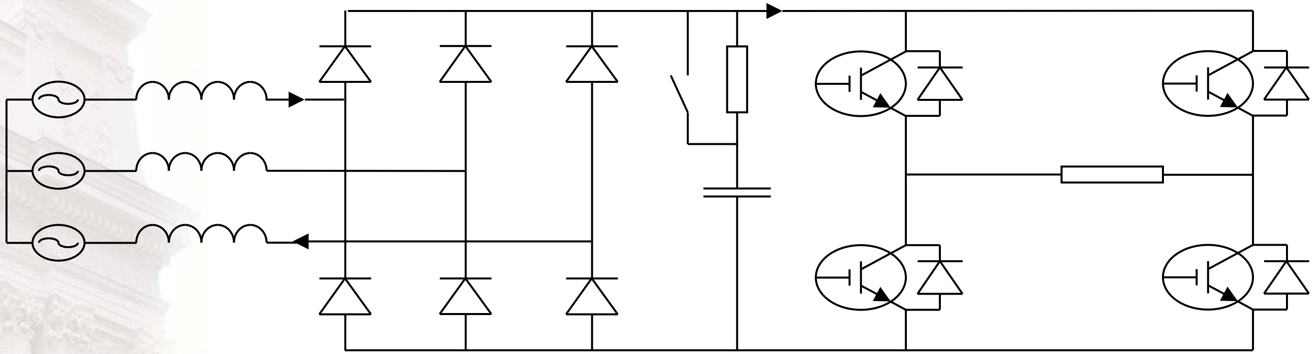
Exam 2014-05-30

Exam 20140530 1a

The four quadrant DC-DC converter

Draw a four quadrant DC DC converter with a three phase diode rectifier connected to the power grid. The Dc link capacitor and protection against too high inrush currents should be included in the drawing. The transistors are of IGBT-type. (2 p.)

Solution Exam 2014-05-30 1a



Exercise Exam 20140530 1b

The three-phase grid, to which the three phase diode rectifier is connected, has the line-to-line voltage 400 V_{rms} at 50 Hz. Calculate the dc output voltage and the maximum dc link voltage from the rectifier.

(1 p.)

Solution Exam 2014-05-30 1b

Maximum dc voltage

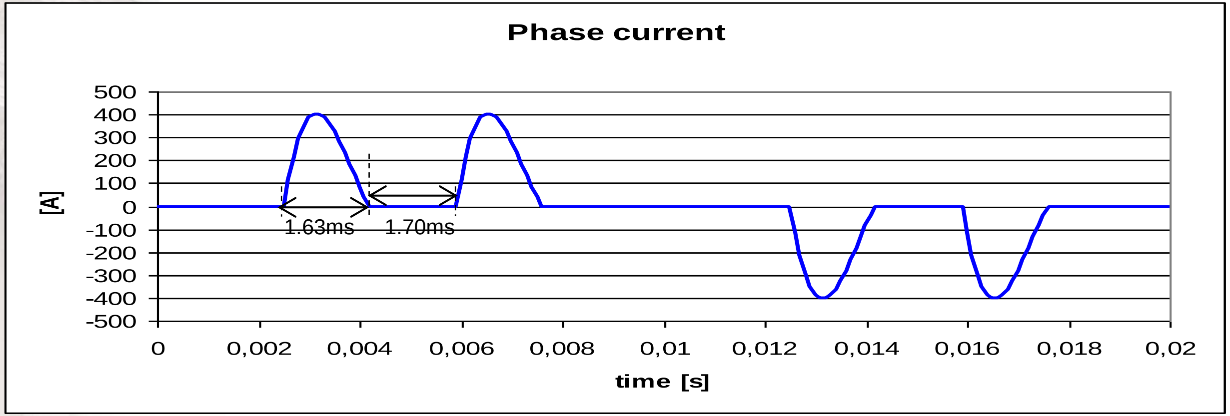
$$U_{dc\max} = 400 \cdot \sqrt{2}\ V = 566\ V$$

Average dc voltage

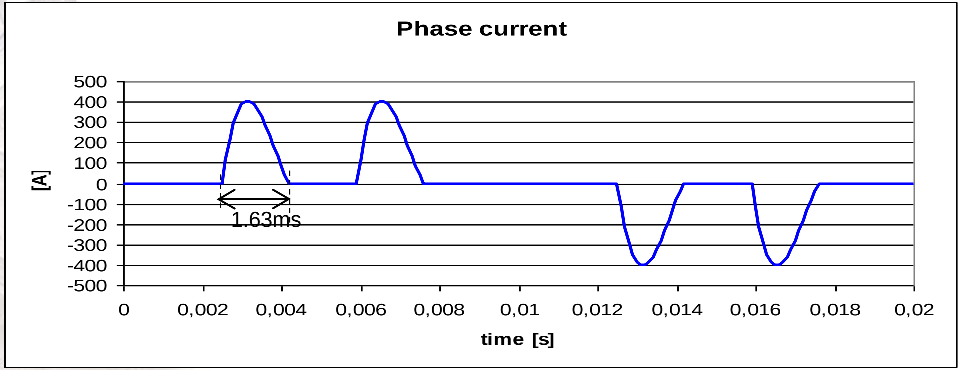
$$U_{dc\_ave} = \frac{3}{\pi} \cdot 400 \cdot \sqrt{2}\ V = 540\ V$$

Exam 20140530 1c

Calculate the rms-current and the average current through one rectifying diode (see figure 1). Calculate the rectifier diode losses. The diode threshold voltage is 1.1 V and the differential resistance is 2.0 mohm. (2 p.)



Solution Exam 2014-05-30 1c



Rectifier diode	
Threshold voltage	*1.1 V*
Differential resistance	*2.0 mohm*
I_{rms}	*114.2 A*
Average current	*41,5 A*

$$I_{diode\,rms} = \sqrt{\frac{2 \cdot 0.00163}{0.02} \cdot \left(\frac{400}{\sqrt{2}}\right)^2} = 114.2A$$

$$I_{diode\,ave} = \left\{ Average\,of\ \sin us = \frac{\int_0^\pi \sin(x)dx}{\pi} = \frac{(\cos(0) - \cos(\pi))}{\pi} = \frac{2}{\pi} \approx 0.637 \right\} = \frac{2 \cdot 0.00163}{0.02} \cdot 0.637 \cdot 400 = 41.5A$$

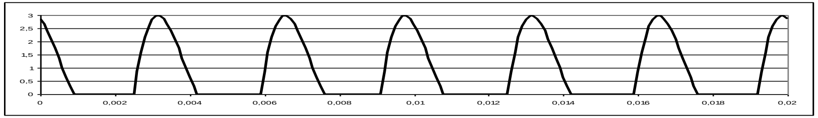
Rectifier diode power loss

$$P_{rectifier\ diode} = V_{threshold} \cdot I_{ave} + R_{diff} \cdot I_{rms}^2 = 1.1 \cdot 41.5 + 0.002 \cdot 114.2^2 = 71.7\ W$$

Exam 20140530 1d

Calculate the IGBT component losses of each IGBT in the four quadrant converter.
The duty cycle of the converter is 70%.
The switching frequency is 2.5 kHz.
The threshold voltage of the IGBT transistor equals 1.6 V
and its differential resistance equals 1.0 mohm.
The turn-on loss of the IGBT transistor equals 65 mJ and
its turn-off loss equals 82 mJ.
These turn-on and turn-off losses are nominal values
at 900 V dclink voltage and 180 A turn-on and turn-off current.
The threshold voltage of the IGBT diode equals 1.0 V and
the differential resistance of this diode equals 10 mohm.
The IGBT diode turn-on can be neglected and its
turn-off losses equals 25 mJ, at 900 V dclink voltage and 180 A. (3 p.)

Solution Exam 2014-05-30 1d



Dc current to the dc link $I_{dc} = \dfrac{6 \cdot 0.00163 \cdot 400}{0.02} \cdot 0.637 = 124.6A$

Duty cycle	*70%*
IGBT and diode on state current	*124.6/0.7= 178 A*
Conduction percentage of IGBT transistor (incl freewheeling)	*70+30/2=85%*
Conduction percentage of IGBT diode (when freewheeling) 30/2=15%	
Switching frequency	*2,5 kHz*
IGBT transistor	
Threshold voltage	*1.6V*
Differential resistance	*1.0 mohm*
On state voltage at 178 A	*1.78 V*
Turn on energy at 900 V and 180 A	*65 mJ*
Turn off energy at 900 V and 180 A	*82 mJ*
IGBT diode	
Threshold voltage	*1.0 V*
Differential resistance	*10. mohm*
On state voltage at 178 A 2.78 V	
Turn on energy at 900 V and 180 A	*0 mJ*
Turn off energy at 900 V and 180 A	*25 mJ*

Power loss

$$\begin{cases} P_{trans\_loss} = 1.78 \cdot 178 \cdot 0.85 + 2500 \cdot \dfrac{(65+82) \cdot 10^{-3} \cdot 540 \cdot 178}{900 \cdot 180} = 487W \\ P_{diode\_loss} = 2.78 \cdot 178 \cdot 0.15 + 2500 \cdot \dfrac{25 \cdot 10^{-3} \cdot 540 \cdot 178}{900 \cdot 180} = 111.3W \end{cases}$$

Exam 20140530 1e

Which is the junction temperature of the IGBT transistor and of the IGBT diode,
and which is the junction temperature of the rectifying diodes?
The thermal resistance of the heatsink equals 0.024 K/W?
The thermal resistance of the IGBT transistor equals 0.07 K/W?
The thermal resistance of the IGBT diode equals 0.16 K/W?
The thermal resistance of the rectifier diode equals 0.14 K/W?
The ambient temperature is 35 ^{o}C.
The rectifier diodes and the four quadrant converter IGBTs share the heatsink. (2 p.)

Solution Exam 2014-05-30 1e

Rectifier diode (6)	IGBT diode (2)	IGBT transistor (2)
Loss each 71.7W	Loss each 111.3 W	Loss each 487 W
Rth diode 0.14 C/W	Rth diode 0.16 C/W	Rth trans 0.07C/W
Temp diff 10.0 ^oC	Temp diff 17.8 ^oC	Temp diff 34.1 ^oC

Heatsink

Contribution fron 6 rectifier diodes and from two IGBT.

Ambient temperature	*35 ^oC*
Total loss to heatsink	*6*71.7+2*487+2*111.3=1627 W*
Rth heatsink	*0.024 C/W*
Temperature heatsink	*1627 *0.024+35=74 ^oC*

Junction temperature

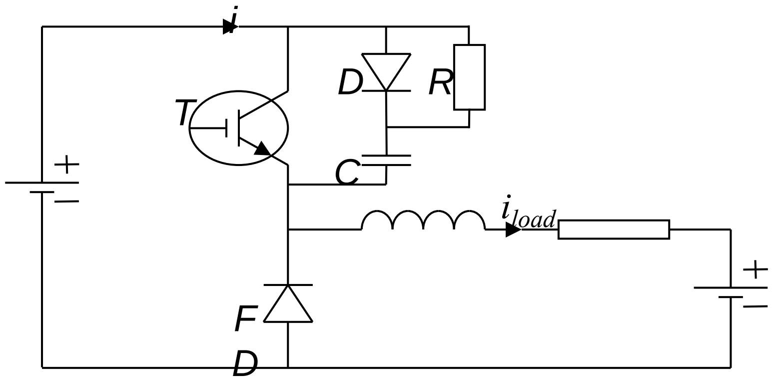
Rectifier diode	*74 +10.0 = 84 ^oC*
IGBT diode	*74 +17.8 = 92 ^oC*
IGBT transistor	*74 +34.1 = 108 ^oC*

Exam 20140530 2a

Snubbers

Draw an IGBTequipped step down chopper (buck converter) with an RCD snubber.
The dclink voltage on the supply side is 200V and the load voltage is 150 V.
Give a detailed description of how the RCD charge-discharge snubber should operate.
Explain why the snubbers are needed (2 p.)

Solution Exam 2014-05-30 2a



The buck converter with RCD snubber

<u>At turn off</u> of transistor T, the current i commtutates over to the capacitor C via diode D. The capacitor C charges until the potential of the transistor emitter reduces till the diode FD becomes forward biased and thereafter the load current i_{load} flows through diode FD and the current i=0.

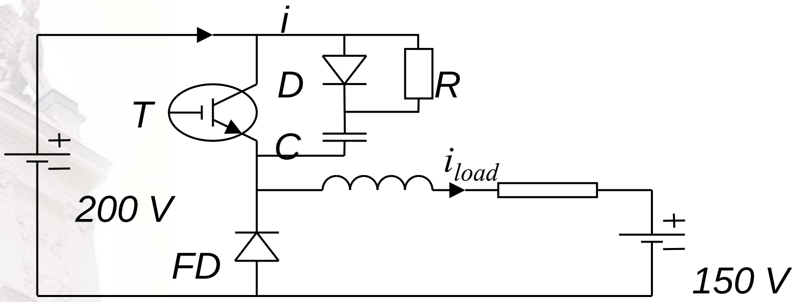
<u>A turn on</u> of the transistor T, the capacitor C is discharged via the transistor T and resistor R. The diode FD becomes reverse biased and the current i commutates to the transistor T.

Exercise Exam 20140530 2b

Snubbers

Calculate the snubber capacitor for the commutation time 0.01 ms.
The load current is 12 A, assumed constant during the commutation.
Calculate the snubber resistor so the discharge time (3 time constants) of the snubber capacitor is less than the IGBT on state time.
The switch frequency is 1.5 kHz (4 p.)

Solution Exam 2014-05-30 2b



Load current	*12 A*
Supply voltage	*200 V*
Load voltage	*150 V*
Commutation time	*0.01 ms*
Switching frequency	*1.5 kHz*

At turn off of transistor T, the capacitor C charges until the potential of the transistor emitter reduces till the diode FD becomes forward biased and thereafter the load current commutates to the freewheeling diode.

$$i = C \cdot \frac{du}{dt} \;\Rightarrow\; C = \frac{i \cdot dt}{du} = \frac{12 \cdot 10 \cdot 10^{-6}}{200} = 0.6\,\mu F$$

A turn on of the transistor the current i commutates to the transistor T, and the capacitor C is discharged via the the transistor T and resistor R. As the load voltage is 150V the duty cycle is 75%. The switching frequency is 1.5 kHz and the on state time is 0.5 ms, and thus the time constant =0.17 ms

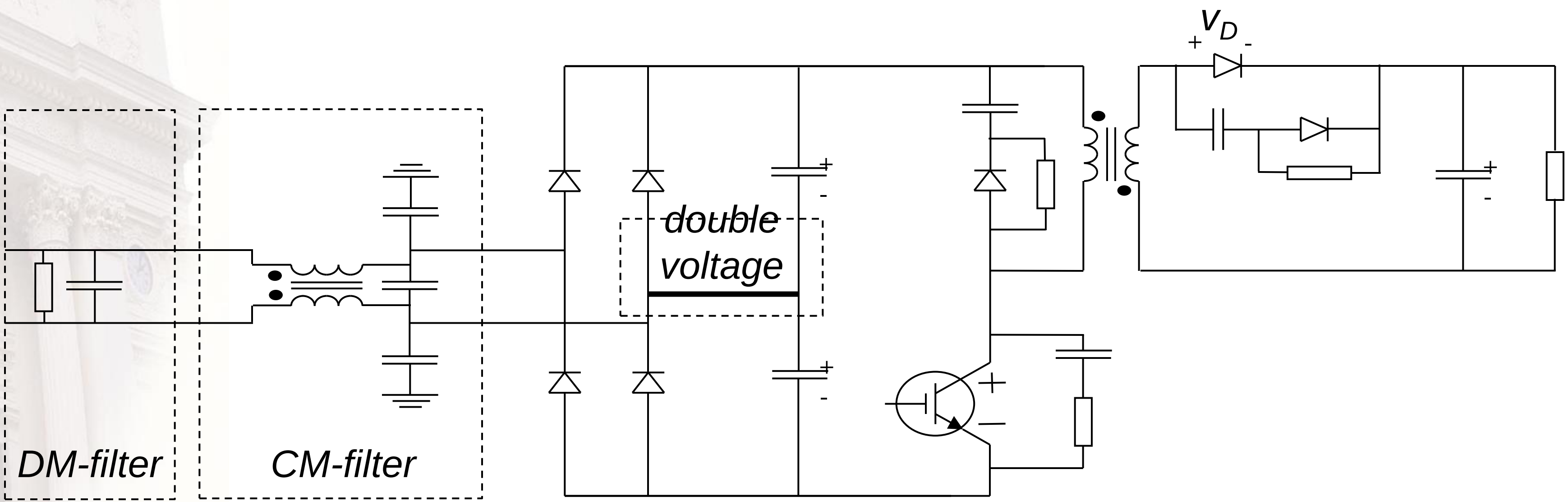
$$\tau = C \cdot R \;\Rightarrow\; R = \frac{\tau}{C} = \frac{170 \cdot 10^{-6}}{0.6 \cdot 10^{-6}} = 283\Omega$$

Exam 20140530 2c

Snubbers

Draw the main circuit of a flyback converter. The circuit should include DM-filter (differential mode) ,CM (common mode) filter, rectifier, dc link capacitors,
alternative connection for voltage doubling connection, switch transformer
(one primary and one secondary winding is enough), switch transistor, flyback
diode and a simple output filter, The circuit should also include snubbers. (2 p.)

Solution Exam 2014-05-30 2c



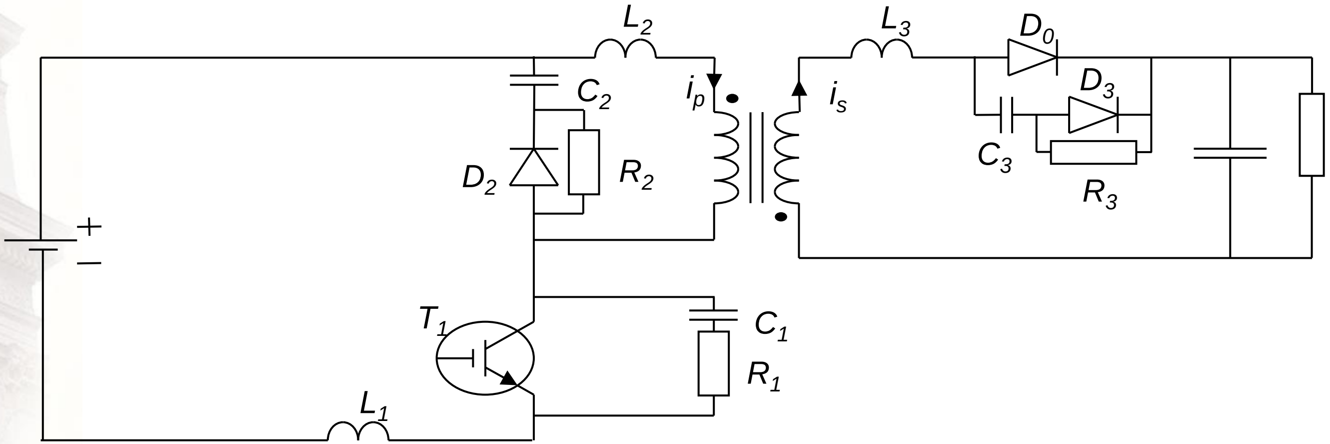
Exam 20140530 2d

Snubbers

Describe, in detail, the operation of the flyback converter snubbers you have used. Describe in detail how the current is flowing in the snubber and the voltages n the snubber (2 p.)

Solution Exam 2014-05-30 2d

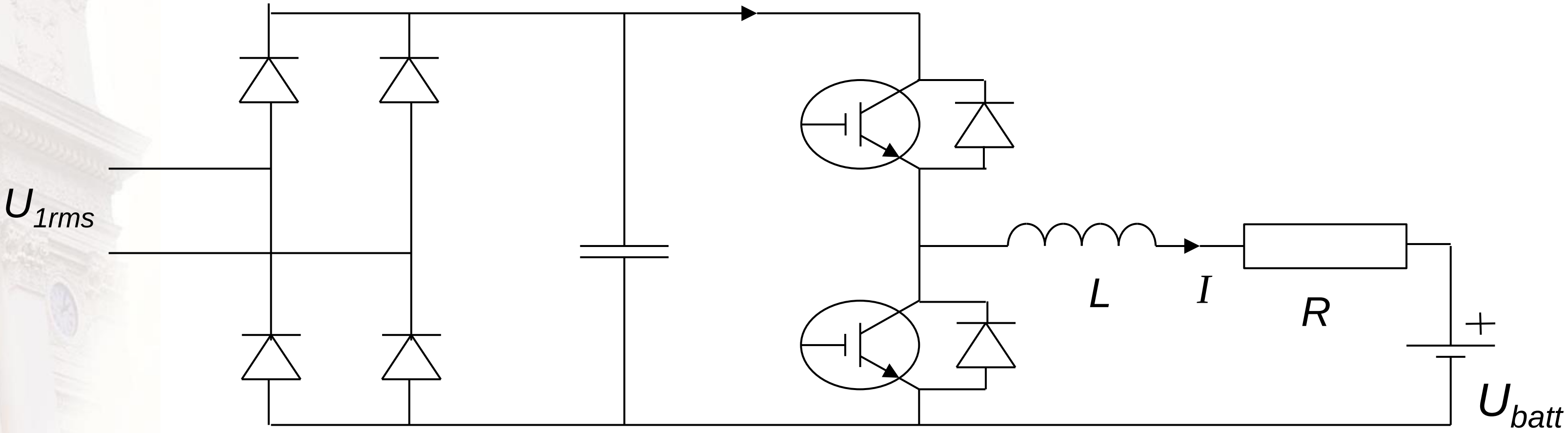
Fly-back converter withSnubber operation



For the description of the snubber operation the stray inductance L_1 between the switch transistor and the supply/dclink, and the transformer leakage inductance, L_2 on primary side and L_3 on secondary side are added as discrete component in the circuit drawing above.

Exam 2014-05-30 3_1

The buck converter as battery charger
A battery charger is supplied from a symmetrical single phase system.
A dc voltage is created by a two pulse diode bridge and a 2-quadrant dc-converter is used for the charge current control.



Data: U_{1rms}= the phase-voltage rms value = 220 V, 50 Hz.
The switching frequency is f = 4 kHz.
L = 4 mH and R=0 Ohms.
U_{batt} = 100 V and is approximated to be independent of the charge current.

The buck converter as battery charger

a) What dc link voltage *Ud* will you get I) when the charging current is zero and II) when the charging current is non-zero with a perfectly smooth rectified current ? (2p)

b) Start with the electrical equation for the load and derive a suitable current control algorithm, giving all approximations you use. (4p)

c) Draw a current step from 0 A till 10 A in the load current. The modulating wave (*um*), the voltage reference (*u**), the output voltage (*u*) and current (*ibatt*) must be shown. Indicate the sampling frequency you use in relation to the switching frequency. (4p)

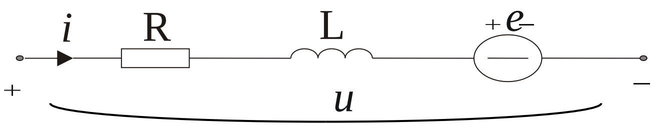
Solution Exam 2014-05-30 3a

Average dc voltage with average dc current $U_{dc\_ave} = \frac{2}{\pi} \cdot 220 \cdot \sqrt{2}\ V = 198\ V$

Max dc voltage with zero dc current $U_{dc\_\max} = 220 \cdot \sqrt{2} V = 311 V$

Solution Exam 2014-05-30 3b_1

Current controller with fast computer



$$u = R \cdot i + L \cdot \frac{di}{dt} + e$$

$$\frac{\int_{k \cdot T_s}^{(k+1)T_s} u \cdot dt}{T_s} = \frac{R \cdot \int_{k \cdot T_s}^{(k+1)T_s} i \cdot dt + L \cdot \int_{k \cdot T_s}^{(k+1)T_s} \frac{di}{dt} \cdot dt + \int_{k \cdot T_s}^{(k+1)T_s} e \cdot dt}{T_s}$$

$$\bar{u}(k, k+1) = R \cdot \bar{i}(k, k+1) + L \cdot \frac{i(k+1) - i(k)}{T_s} + \bar{e}(k, k+1)$$

Solution Exam 2014-05-30 3b_2

$$\bar{u}(k,k+1) = u^*(k) \qquad (a)$$

$$i(k+1) = i^*(k) \qquad (b)$$

$$\bar{i}(k,k+1) = \frac{i^*(k) + i(k)}{2} \qquad (c)$$

$$\bar{e}(k,k+1) = e(k) \qquad (d)$$

$$i(k) = \sum_{n=0}^{n=k-1} \left(i^*(n) - i(n) \right) \qquad (e)$$

$$u^*(k) = \{R = 0\} = L \cdot \frac{i^*(k) - i(k)}{T_s} + e(k) = \frac{L}{T_s} \cdot \underbrace{\left(i^*(k) - i(k) \right)}_{\text{Proportional}} + \underbrace{e(k)}_{\substack{Feed \\ forward}}$$

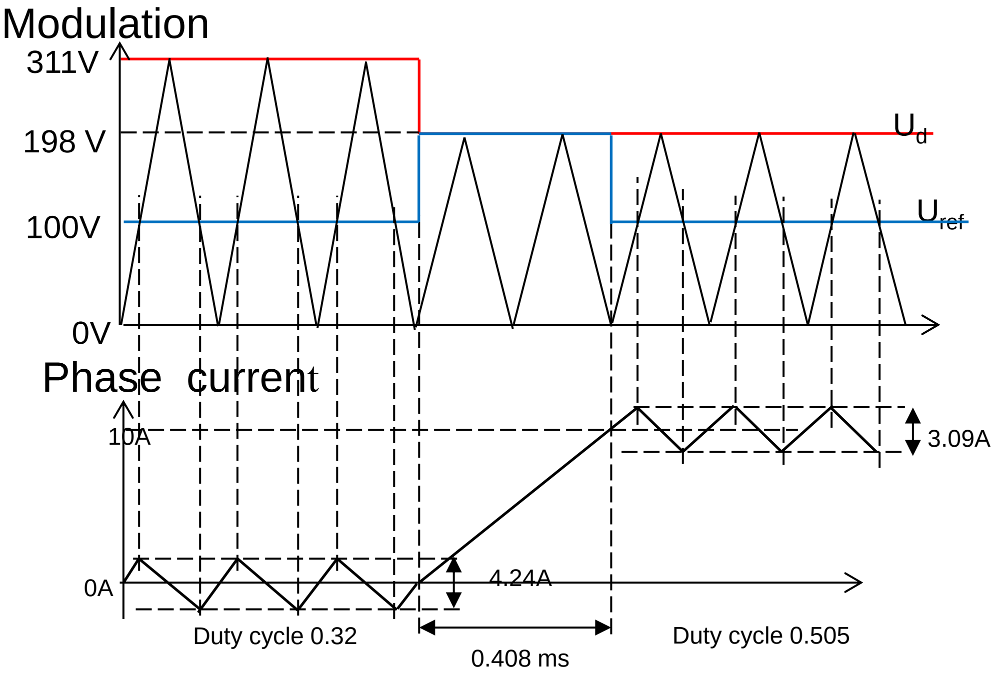
$$u^*(k) = \{R = 0\} = L \cdot \frac{i^*(k) - i(k)}{T_s} + e(k) = \frac{L}{T_s} \cdot \underbrace{\Delta i}_{\text{Proportional}} + \underbrace{e(k)}_{\substack{Feed \\ forward}}$$

Solution Exam 2014-05-30 3c

Constant 0 A
*Rectifier dc-voltage 220*1.414=311 V*
Voltage ref with const 0 A =100 V
Duty cycle 100/311=0.32
*On pulse 0.25 *0.32=0.080 ms*
*Current ripple =(311-100)/0.004*0.00008=4.24A*

Load current 0 to 10 A
*Rectifier dc-voltage 2/3.14*220*1.414=198 V*
Inductive voltage drop at current step =98 V Time to reach 10 A
t=10.004/98=0.408 ms*
More than one sample time, set duty cycle =1

Constant 10 A
*Rectifier dc-voltage 2/3.14*220*1.414=198 V*
Duty cycle with 10 A = 100/198=0.505
*On pulse 0.25 *0.505=0.126 ms*
Voltage ref at const 10 A=100 V
Inductive voltage drop at current step =98 V
Current ripple=(198-100)/.004.000126=3.09 A*



Three phase system and 4QC

A symmetric three phase voltage:

$$\begin{cases} e_a = \hat{e} \cdot \cos(\omega \cdot t) \\ e_b = \hat{e} \cdot \cos\left(\omega \cdot t - \frac{2\pi}{3} \right) \\ e_c = \hat{e} \cdot \cos\left(\omega \cdot t - \frac{4\pi}{3} \right) \end{cases}$$

Show that these voltages form a rotating vector with constant length and constant speed in the complex (α,β) frame. (5 p.)

Solution Exam 2014-05-30 4a

$$\begin{cases} e_a = \hat{e}\cdot COS(\omega\cdot t) \\ e_b = \hat{e}\cdot COS(\omega\cdot t - \frac{2\pi}{3}) \\ e_c = \hat{e}\cdot COS(\omega\cdot t - \frac{4\pi}{3}) \end{cases}$$

$$\vec{e} = \sqrt{\frac{2}{3}}\cdot\left(e_a + e_b\cdot e^{j\frac{2\pi}{3}} + e_c\cdot e^{j\frac{4\pi}{3}}\right) = \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t) + \cos\left(\omega t - \frac{2\pi}{3}\right)\cdot\left(-\frac{1}{2} + j\cdot\frac{\sqrt{3}}{2}\right) + \cos\left(\omega t - \frac{4\pi}{3}\right)\cdot\left(-\frac{1}{2} - j\cdot\frac{\sqrt{3}}{2}\right)\right] =$$

$$= \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t) + \left\{\cos(\omega t)\cdot\cos\left(\frac{2\pi}{3}\right) + \sin(\omega t)\cdot\sin\left(\frac{2\pi}{3}\right)\right\}\cdot\left(-\frac{1}{2} + j\cdot\frac{\sqrt{3}}{2}\right) + \left\{\cos(\omega t)\cdot\cos\left(\frac{4\pi}{3}\right) + \sin(\omega t)\cdot\sin\left(\frac{4\pi}{3}\right)\right\}\cdot\left(-\frac{1}{2} - j\cdot\frac{\sqrt{3}}{2}\right)\right] =$$

$$= \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t) + \left\{\cos(\omega t)\cdot\left(-\frac{1}{2}\right) + \sin(\omega t)\cdot\left(\frac{\sqrt{3}}{2}\right)\right\}\cdot\left(-\frac{1}{2} + j\cdot\frac{\sqrt{3}}{2}\right) + \left\{\cos(\omega t)\cdot\left(-\frac{1}{2}\right) + \sin(\omega t)\cdot\left(-\frac{\sqrt{3}}{2}\right)\right\}\cdot\left(-\frac{1}{2} - j\cdot\frac{\sqrt{3}}{2}\right)\right] =$$

$$= \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t)\cdot\left(1 + \frac{1}{4} - j\cdot\frac{\sqrt{3}}{4} + \frac{1}{4} + j\cdot\frac{\sqrt{3}}{4}\right) + \sin(\omega t)\cdot\left(-\frac{\sqrt{3}}{4} + j\cdot\frac{3}{4} + \frac{\sqrt{3}}{4} + j\cdot\frac{3}{4}\right)\right] = \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t)\cdot\left(\frac{3}{2}\right) + \sin(\omega t)\cdot\left(j\cdot\frac{3}{2}\right)\right] =$$

$$= \hat{e}\cdot\sqrt{\frac{3}{2}}\cdot[\cos(\omega t) + j\cdot\sin(\omega t)] = \hat{e}\cdot\sqrt{\frac{3}{2}}\cdot e^{j\omega t}$$

Alternative solution

$$\vec{e} = \sqrt{\frac{2}{3}}\cdot\left(e_a + e_b\cdot e^{j\frac{2\pi}{3}} + e_c\cdot e^{j\frac{4\pi}{3}}\right) = \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t) + \cos\left(\omega t - \frac{2\pi}{3}\right)\cdot e^{j\frac{2\pi}{3}} + \cos\left(\omega t - \frac{4\pi}{3}\right)\cdot e^{j\frac{4\pi}{3}}\right] = \left\{\cos(x) = \frac{e^{jx} + e^{-jx}}{2}\right\} =$$

$$= \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\frac{e^{j\omega t} + e^{-j\omega t}}{2} + \frac{e^{j\left(\omega t - \frac{2\pi}{3}\right)} + e^{-j\left(\omega t - \frac{2\pi}{3}\right)}}{2}\cdot e^{j\frac{2\pi}{3}} + \frac{e^{j\left(\omega t - \frac{4\pi}{3}\right)} + e^{-j\left(\omega t - \frac{4\pi}{3}\right)}}{2}\cdot e^{j\frac{4\pi}{3}}\right] = \hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\frac{e^{j\omega t} + e^{-j\omega t}}{2} + \frac{e^{j\omega t - j\frac{2\pi}{3} + j\frac{2\pi}{3}} + e^{-j\omega t + j\frac{2\pi}{3} + j\frac{2\pi}{3}}}{2} + \frac{e^{j\omega t - j\frac{4\pi}{3} + j\frac{4\pi}{3}} + e^{-j\omega t + j\frac{4\pi}{3} + j\frac{4\pi}{3}}}{2}\right] =$$

$$= \hat{e}\cdot\frac{1}{2}\cdot\sqrt{\frac{2}{3}}\cdot\left[e^{j\omega t} + e^{-j\omega t} + e^{j\omega t} + e^{-j\omega t}\cdot e^{j\frac{4\pi}{3}} + e^{j\omega t} + e^{-j\omega t}\cdot e^{j\frac{8\pi}{3}}\right] = \hat{e}\cdot\frac{1}{2}\cdot\sqrt{\frac{2}{3}}\cdot\left[3\cdot e^{j\omega t} + e^{-j\omega t}\cdot\underbrace{\left(1 - \frac{1}{2} + \frac{\sqrt{3}}{2} - -\frac{1}{2} - \frac{\sqrt{3}}{2}\right)}_{=0}\right] = \hat{e}\cdot\sqrt{\frac{3}{2}}\cdot e^{j\omega t}$$

Exam 2014-05-30 4b

In a 4QC dc-dc converter using PWM bipolar voltage switching, the the bridge load consist of a constant voltage E (e.g. the back emf of a dc-motor) and an inductor L_a, the inductor resistance can be neglected. The switching frequency is f_s, and the dc-link voltage is V_d

Calculate the maximum peak-to-peak load current ripple, .expressed in V_d, L_a and f_s,. (5 p.)

Solution Exam 2014-05-30 4b

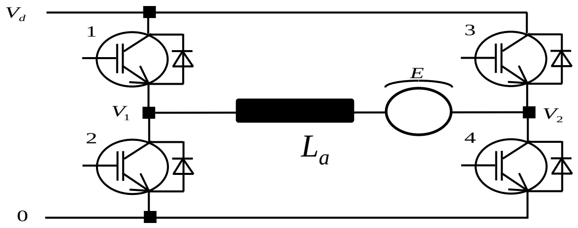


Fig 1

Control ratio x

On – pulse duration $\Delta t = x \cdot T_{s\_per} = \dfrac{x}{f_s}$

Phase voltages $V_{1\_avg} = x \cdot V_d$

$V_{2\_avg} = (1 - x) \cdot V_d = V_d - x \cdot V_d$

Voltage over motor $e = V_{1\_avg} - V_{2\_avg} = x \cdot V_d - V_d + x \cdot V_d = 2 \cdot V_d \cdot x - V_d$

At current rise, switch 1 and 4 are turned – on $V_1 = V_d$

$V_2 = 0$

Voltage over inductor $V_L = V_1 - e - V_2 = V_d - e = V_d - 2 \cdot V_d \cdot x + V_d = 2 \cdot V_d \cdot (1 - x)$

Current ripple via equation $V_L = L\dfrac{di}{dt} \Rightarrow \Delta i = \dfrac{V_L \cdot \Delta t}{L} \Rightarrow \Delta i = \dfrac{2 \cdot V_d \cdot (1 - x)}{L_a} \cdot \dfrac{x}{f_s} = \dfrac{2 \cdot V_d (x - x^2)}{f_s \cdot L_a}$

it's derivative $\dfrac{\partial(\Delta i)}{\partial x} = \dfrac{2 \cdot V_d}{f_s \cdot L_a} \cdot (1 - 2x) \Rightarrow \dfrac{\partial(\Delta i)}{\partial x} = 0 \text{ when } x = 0.5$

it's second derivative $\dfrac{\partial^2(\Delta i)}{\partial x^2} = -\dfrac{4 \cdot V_d}{f_s \cdot L_a} < 0 \Rightarrow \max \text{ at } x = 0.5$

Phase voltages at max $V_{1\_avg} = 0.5 \cdot V_d = 0.5 \cdot V_d$

$V_{2\_avg} = (1 - 0.5) \cdot V_d = 0.5 \cdot V_d$

$e = V_{1\_avg} - V_{2\_avg} = 0.5 \cdot (V_d - V_d) = 0 = \dfrac{0}{V_d}$

Max current ripple $\Delta i_{\max} = \dfrac{2 \cdot V_d \cdot (1 - 0.5)}{L_a} \cdot \dfrac{0.5}{f_s} = \dfrac{V_d}{2 f_s L_a}$

Permanent magnetized motor

A 50 kW motor drive is to be designed. The motor will run from a Power Electronic Converter with 500 V DC link. The bases speed must be 4000 rpm and the maximum speed 12000 rpm. The motor has 18 poles.

a) What is the highest output voltage ($U_{phase\text{-}to\text{-}phase\_rms}$) that you would use? (2p)
b) What will be the phase current in this case? (2p)
c) What is the lowest sampling frequency that the controller must use to run this motor? (2p)
d) What is the lowest switching frequency that the modulation can use? (2p)
e) What will be the motor torque at base speed and maximum speed? (2p)

All your answers must be accompanied with your calculations and motivations!

Solution Exam 20140530 5

$$5a) \quad Phase-to-phase\,voltage\;\; rms \;\; U_{rms\_line\_to\_line} = \frac{500}{\sqrt{2}} = 354V$$

$$5b) \quad 50000 = \{assume\ \cos\varphi = 0.9\} = \sqrt{3}\cdot 354\cdot I\cdot 0.9 \Rightarrow I = \frac{50000}{\sqrt{3}\cdot 354\cdot 0.9} = 90.7\,A$$

$$5c) \quad Mech\;\; freq = \frac{12000}{60} = 200\,Hz$$

$$Elec.freq = \{18\ poles, 9\ polepairs\} = 9\cdot 200\,Hz = 1800\,Hz$$

$$Sampling\;\; freq, see\, solution\, 5d.$$

$$5d) \quad At\, least\, switching\;\; freq = 6\cdot Elec.freq = 10800\,Hz$$

$$Two\;\; sample\, per\, switch.\, freq\;\, period \Rightarrow sampling\;\; freq = 21600\,Hz$$

$$5e) \quad Torque_{\,4000rpm} = \{P = T\cdot\omega\} = \frac{P}{\omega} = \frac{50000}{2\pi\cdot\frac{4000}{60}} = 119\,Nm$$

$$Torque_{\,12000rpm} = \frac{P}{\omega} = \frac{50000}{2\pi\cdot\frac{12000}{60}} = 39.8\,Nm$$

Exam 2017-05-30

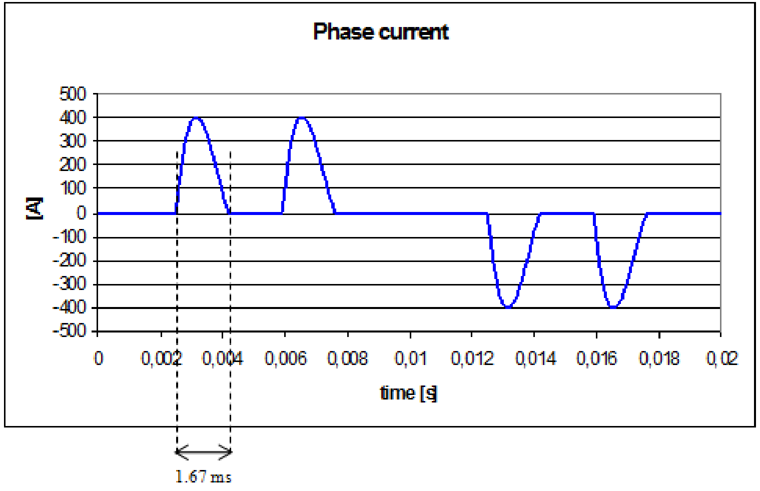
Exam 2017-05-30, 1a-c

The DC-DC buck converter

a) Draw a 1QC buck converter connected to the dc side of a three phase diode rectifier, which is connected to the power grid. The dclink capacitor and protection against too high inrush currents should be included in the drawing. The transistor is of IGBT-type. (1 p.)

b) The three-phase grid, to which the three phase diode rectifier is connected, has the line-to-line voltage 400 V_{rms} and the frequency 50 Hz. Calculate the dc output voltage and the maximum dc link voltage from the rectifier. (1 p.)

c) Calculate the rms-current and the average current through one rectifying diode (see figure 1). Calculate the rectifier diode losses. The diode threshold voltage is 0,9 V and the differential resistance is 2.0 mohm. (2 p.)



Exam 2017-05-30, 1d

The DC-DC buck converter

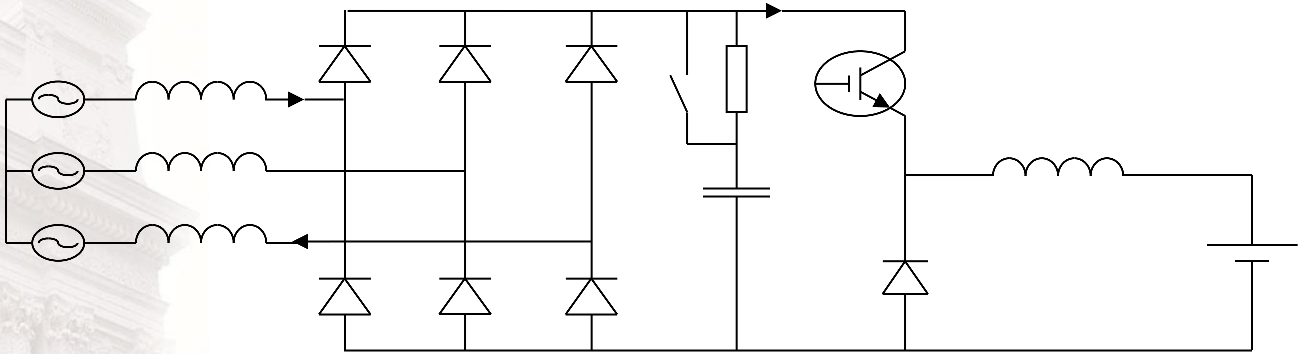
d) Calculate the losses of the IGBT transistor and of the free wheeling diode in the buck converter. The buck converter phase inductor is 1 mH, and its resistance can be neglected.
Draw a time diagram with the buck converter phase current versus time during one period of the switching frequency.
The load on the low voltage side of the buck converter is a battery with the voltage 400 V_{dc}.
The switching frequency is 2 kHz.
The threshold voltage of the IGBT transistor equals 1.1 V and its differential resistance equals 1.0 mohm. The turn-on loss of the IGBT transistor equals 60 mJ and its turn-off loss equals 80 mJ. These turn-on and turn-off losses are nominal values at 900 V dclink voltage and 180 A turn-on and turn-off current.
The threshold voltage of the free wheeling diode equals 1.3 V and the differential resistance of this diode equals 2 mohm. The free wheeling diode turn-on losses can be neglected and its turn-off losses equals 25 mJ, at 900 V dclink voltage and 180 A turn off current. (4 p.)

Exam 2017-05-30, 1e

The DC-DC buck converter

e) Which is the junction temperature of the IGBT transistor and of the free wheeling diode, and which is the junction temperature of the rectifying diodes?
The thermal resistance of the heatsink equals 0.065 K/W?
The thermal resistance of the IGBT transistor equals 0.078 K/W?
The thermal resistance of the free wheeling diode equals 0.19 K/W?
The thermal resistance of the rectifier diode equals 0.21 K/W?
The ambient temperature is 35 ^{o}C.
The rectifier diodes and the buck converter transistor and diode share the heatsink. (2 p.)

Solution Exam 2017-05-30 1a



Solution Exam 2017-05-30 1b

Maximum dc voltage $$U_{dc\max} = 400 \cdot \sqrt{2}\ V = 566\ V$$

Average dc voltage $$U_{dc\_ave} = \frac{3}{\pi} \cdot 400 \cdot \sqrt{2}\ V = 540\ V$$

Solution Exam 2017-05-30 1c

Data
Rectifier diode Threshold voltage 0.9 V
Differential resistance 2.0 mohm

$$I_{diode\,rms\_onehalf\sin us} = \left(\frac{400}{\sqrt{2}}\right) A = 282.8\,A$$

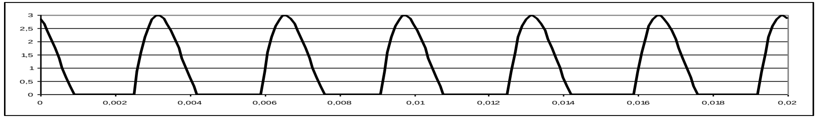
$$I_{diode\,rms} = \sqrt{\frac{2 \cdot 0.00163}{0.02} \cdot 282.8^2} = 114.2A$$

$$I_{diode\,ave} = \left\{ Average of \sin us = \frac{\int_0^{\pi} \sin(x)dx}{\pi} = \frac{(\cos(0) - \cos(\pi))}{\pi} = \frac{2}{\pi} \approx 0.637 \right\} = \frac{2 \cdot 0.00163}{0.02} \cdot 0.637 \cdot 400 = 41.5A$$

Rectifier diode power loss

$$P_{rectifier\ diode} = V_{threshold} \cdot I_{ave} + R_{diff} \cdot I_{rms}^2 = 0.9 \cdot 41.5 + 0.002 \cdot 114.2^2 = 63.4\ W$$

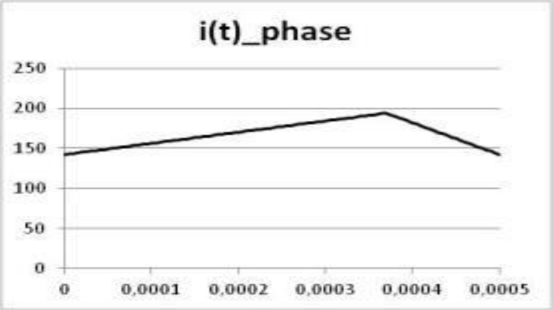
Solution Exam 2017-05-30 1d_1

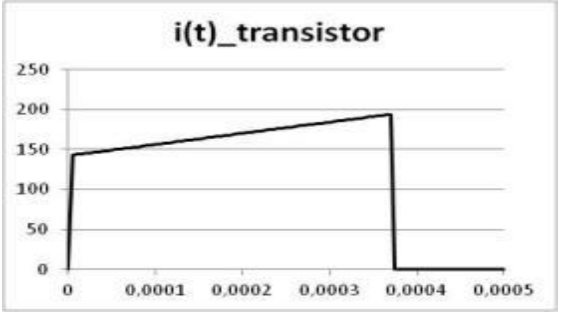


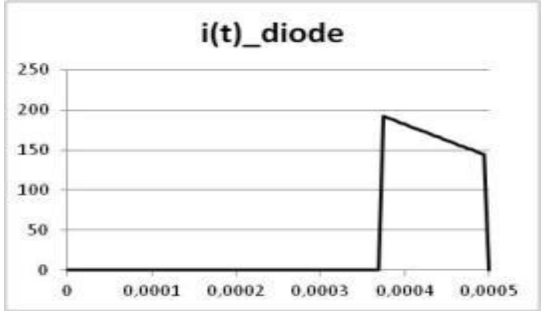
$$I_{dc} = \frac{6 \cdot 0.00163 \cdot 400}{0.02} \cdot 0.637 = 124.6A$$

Thc current to the dc link

Duty cycle	400/540=74%
Average transistor current	124.6/0.74=168.2 A
Switching frequency period time	1/2000=0.0005 s
Duration of transistor on	0.74/0.0005=0.00037 s
Current ripple, equ U=L*di/dt , Δi=U*Δt/L=	Δi=(540-400)*0.00037/0.001=51.8 A
Low voltage side max phase current	Imax=168.2+51.8/2=194.1 A
Low voltage side max phase current	Imin=168.2-51.8/2=142.3A

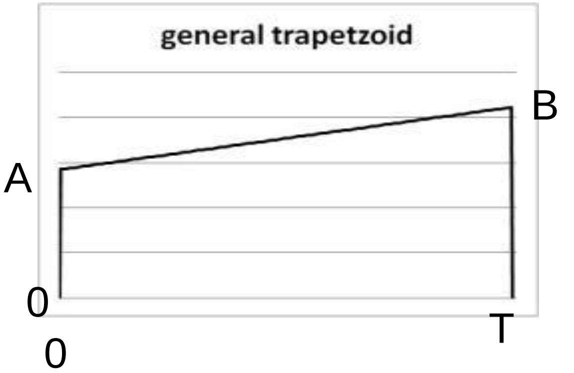






Solution Exam 2017-05-30 1d_2

Find a general expression for RMS from a time domain trapezoid shaped current



$$Equation\ \ for\ the\ \ straight\ line\ \ i(t) = \frac{(B-A)}{T}\cdot t + A$$

$$I_{rms} = \sqrt{\frac{\int_0^T\left(\frac{(B-A)}{T}\cdot t + A\right)^2 dt}{T}} = \sqrt{\left(\frac{\left(B^2 + A^2 - 2AB\right)\cdot T^3}{3T^2\cdot T} + \frac{A^2T}{T} + 2\cdot A\cdot\frac{(B-A)\cdot t^2}{2T\cdot T}\right)} =$$

$$= \sqrt{\left(\frac{B^2 + A^2 - 2AB + 3A^2 + 3AB - 3A^2}{3}\right)} = \sqrt{\left(\frac{A^2 + B^2 + AB}{3}\right)}$$

$$\begin{cases} I_{\min} = 142.3\,A \\ I_{\max} = 194.1\,A \end{cases}$$

$$I_{rms\_transistor} = \sqrt{\left(\frac{142.3^2 + 194.1^2 + 142.3\cdot 194.1}{3}\right)\cdot 0.74} = 145.3\,A \qquad I_{avg\_transistor} = \left(\frac{142.3 + 194.1}{2}\right)\cdot 0.74 = 124.5\,A$$

$$I_{rms\_diode} = \sqrt{\left(\frac{142.3^2 + 194.1^2 + 142.3\cdot 194.1}{3}\right)\cdot 0.26} = 86.1\,A \qquad I_{avg\_diode} = \left(\frac{142.3 + 194.1}{2}\right)\cdot 0.26 = 43.7\,A$$

	threshold volrage[V]	Rdiff[mohm]	Turn-on[mJ]	Turn off[mJ]	Switch losses at voltage[V]	and at current[A]
Transistor	1.5	1.0	60	80	900	180
Diode	1.0	2.0	0	25	900	180

$$\begin{cases} P_{trans\_loss} = 1.5\cdot 124.5 + 0{,}001\cdot 145.3^2 + 2000\cdot\frac{(0.060\cdot 142.3 + 0.080\cdot 194.1)\cdot 540}{900\cdot 180} = 368\,W \\ P_{diode\_loss} = 1.0\cdot 43.7 + 0{,}002\cdot 86.1^2 + 2000\cdot\frac{0.025\cdot 142.3\cdot 540}{900\cdot 180} = 82.2\,W \end{cases}$$

Solution Exam 2017-05-30 1e

Rectifier diode (6)	IGBT diode	IGBT transistor
Loss each 63.4W	Loss each 82.2 W	Loss each 368 W
Rth diode 0.25 C/W	Rth diode 0.4 C/W	Rth trans 0.2C/W
Temp diff 15.8 ^{o}C	Temp diff 32.9 ^{o}C	Temp diff 73.6 ^{o}C

Heatsink

Contribution fron 6 rectifier diodes and from one IGBT and one diode.

Ambient temperature	35 ^{o}C
Total loss to heatsink	6*63.4+368+82.2=831 W
Rth heatsink	0.07 C/W
Temperature heatsink	831 *0.07+35=93 ^{o}C

Junction temperature

Rectifier diode	93 +15.8 = 109 ^{o}C
IGBT diode	93 +32.9 = 126 ^{o}C
IGBT transistor	93 +73.6 = 167 ^{o}C

Exam 2017-05-30, 2

Snubbers and semiconductor

a) Draw an IGBTequipped step down chopper (buck converter) with an RCD snubber. Give a detailed description of how the RCD charge-discharge snubber operates at turn on and at turn-off. Explain why the snubbers are needed (2 p.)

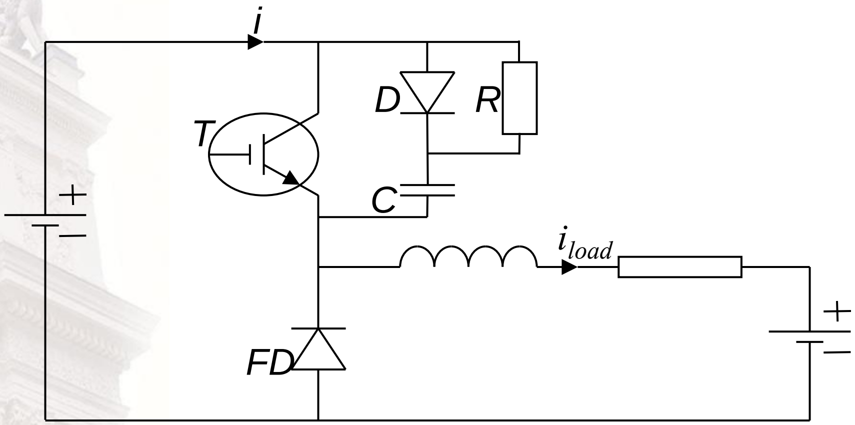
b) The DC link voltage on the supply side is 250V and the load voltage is 200 V.
Calculate the snubber capacitor for the commutation time 0.015ms.
The load current is 17 A, assumed constant during the commutation.
Calculate the snubber resistor so the discharge time (3 time constants) of the snubber capacitor is less than the IGBT on state time.
The switch frequency is 2 kHz (3 p.)

c) Draw a figure with the diffusion layers in a (n-channel) MOSFET (2 p.)

d) Where in the (n-channel) MOSFET diffusion layers structure can an unwanted NPN-transistor be found, and where can the anti-parallel diode be found? (2 p.)
What in the MOSFET layout reduces the risk that this unwanted transistor is turned on?

e) Which layer is always present in a power semiconductor? How is it doped? (1 p.)

Solution Exam 2017-05-30, 2a



The buck converter with RCD snubber
<u>At turn off</u> of transistor T, the current i commtutates over to the capacitor C via diode D. The capacitor C charges until the potential of the transistor emitter reduces till the diode FD becomes forward biased and thereafter the load current i_{load} flows through diode FD and the current i=0.
<u>A turn on</u> of the transistor T, the capacitor C is discharged via the transistor T and resistor R. The diode FD becomes reverse biased and the current i commutates to the transistor T.

Exam 2017-05-30 2b, 1

The DC link voltage on the supply side is 250V and the load voltage is 200 V.

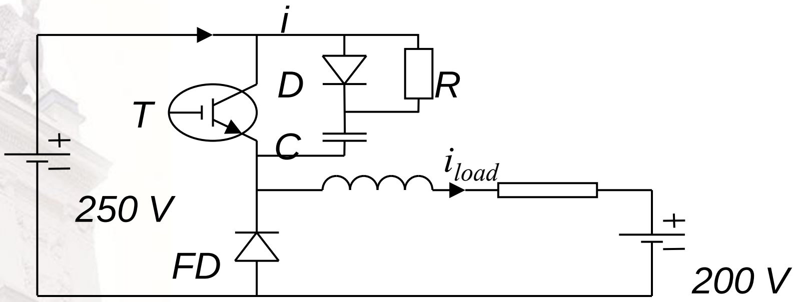
Calculate the snubber capacitor for the commutation time 0.015ms.
The load current is 17 A, assumed constant during the commutation.

Calculate the snubber resistor so the discharge time (3 time constants) of the snubber capacitor is less than the IGBT on state time.

The switching frequency is 2 kHz

(3 p.)

Solution Exam 2017-05-30 2b, 2



Load current	17 A
Supply voltage	250 V
Load voltage	200 V
Commutation time	0.015 ms
Switching frequency	2 kHz

At turn off of transistor T, the capacitor C charges until the potential of the transistor emitter reduces till the diode FD becomes forward biased and thereafter the load current commutates to the freewheeling diode.

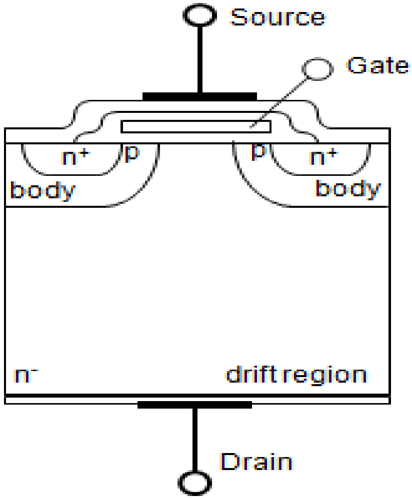
$$i = C \cdot \frac{du}{dt} \quad \Rightarrow \quad C = \frac{i \cdot dt}{du} = \frac{17 \cdot 15 \cdot 10^{-6}}{250} = 1.0\,\mu F$$

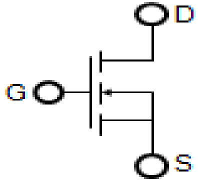
A turn on of the transistor the current i commutates to the transistor T, and the capacitor C is discharged via the the transistor T and resistor R. As the load voltage is 200V the duty cycle is 80%. The switching frequency is 2 kHz and the on state time is 0.5*0.8=0.4 ms, and thus the time constant =0.12 ms

$$\tau = C \cdot R \quad \Rightarrow \quad R = \frac{\tau}{C} = \frac{120 \cdot 10^{-6}}{1 \cdot 10^{-6}} = 120\,\Omega$$

Solution Exam 20170530 2c

The MOSFET

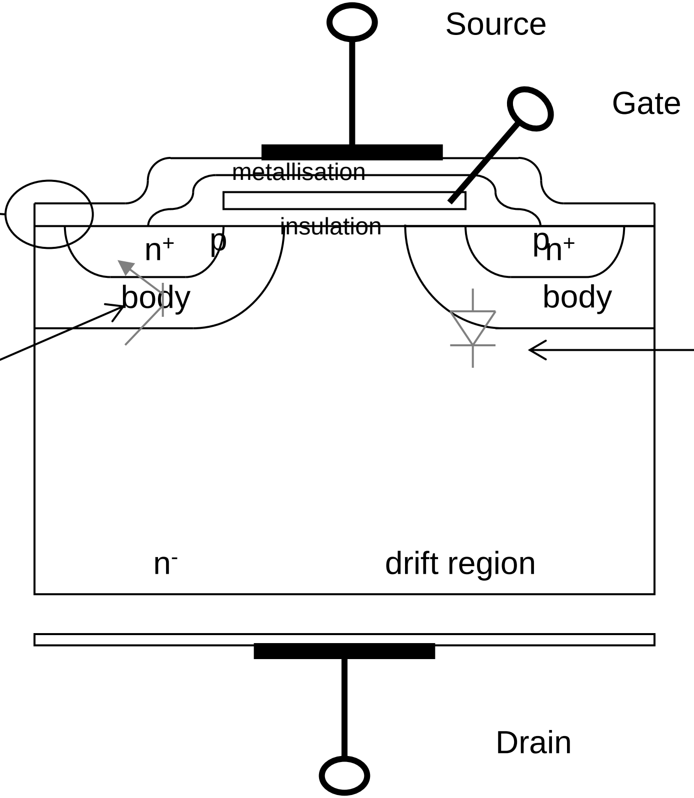




Solution Exam 20170530 2d

The metallisation short circuits the emitter and the base of the unwanted transistor to reduce the risk for its turning on

The transistor



The diode

Solution Exam 20170530 2e

Depletion region n^-

- *The depletion region, is an insulating region within a conductive, doped semiconductor material where the mobile charge carriers have been diffused away, or have been forced away by an electric field.*
- *The only elements left in the depletion region are ionized donor or acceptor impurities.*
- *The depletion region is so named because it is formed from a conducting region by removal of all free charge carriers, leaving none to carry a current.*

Three phase system

a) A symmetric three phase voltage:

$$\begin{cases} e_a = \hat{e} \cdot \cos(\omega \cdot t) \\ e_b = \hat{e} \cdot \cos\left(\omega \cdot t - \frac{2\pi}{3}\right) \\ e_c = \hat{e} \cdot \cos\left(\omega \cdot t - \frac{4\pi}{3}\right) \end{cases}$$

b) Show that these voltages form a rotating vector with constant length and constant speed in the complex (α,β) frame. (5 p.)

c) Draw the circuit of a current control block for a generic three phase RLE load. The drawing shall include three phase converter, reference and load current measurement. It must be clear in which blocks the different frame transformations occur. (5 p.)

Solution Exam 2017-05-30 3a

$$\begin{cases} e_a = \hat{e}\cdot COS(\omega\cdot t) \\ e_b = \hat{e}\cdot COS(\omega\cdot t-\frac{2\pi}{3}) \\ e_c = \hat{e}\cdot COS(\omega\cdot t-\frac{4\pi}{3}) \end{cases}$$

$$\vec{e}=\sqrt{\frac{2}{3}}\cdot\left(e_a+e_b\cdot e^{j\frac{2\pi}{3}}+e_c\cdot e^{j\frac{4\pi}{3}}\right)=\hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t)+\cos\left(\omega t-\frac{2\pi}{3}\right)\cdot\left(-\frac{1}{2}+j\cdot\frac{\sqrt{3}}{2}\right)+\cos\left(\omega t-\frac{4\pi}{3}\right)\cdot\left(-\frac{1}{2}-j\cdot\frac{\sqrt{3}}{2}\right)\right]=$$

$$=\hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t)+\left\{\cos(\omega t)\cdot\cos\left(\frac{2\pi}{3}\right)+\sin(\omega t)\cdot\sin\left(\frac{2\pi}{3}\right)\right\}\cdot\left(-\frac{1}{2}+j\cdot\frac{\sqrt{3}}{2}\right)+\left\{\cos(\omega t)\cdot\cos\left(\frac{4\pi}{3}\right)+\sin(\omega t)\cdot\sin\left(\frac{4\pi}{3}\right)\right\}\cdot\left(-\frac{1}{2}-j\cdot\frac{\sqrt{3}}{2}\right)\right]=$$

$$=\hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t)+\left\{\cos(\omega t)\cdot\left(-\frac{1}{2}\right)+\sin(\omega t)\cdot\left(\frac{\sqrt{3}}{2}\right)\right\}\cdot\left(-\frac{1}{2}+j\cdot\frac{\sqrt{3}}{2}\right)+\left\{\cos(\omega t)\cdot\left(-\frac{1}{2}\right)+\sin(\omega t)\cdot\left(-\frac{\sqrt{3}}{2}\right)\right\}\cdot\left(-\frac{1}{2}-j\cdot\frac{\sqrt{3}}{2}\right)\right]=$$

$$=\hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t)\cdot\left(1+\frac{1}{4}-j\cdot\frac{\sqrt{3}}{4}+\frac{1}{4}+j\cdot\frac{\sqrt{3}}{4}\right)+\sin(\omega t)\cdot\left(-\frac{\sqrt{3}}{4}+j\cdot\frac{3}{4}+\frac{\sqrt{3}}{4}+j\cdot\frac{3}{4}\right)\right]=\hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t)\cdot\left(\frac{3}{2}\right)+\sin(\omega t)\cdot\left(j\cdot\frac{3}{2}\right)\right]=$$

$$=\hat{e}\cdot\sqrt{\frac{3}{2}}\cdot[\cos(\omega t)+j\cdot\sin(\omega t)]=\hat{e}\cdot\sqrt{\frac{3}{2}}\cdot e^{j\omega t}$$

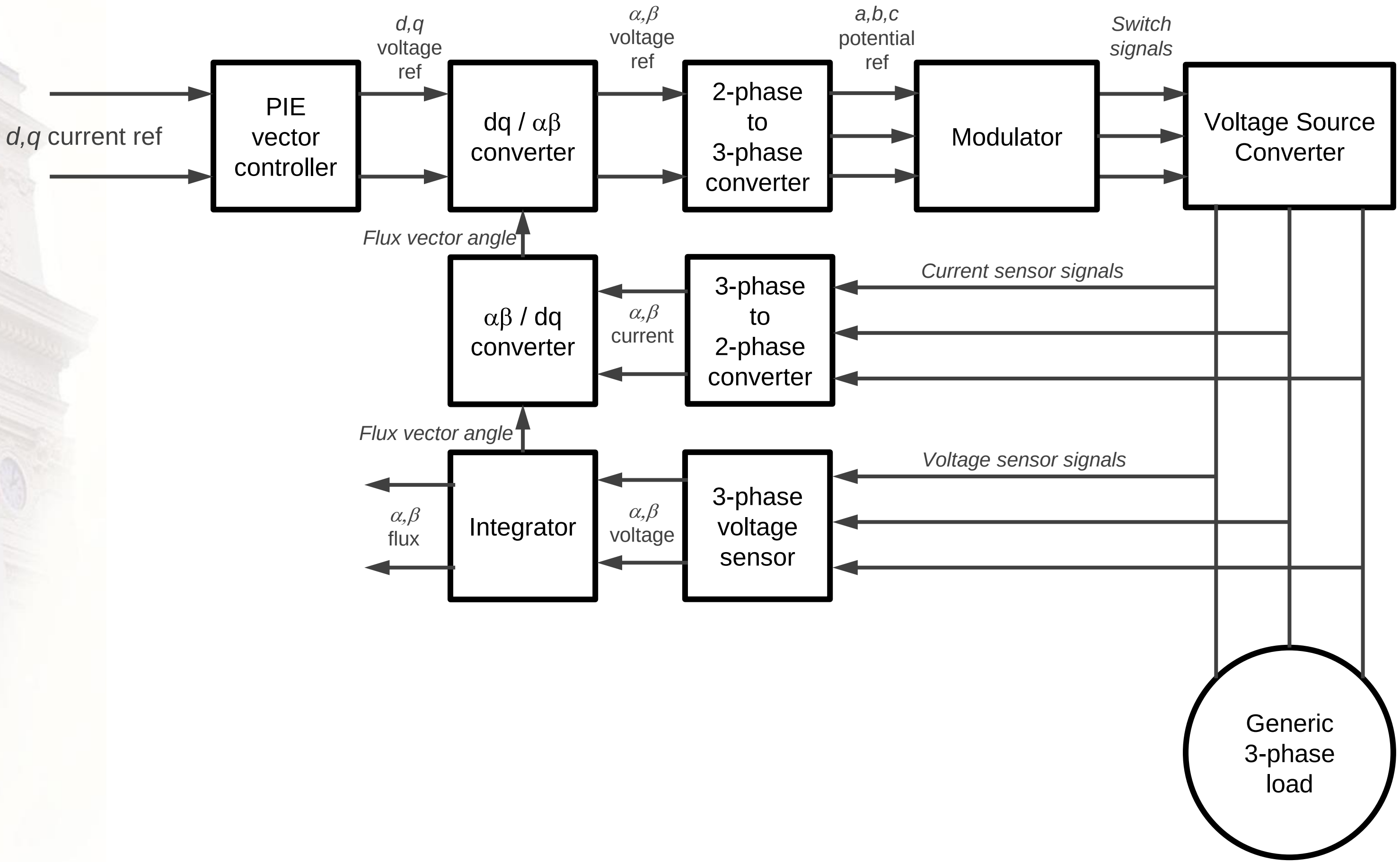
Alternative solution

$$\vec{e}=\sqrt{\frac{2}{3}}\cdot\left(e_a+e_b\cdot e^{j\frac{2\pi}{3}}+e_c\cdot e^{j\frac{4\pi}{3}}\right)=\hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\cos(\omega t)+\cos\left(\omega t-\frac{2\pi}{3}\right)\cdot e^{j\frac{2\pi}{3}}+\cos\left(\omega t-\frac{4\pi}{3}\right)\cdot e^{j\frac{4\pi}{3}}\right]=\left\{\cos(x)=\frac{e^{jx}+e^{-jx}}{2}\right\}=$$

$$=\hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\frac{e^{j\omega t}+e^{-j\omega t}}{2}+\frac{e^{j\left(\omega t-\frac{2\pi}{3}\right)}+e^{-j\left(\omega t-\frac{2\pi}{3}\right)}}{2}\cdot e^{j\frac{2\pi}{3}}+\frac{e^{j\left(\omega t-\frac{4\pi}{3}\right)}+e^{-j\left(\omega t-\frac{4\pi}{3}\right)}}{2}\cdot e^{j\frac{4\pi}{3}}\right]=\hat{e}\cdot\sqrt{\frac{2}{3}}\cdot\left[\frac{e^{j\omega t}+e^{-j\omega t}}{2}+\frac{e^{j\omega t-j\frac{2\pi}{3}+j\frac{2\pi}{3}}+e^{-j\omega t+j\frac{2\pi}{3}+j\frac{2\pi}{3}}}{2}+\frac{e^{j\omega t-j\frac{4\pi}{3}+j\frac{4\pi}{3}}+e^{-j\omega t+j\frac{4\pi}{3}+j\frac{4\pi}{3}}}{2}\right]=$$

$$=\hat{e}\cdot\frac{1}{2}\cdot\sqrt{\frac{2}{3}}\cdot\left[e^{j\omega t}+e^{-j\omega t}+e^{j\omega t}+e^{-j\omega t}\cdot e^{j\frac{4\pi}{3}}+e^{j\omega t}+e^{-j\omega t}\cdot e^{j\frac{8\pi}{3}}\right]=\hat{e}\cdot\frac{1}{2}\cdot\sqrt{\frac{2}{3}}\cdot\left[3\cdot e^{j\omega t}+e^{-j\omega t}\cdot\underbrace{\left(1-\frac{1}{2}+\frac{\sqrt{3}}{2}--\frac{1}{2}-\frac{\sqrt{3}}{2}\right)}_{=0}\right]=\hat{e}\cdot\sqrt{\frac{3}{2}}\cdot e^{j\omega t}$$

Solution Exam 20170530 3b



Exam 2017-05-30 4

The buck converter as battery charger

a) A DC/DC Converter has a DC link voltage of 100 V and can be either a 2Q or a 4Q converter supplying a load consisting of a 625 mH inductance in series with a 20 V back emf. The converter is carrier wave modulated with a 4 kHz modulation frequency and equipped with a current controller. A current step from 0 to 12 A is made and then back to 0 A again after 4 modulation periods.

b) Calculate the voltage reference for a few modulation periods before the positive step, for the positive step, for the time in between the steps, for the negative step and for a few modulation periods after the negative step in the 2Q case. (3p)

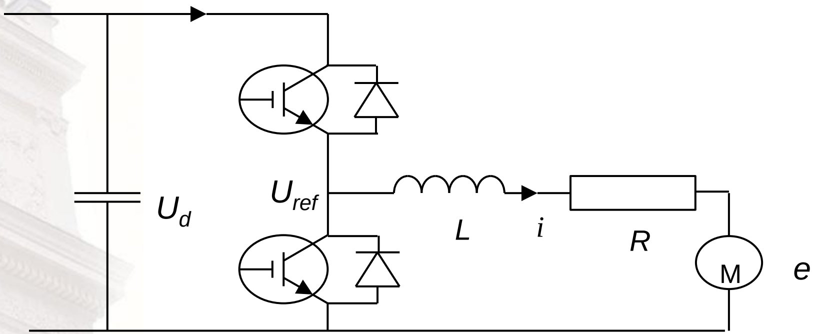
c) Draw the current to the load in the 2Q case, from two modulation periods before the positive current step to two modulation periods after the negative step. (2p)

d) Calculate the voltage reference for a few modulation periods before the positive step, for the positive step, for the time in between the steps, for the negative step and for a few modulation periods after the negative step in the 4Q case. (3p)

e) Draw the current to the load in the 4Q case, from two modulation periods before the positive current step to two modulation periods after the negative step. (2p)

f) In both b) and d) the current ripple must be correctly calculated.

Solution Exam 2017-05-30 4a



Data: f_{sw} = 4 kHz.
L = 0.625 Mh
R=0
U_d = 100 V
e = 20 V

Equation $$U = L \cdot \frac{di}{dt} + e$$

Before the pos. current step i=0 A, constant, di/dt=0	U_{ref}=e=20 V
At the positive current step, use max voltage	U_{ref}=U_d=100 V
At the constant current 12 A, di/dt=0, and R=0	U_{ref}=e=20 V
At the negative current step, use zero voltage	U_{ref}=0 V
After the neg. Current step i=0 A, constant, di/dt=0	U_{ref}=e=20 V

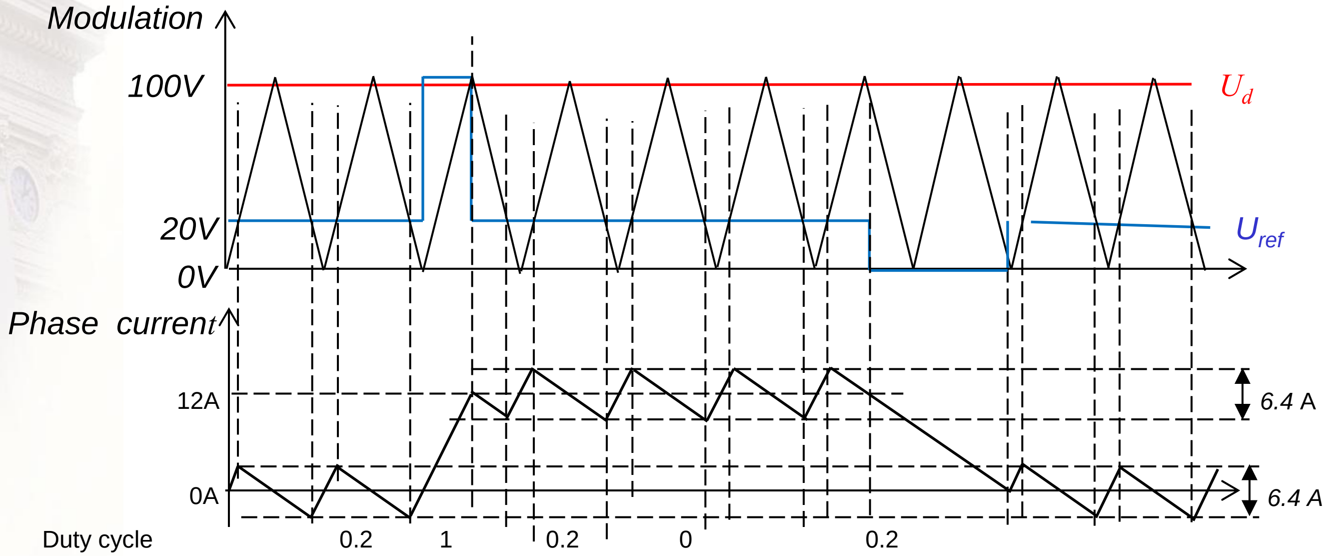
Solution Exam 2017-05-30 4b

$$Duty\ cycle \quad D = \frac{20}{100} = 0.2$$

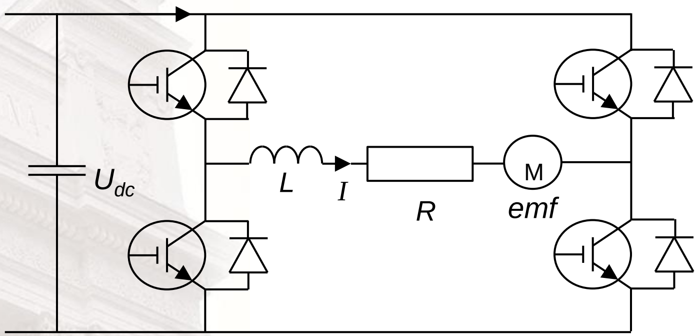
$$U = L \cdot \frac{di}{dt} + e \approx L \cdot \frac{\Delta i}{\Delta t} + e \Rightarrow \Delta i = \frac{(U - e)}{L} \cdot \Delta t$$

See figure, time for current rise = Duty cycle times the switching frequency period

$$Current\ ripple \quad \Delta i = \frac{(U-e)}{L} \cdot \Delta t = \left\{ \Delta t = \frac{1}{f_{sw}} \cdot D \right\} = \frac{(100-20)}{0.625 \cdot 10^{-3}} \cdot \frac{D}{4 \cdot 10^{3}} = 6.4\,A$$



Solution Exam 2017-05-30 4c



Data: f_{sw} = 4 kHz.
L = 0.625 Mh
R=0
U_{dc} = 100 V
e = 20 V

Equation $$U = L \cdot \frac{di}{dt} + e$$

Before the pos. current step i=0 A, constant, di/dt=0 — U_{ref}=e=20 V
At the positive current step, use max voltage — U_{ref}=U_{dc}=100 V
At the constant current 12 A, di/dt=0, and R=0 — U_{ref}=e=20 V
At the negative current step, use minimum voltage — U_{ref}=-100 V
After the neg. current step i=0 A, constant, di/dt=0 — U_{ref}=e=20 V

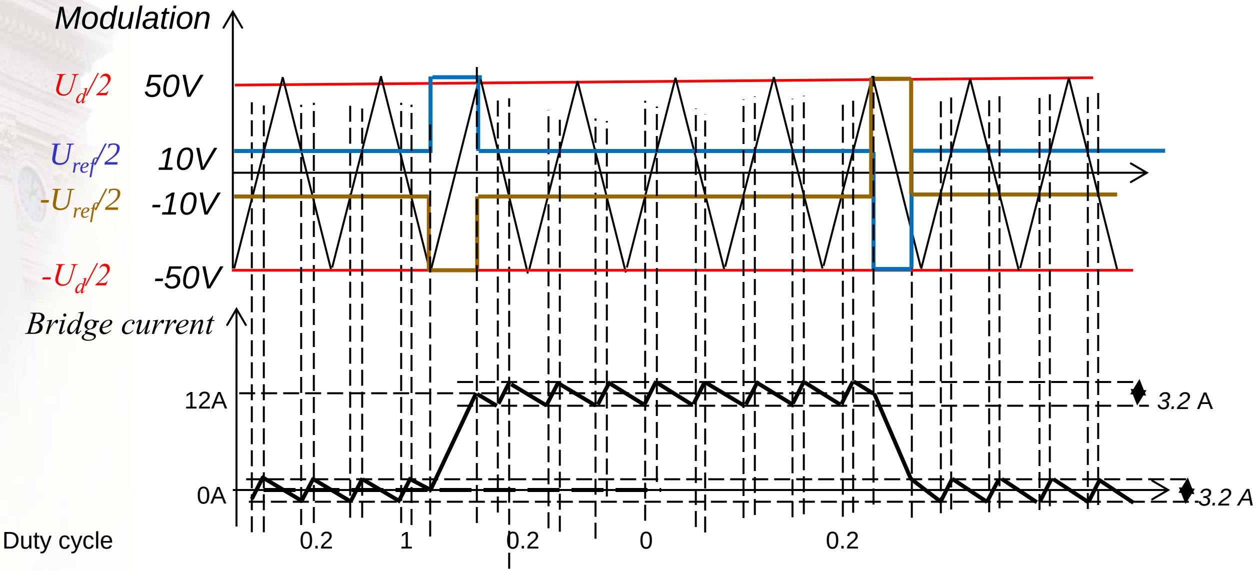
Solution Exam 2017-05-30 4d

$$Duty\ cycle \quad D=\frac{20}{100}=0.2$$

$$U=L\cdot\frac{di}{dt}+e\approx L\cdot\frac{\Delta i}{\Delta t}+e \Rightarrow \Delta i=\frac{(U-e)}{L}\cdot\Delta t$$

*See figure. Time for current rise = Duty cycle * half the switching frequency period*

$$Current\ ripple\ \Delta i=\frac{(U-e)}{L}\cdot\Delta t=\left\{\Delta t=\frac{0.5\cdot D}{f_{sw}}\right\}=\frac{(100-20)}{0.625\cdot 10^{-3}}\cdot\frac{0.1}{4\cdot 10^{3}}=3.2\,A$$



Permanently magnetized synchronous machine

A 120 kW motor drive is to be designed. The motor will run from a Power Electronic Converter with 800 V DC link. The bases speed must be 5000 rpm and the maximum speed 12000 rpm.

a) What is the rated torque of the machine (2p)
b) What is the RMS Phase-to-phase voltage at rated power? (2p)
c) What is the rated phase current of the machine? (2p)

All your answers must be accompanied with your calculations and motivations!

Exam 2017-05-30 5_2

Permanent magnetized motor

d) The electric frequency of the machine at base speed is 250 Hz.
What is the lowest sampling frequency that you would choose to control the machine? (2p)

e) What is a suitable switching frequency for the converter? (2p)

All your answers must be accompanied with your calculations and motivations!

Solution 20170530 5

$$5a)\quad Torque\,of\ the\,machine\ T=\frac{P}{\omega}=\frac{120000}{\frac{5000}{60}\cdot 2\pi}=229\,Nm$$

$$5b)\quad Phase-to-phase\,voltage\ \ rms\ \ U_{LL\_rms}=\frac{800}{\sqrt{2}}=566V$$

$$5c)\quad 120000=\{assume\ \cos\varphi=0.95\}=\sqrt{3}\cdot 566\cdot I\cdot 0.95\Rightarrow I=\frac{120000}{\sqrt{3}\cdot 566\cdot 0.95}=129\,A$$

$$5d)\quad The\,base\,speed\ is\ where\,the\,top\ power\ is\ achieved.\,At\ this\ speed\ the\,electric\ \ frequency$$

$$of\ the\,motor\,equals\,250\,Hz,\,and\ its\,the\,mechanical\ \ frequency=\frac{5000}{60}=83.33\,Hz$$

$$The\,relation\ between\ the\,electric\ and\ the\,mechanical\ \ frequency=\frac{250}{83.33}\,a=3$$

$$gives\,the\,result\ that\ the\,motor\ has\ 6-poles.$$

$$At\ the\,top\ motor\ speed\ 12000\,rpm\,the\,electric\ \ frequency=3\cdot\frac{12000}{60}=600\,Hz.$$

$$Switching\ \ freq=\{at\ least\ one\ switching\ \ period\ \ per\ hexagon\ side\}=6\cdot 600=3600\,Hz$$

$$Sampling\ \ freq=\{2\,samples\ per\ switching\ \ frequency\ \ period\}=2\cdot 3600\,Hz=7200\,Hz$$

$$5e)\quad See\,5d.\,Switching\ \ freq=3600\,Hz$$

Exam 2017-05-30

Formulas:

$$\vec{s} = K \cdot \left[s_a + s_b \cdot e^{j \cdot \frac{2 \cdot \pi}{3}} + s_c \cdot e^{j \cdot \frac{4 \cdot \pi}{3}} \right] = K \cdot \left[\frac{3}{2} \cdot s_a + j \cdot \frac{\sqrt{3}}{2} (s_b - s_c) \right] = s_\alpha + j \cdot s_\beta$$

Power invariant

Three phase –> two phase conversion

$$s_\alpha = \sqrt{\frac{3}{2}} \cdot s_a$$

$$s_\beta = \frac{1}{\sqrt{2}} \cdot (s_b - s_c)$$

Power invariant

Two phase –> three phase conversion

$$s_a = \sqrt{\frac{2}{3}} \cdot s_\alpha$$

$$s_b = -\frac{1}{\sqrt{6}} s_\alpha + \frac{1}{\sqrt{2}} \cdot s_\beta$$

$$s_c = -\frac{1}{\sqrt{6}} s_\alpha - \frac{1}{\sqrt{2}} \cdot s_\beta$$